Journal of the Nigerian Association of Mathematical Physics
***Volume* 14 (May, 2009), pp 77 – 80**
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The effect of non-linear wave in front of vertical wall using bi-parametric approach

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Abstract

The modification of free surface displacement and fluctuating wave pressure of waves in front of a vertical wall are examined, using the new theoretical approach of a bi-parametric distribution, proposed by Ejinkonye* [1] *to investigate the effect of non-linearity for the mechanics of the sea waves. The most probable value of the wave steepness is assumed to be $\varepsilon = 0.055$. From the subsequent calculation carried out, it was found that on deep water the parameter α_2 tends to zero and α_1 tends to ε, which is twice as much as the value of α for the progressive waves on deep water. Moreover, for a fixed kd, this theory suggests that the non-linear effects increase while approaching the bottom, which is valid for the mechanics of the sea waves.

Keywords
Free surface displacement, fluctuating wave pressure, narrow spectrum, vertical wall.

1.0 Introduction

The problem of water waves whose passage is obstructed by an object had been intensively studied. In this consideration, the equation of Morison (1950) [2] is of interest. This author is able to decompose the wave force on the object (pile) into two. That is, those due to drag and inertia. The general application was essentially on the cylindrical off shore structure.

However, Okeke and Asor [3] developed a theory for on-shore wave field. The theory was used to calculate effectively the amplitude spectrum of an incident wave in shallow water.

There are other theorems that proved very effective in calculating wave forces on structure. The most widely used among them is that of Rahman and Frocide-Krylov [4]. As for the non-linearity effects, it is emphasized that the probability of exceedance of the absolute minimum of the fluctuating wave pressure beneath the sea surface usually is markedly greater than the probability of exceedance of the absolute maximum, especially if the waves are subject to reflection Arena and Fedele, [5].

However, the approach in this study is purely statistical. In this consideration, we consider the statistical properties of the spectrum of the free-surface profile and fluctuating wave bottom pressure for high-wave in front of a vertical wall.

From this, subsequent calculations are carried out on the related free surface displacement and fluctuating wave pressure, both for waves in front of a vertical wall. The expressions of the parameters α_1 and α_2, enable us to easily predict the effects of non-linearity,

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which is valid for most of the second order processes in the mechanics of the sea waves, except for concerned the special case which relates to the fluctuating wave pressure in front of a vertical wall near the seabed.

1.1 Specification

The reference from (*x*,*y*) has the *x*-axis horizontal and the *y*-axis vertical, with origin on the mean water level. The bottom depth is d and the wave steepness ε (ε = *ka*, where *k* is the wave number and α is the wave height).

In the case of the wind generated surface waves, in front of vertical wall which typically lies between 0.05 and 0.08 a most generally acceptable characteristic value is ε = 0.055.

2.0 The free surface displacement in front of a vertical wall

Here, we consider the profile of wave field moving towards in front of a vertical wall, located at the abscissa *x* = 0. The free surface displacement to the second-order, for an infinitely narrow spectrum, is given by Arena and Fedele [5].

$$\eta(x,t) = 2a\cos(kx)\cos(X) + 2ka^2 f_{\eta 1}\cos(2kx)\cos(2X) + 2ka^2 f_{\eta}\cos(2kx) \qquad (2.1)$$

where $\chi = w_o t + v$ and (2.1) may be rewritten as

$$\eta(x,t) = 2\sigma\cos(kx)z_1 + 2\sigma\varepsilon\cos(2kx)(f_{\eta 1} + f_{\eta 2})z_1^2 + 2\sigma\varepsilon\cos(2kx)(-f_{\eta 1} + f_{\eta 2})z_2^2 \qquad (2.2)$$

