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On the non-linear wave in an undisturbed wave field

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Abstract

This work is based on the application of free surface displacement and fluctuating wave pressure in an undisturbed water surface. It is shown that the probability of exceeding wave crest is higher than that of the trough. Using the parameters α_1 and α_2, it is shown that for a fixed kd the effects of non-linearity decrease on approaching the bottom (sea floor) for which α_1 decreases as ky decreases and α_2 increases as ky decreases. These will be valid for most of the second-order processes in the mechanics of the sea waves.

***Keywords*:**
Freed surface displacement, fluctuating wave, undisturbed field, narrow spectrum, wave crest, wave trough.

1.0 Introduction

If the nonlinear effects are not negligible, the probability density function $P(\eta)$ of the surface displacement tends to deviate from Gaussian. In particular, second order non-linearities make crests to be more probable than deep troughs. This implies that the skewness of $P(\eta)$ is not zero Longuet-Higgins, [1]. Later, Tayfun [2, 3] obtained the probability density function in the probability of the exceedances of the crest (absolute maximum) for the free surface displacement in an undisturbed wave field. Tung and Huang [4], investigated the crest-trough symmetry, deriving the probability distributions of both the second order crest and trough under the hypothesis of narrow-band spectrum. Arena and Fedele [5] obtained the crest and the trough distributions of a general nonlinear narrow-band stochastic family; which includes many processes in the mechanics of the sea waves.

Here, we consider the narrow- banded 'free surface displacement' and 'fluctuating wave pressure' and concern ourselves with the progressive random waves in an undisturbed field and probable reflection of the waves.

We are able to show that for a fixed *kd the* effects of non linearity decrease on approaching the bottom (sea floor) for which α_1 decreases as *ky* decreases.

1.1 Specification

Here, the coordinate axis (*x,y*), has the *x*-axis horizontal and it is perpendicular to the wave front; the *y*-axis is vertical, with origin on the mean water level. The bottom depth is *d*,

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measured from the undisturbed water level. The steepness ε is given by $\varepsilon = k\sigma$, where k is the wave number and σ is the height of the dominant-wave mode or the standard deviation in the linear process. The typical value of ε is between 0.05 and 0.08 (a very characteristic value is $\varepsilon = 0.055$)

2.0 The free surface displacement in an undisturbed field

The free surface displacement in an undisturbed field at any fixed point x, to the first- order solution in a Stokes expression, is a stochastic stationary Gaussian process. The second – order free surface displacement, is given by Boccotti, [6]

$$\eta(x,t) = a \cos \alpha + ka^2 f_{\eta 1} \cos(2x), \tag{2.1}$$

where

$$f_{\eta 1}(kd) = \frac{[2 + coah(2kd)]\cosh(kd)}{4\sin^3(kd)} \tag{2.2}$$

This is the case when the spectrum is narrow-banded

$$x = kx - \square_0 t + v_1 \tag{2.3}$$

where the phase function υ_1 is a stochastic variable uniformly distributed in $(0,2\pi)$. At any fixed point x the equation (2.3) is rewritten as

$$\chi = \omega_0 t + \upsilon \tag{2.4}$$

where

$$\upsilon = -kx - \upsilon_1 \tag{2.5}$$

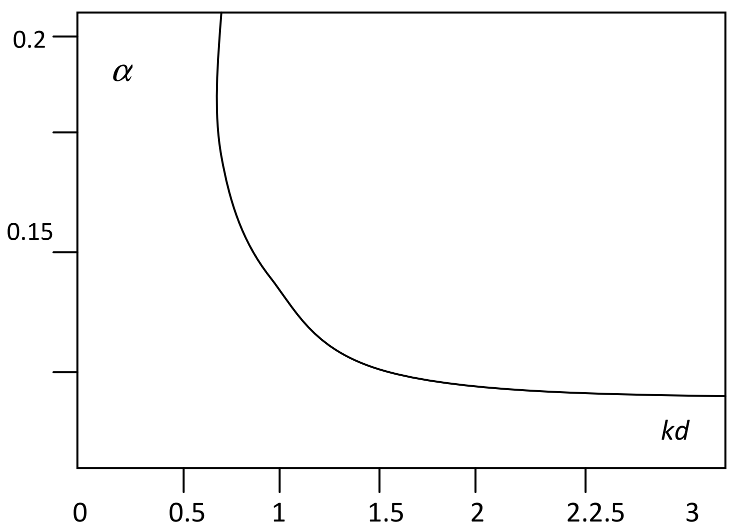

Figure 2.1: The parameters α(equation (2.1) for the free surface displacement in an undisturbed wave field as function of *kd* (the steepness ε has been assumed equal to 0,.055

is a stochastic variable uniformly distributed in $(0,2\pi)$, like the v_1. Therefore, the zero mean value process for the wave profile (2.1) is written as follows

$$\eta(x,t) = \sigma Z_1 + \sigma\varepsilon f_{n1}(Z_1^2 - Z_2^2) \tag{2.6}$$

The functions Z_1 and Z_2 are given by Arena and Fedele [5] in the form

$$Z_1 = \frac{aCos(X)}{\sigma}, \quad Z_2 = \frac{a\,Sin(X)}{\sigma}$$

Finally, defining

$$F = 1,\, G = \varepsilon f \eta_1,\, H = -G \qquad (2.7)$$

$$\psi(x,y,t) = f(x,y)a\cos[X(t)] + g(x,y)a^2\cos^2[X(t)] + h(x,y)a^2\sin^2[X(t)], \qquad (2.8)$$

We see that the process (2.7) belongs to the stochastic distribution (2.8), with parameters Ejinkonye [7], $\alpha = \alpha_1,\, G = \varepsilon f \eta_1,\, \alpha_2 = -\alpha_1$. Therefore from Figure 2.1 the parameter α is always greater than zero.