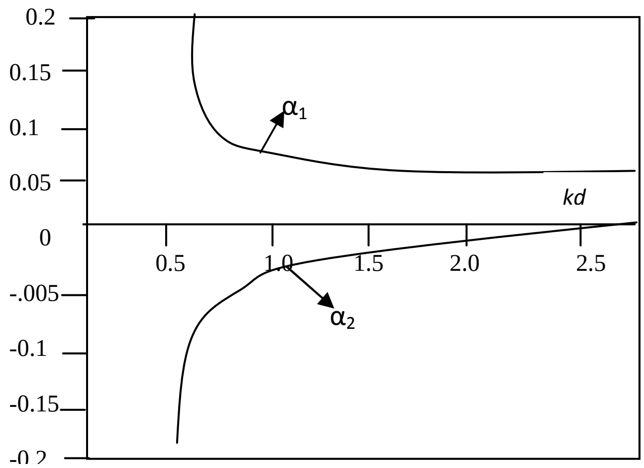

Figure 2.1: The parameters α_1 and α_2 (equation 2.5)) for the free surface displacement at a vertical wall (where *x* = 0) as function of *kd* (the stepness ε has been assumed equal to 0.055).

where:
$$z_1 = \frac{a\,Cos\,(X)}{\sigma}, \quad z_2 = \frac{a\,Sin\,(X)}{\sigma}$$

$$f_{\eta_1}(kd) = \frac{[2 + Cosh\,(2kd)]Cosh\,(kd)}{4Sinh^3(kd)} \qquad (2.3a)$$

$$f_{\eta 2}(kd)) = \frac{1}{2\tanh(2kd)} \qquad (2.3b)$$

This process belongs to the stochastic distribution (Ejinkonye [1] by defining

$$F = 2\cos(kx), \quad G = 2\varepsilon\cos(2kx)(f_{\eta 1} + f_{\eta 2}), \; H = 2\varepsilon\cos(2kx)(- f_{\eta 1} + f_{\eta 2}) \qquad (2.4)$$

And therefore it has parameters as the following expressions

$$\alpha_1 = \varepsilon(f_{\eta 1} + f_{\eta 2})\frac{\cos(2kx)}{|\cos(kx)|}, \qquad (2.5a)$$

$$\alpha_2 = \varepsilon\left(-f_{\eta 1} + f_{\eta 2}\right)\frac{\cos(2kx)}{|\cos(kx)|} \qquad (2.5b)$$

Observe that for $kx = \frac{\pi}{2} + n\pi \quad (\text{with } n = 0, \pm 1, \pm 2, \ldots)$, the linear term is zero: therefore, the process is only second-order. In this case, the process does not belong to the stochastic family (2.6) because $\alpha_1, \alpha_2 \to \infty$

$$\psi(x,y,t) = f(x,y)a\cos[X(t)] + g(x,y)a^2\cos^2[X(t)] + h(x,y)a^2\sin^2[X(t)], \qquad (2.6)$$

Figure 2.1 shows the family parameters α_1 and α_2 (equation 2.5) at the wall (where $x = 0$), as function of kd. The parameter α_1 is positive. Furthermore, the effects of non-linearity for surface waves on a vertical wall are greater than for surface waves in an undisturbed wave field. As an example: on deep water the parameter α_2 tends to zero and α_1 tends to ε (which is twice as much as the value of α for progressive waves on deep water). As for the free surface displacement, the fluctuating wave pressure does not belong to the stochastic distribution (2.6), for $kx = \pi/2$ (with $n = 0, \pm 1, \pm 2, \cdots$)

3.0 The fluctuating wave pressure in front of a vertical wall

The second-order fluctuating wave pressure in front of a vertical wall, for an infinitely narrow spectrum, is given by:

$$\eta_{\Delta P}(x,y,t) = 2\sigma f_{ph1}\cos(kx)Z_1 + 2\sigma\varepsilon\left[f_{ph2}\cos(2kx) - f_{ph3} + f_{ph4} + f_{ph5}\cos(2kx)\right]Z_1^2 - 2\sigma\varepsilon\left[f_{ph2}\cos(2kx) + f_{ph3} + f_{ph4} - f_{ph5}\cos(2kx)\right]Z_2^2 \qquad (3.1)$$

where

$$f_{ph_1}(ky,kd) = \frac{Cosh[k(y+d)]}{Cosh(kd)} \qquad (3.2)$$

$$f_{ph2}(ky,kd) = \frac{3\cosh[2k(y+d)] - \sinh^2(kd)}{3\sinh^3(kd)\cosh(kd)} \qquad (3.3)$$

$$f_{ph3}(ky,kd) = \frac{Cosh[2k(y+d)] - 1}{2Sinh^3(2kd)} \qquad (3.4)$$

$$f_{ph4}(ky,kd) = \frac{\cosh^2[k(y+d)] - \cosh(2kd)}{\sinh(2kd)} \qquad (3.5)$$