As a consequence, for a fixed, the threshold of probability of exceeding the wave crest (absolute maximum) is higher than the wave trough (absolute minimum). Figure 2.1 shows the parameter α (obtained from equation (2.9) for $\varepsilon = 0.055$), as a function of *kd*. On deep water ($kd \Rightarrow \infty$) α tends to $\frac{\varepsilon}{2}$.

2.0 The fluctuating wave pressure in an undisturbed field

The second–order wave profile generated by fluctuating wave pressure, for narrow–band spectrum, is given by Arena and Fedele [5] in the form of

$$\eta_{\Delta\rho}(x,y,t) = af_{ph1}\cos(\chi) + ka^2 f_{ph2}\cos(2x) - ka^2 f_{ph3,} \qquad (3.1)$$

where

$$f_{ph1}(ky,kd) = \frac{\cosh[k(y+d)]}{\cosh(kd)} \qquad (3.2)$$

$$f_{ph2}(ky,kd) = \frac{3\cosh[2k(y+d)] - \sinh^2(kd)}{4\sinh^3(kd)\cosh(kd)} \qquad (3.3)$$

$$f_{ph3}(ky,kd) = \frac{\cosh 2[k(y+d)] - 1}{2\sinh(2kd)} \qquad (3.4)$$

(3.2) is proportional to the linear wave amplitude in the sea with moderate depth. It is assumed that the wave travels along the *x*-axis. As for the free surface displacement (see section 2.0), at any fixed point (*x,y*) the fluctuating wave profile (3.1) may be rewritten as

$$\eta_{\Delta p}(x,y,t) = \sigma f_{ph1}\, Z_1 + \sigma\varepsilon\left(f_{ph2} - f_{ph3}\right) Z_1^2 - \sigma\varepsilon\left(f_{ph2} + f_{ph3}\right) Z_2^2 \qquad (3.5)$$

Finally, by defining

$$F = f_{phi},\;\; G = \varepsilon(f_{ph2} - f_{ph3}),\;\; H = -\varepsilon(f_{ph2} + f_{ph3}), \qquad (3.6)$$

We also see that the process (3.6) belongs to distribution (2.8) with parameters:

$$\alpha_1 = \frac{\varepsilon(f_{ph2} - f_{ph3})}{f_{ph1}}, \quad \alpha_2 = -\frac{\varepsilon(f_{ph2} + f_{ph3}),}{f_{ph1}} \qquad (3.7)$$

From equation (3.7), we deduce that the parameter α_1 is negative for *kd* >1.32. In this case, the fluctuating wave pressure has inverse-behaviour with respect to the free surface displacement: for a fixed, the threshold of probability of exceeding the wave crest (absolute maximum) is lower than the wave trough (absolute minimum).

The non-linear effects decrease by approaching the bottom (for a fixed *kd*), because α_1 decreases as *ky* decreases. The functional representation in this case is presently being investigated.

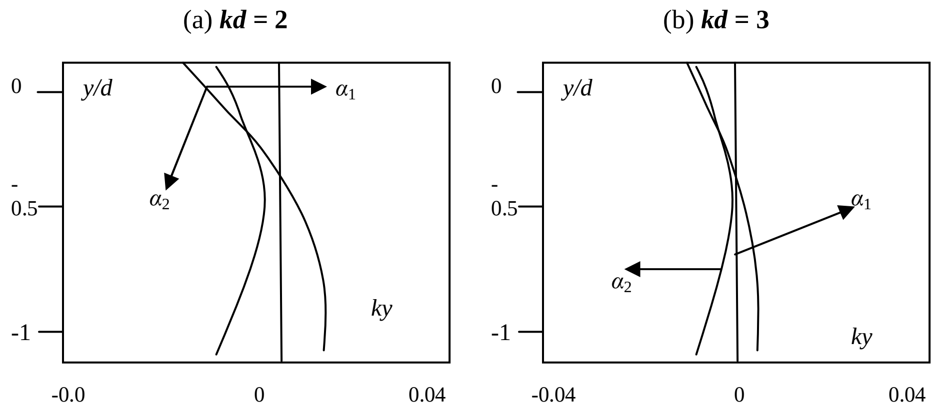

Figure 3.1: The parameters α_1 and α_2 (equation (3.6)) for the fluctuating wave pressure in an undisturbed wave field, as function of y/d (the steppness ε has been assumed equal to 0.055): (a) = kd=2; (b) kd=3

4.0 Conclusion

We have shown that generally the surface waves have the crest greater than the wave trough in terms of amplitude. Further more, for a fixed *kd* the effects of non-linearity decrease on approaching the sea floor for which α_1 decrease as *ky* decreases. The expressions for the parameter α_1 and α_2 enable us to predict the effects of non-linearity and suggest the following:

1. For α fixed, the threshold of probability of exceeding the wave crest (absolute maximum) is higher than the wave trough (absolute minimum).
2. From equation (3.7), the fluctuating wave pressure has inverse behaviour with respect to the free surface displacement, and for α fixed, the threshold of probability of exceeding the wave crest (absolute maximum) is lower than the wave trough (absolute minimum).

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