$$f_{ph5}(kd) = \frac{1}{2\sinh(2kd)} \qquad (3.6)$$

The relationship (3.5) formally belongs to the family (2.4) by defining

$$F = 2f_{phi}\cos(kx); \qquad (3.7)$$

$$G = 2\varepsilon\left[f_{ph2}\cos(2kx) - f_{ph3} + f_{ph4} + f_{ph5}\cos(2kx)\right]; \qquad (3.8)$$

$$H = -2\varepsilon\left[f_{ph2}\cos(2kx) + f_{ph3} + f_{ph4} + f_{ph5}\cos(2kx)\right] \qquad (3.9)$$

The parameters are given respectively by:

$$\alpha_1 = \varepsilon\frac{(f_{ph2} + f_{ph5})\cos(2kx) - f_{ph3} + f_{ph4}}{f_{ph1}|\cos(kx)|} \qquad (3.10)$$

$$\alpha_2 = -\varepsilon \frac{(f_{ph2} - f_{ph5})\cos(2kx) + f_{ph3} + f_{ph4}}{f_{ph1}\left|\cos(kx)\right|} \qquad (3.11)$$

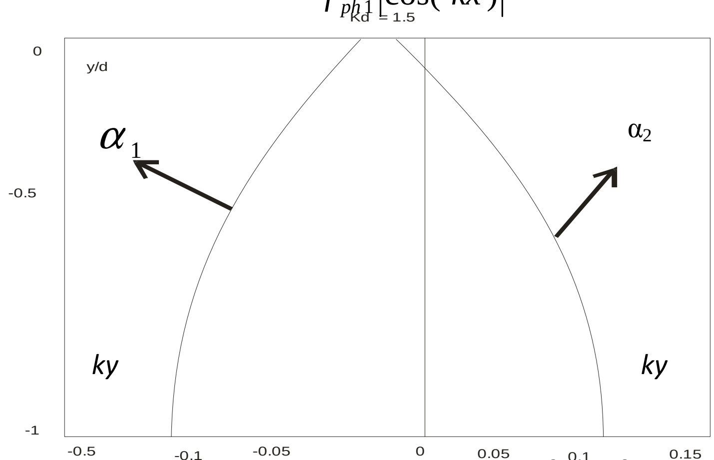

Figure 3.1: ***The parameter*** α_1 *and* α_2 **(equations 3.9 and 3.10)** ***for the fluctuating wave pressure at a vertical wall, as function of y/d. We have assumed*** **ε = 0.055** ***and kd*** **= 1.5**

As an example, Figure 3.1 shows the behaviour of the parameters (α_1 and α_2) at the wall, where $x = 0$ (obtained from equation (3.5)) as function of $\frac{y}{d}$, for ε = 0.055 and for $kd = 1.5$. Since α_1 being is negative for the fluctuation wave pressure, each realization of the stochastic process is a sequence of waves with trough amplitude greater than the crest amplitude. Moreover, for a fixed kd, the non-linear effects increases as we approach the bottom.

In fact, at the bottom (where $y = -d$) for $kd \to \infty$ the fluctuating wave has the limiting expression

$$\eta^{\infty}_{\Delta\rho} = -2\sigma\varepsilon\cos(2x) \qquad (3.6)$$

in which the linear term vanishes and only the second-order terms are not zero. The first linear term goes to zero faster than the second-order term, for kd→∞; this implies that α_1, $\alpha_2 \to \infty$.

4.0 Conclusion

For the fluctuating wave pressure (at a point beneath the sea surface) α_1 is generally less than zero. In this case the trough of the fluctuating wave pressure is greater than the crest and the effect of non-linearity for fluctuating wave pressure on a vertical wall is greater than in an undisturbed field.

Moreover, for a fixed *kd*, this theory suggests that the non-linear effects increase while approaching the bottom, which is valid for the mechanics of the sea waves.

5.0 Acknowledgment

The author is grateful to Professor E.O. Okeke who had read through these manuscripts, made a number of correction and suggestions. I am also grateful to Dr. C.S. Osuala my Head of Department for his encouragements. These are highly appreciated.

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