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Instantaneous source functions of a 3D source subject to edge water drive mechanism

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Abstract

This paper derives necessary instantaneous source functions of some reservoir systems with horizontal wells* (3D *sources*) *experiencing edge water drive. Eleven* (11) *different systems were considered for which thirty six* (36) *instantaneous source functions were derived.

1.0 Introduction

Flow strength in 3D sources, such as horizontal or lateral wells, strongly depend on the nature of the boundaries contributing them. Horizontal or lateral wells are usually drilled in an oil or gas reservoir domain. Hence these sources are obtainable from only four major boundary conditions. These are (1) the mandatory infinite-acting boundary condition, common to all reservoir domains irrespective of boundary kind, (2) a no-flow (sealed) boundary (prescribed flux source), (3) a constant-pressure boundary (prescribed pressure source) and (4) mixed boundary conditions (both 2 and 3).

Derivation of different instantaneous plane sources by infinite summation of their images is discussed in Refs.1 and 2 for these boundary conditions. Following these references therefore, a 3D source or a point source is visualized as the interception of three perpendicular infinite plane sources normal to the principal axes of permeability of the reservoir (Newman's product method [2-4]).

Instantaneous point sources are used to compute the reservoir pressure and rate distributions and can be used to generate other sources by superposition or integration [2,3]. The relevant advantage of this approach is that the functions are transient in nature, that is, they can be evaluated over time. This is popular in modern oil and gas literature and has given birth to today's more advanced reservoir management techniques, such as the use of coaxial pressure and pressure derivatives plots.[5-10].

In an edge water drive reservoir, water encroachment into the reservoir is through a direction parallel to the bedding planes of the reservoir. It manifest through increasing water/oil or water/gas ratio and first in nearby wells in a multiple well field. In contrast, bottom water drive manifest at the same time in all the wells in the same field, given the same reservoir anisotropy and production histories. It should be that apart from these water arrival patterns, influx due to edge water is not different from that due to bottom water.

Although analytical quantification for both influxes were attempted in Refs. [7-8], only one inflow pattern each was considered. In this paper, several influx patterns will be considered for only the case of edge water drive. The main objective therefore is to identify all possible reservoir architecture under edge water influences and to derive all 3D instantaneous sources for the influx patterns. Knowledge of these patterns can assist in deriving both pressure and rate distributions for the reservoirs.

Nomenclature

h: reservoir thickness (along the z-axis), ft
x: well location along the x-axis, ft
y: well location along the y-axis, ft
z: well location along the z-axis, ft
s : source
t: flow time, *hr*
n, l, m: Fourier series parameters
η: hydraulic diffusivity constant, md-psi/cp

Subscripts

e: external
f: slab source thickness
x: x-axis
y: y-axis
z: *z*-axis
w; wellbore

2.0 Horizontal or Lateral well nomenclature

The horizontal or lateral well being considered has the bottom (heel) located at the entrance of the reservoir open to the surface. Therefore, the top (toe) of the well is the last part in contact with the reservoir laterally adjacent to the bottom. Figure 2.1 illustrates this nomenclature.

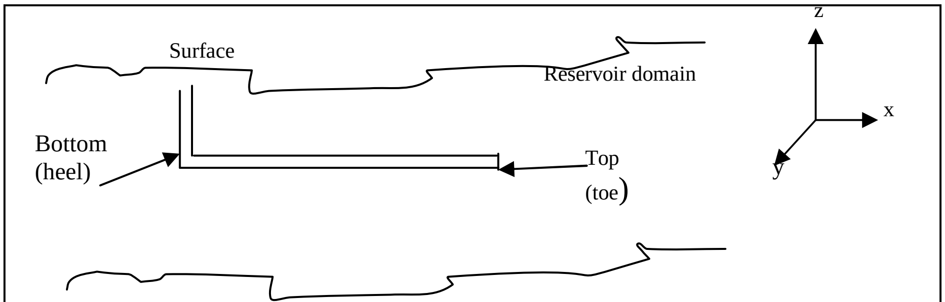


Figure 2.1: Horizontal or Lateral Well Nomenclature

3.0 Description of axes of fluid flow and selection of source functions

According to [Ref.1], instantaneous sources for the well are different when the well is located in the middle and outside the middle of the axis of interest. Therefore, more than one instantaneous source function may be written for one particular axis.

A boundary is said to be infinite only when its influence cannot be felt during a normal transient well test, even though that boundary may exist physically. Well location and completion may create this situation. But in this paper the influence of any aquifer activity is emphasized thus permitting the computation or prediction of reservoir pressures and the character of its perimeter.

Green's and source functions compiled in [Refs.1-4] will be used to prepare instantaneous source functions for the selected models.

A horizontal or lateral well is modeled here either as a point source or a line source. The well width corresponds to the well radius. Except deliberately assumed otherwise the source function from the width of the reservoir is usually regarded as infinite. This is written as

$$s_1(y,t) = \frac{e^{-(y-y_w)^2/4\eta_y t}}{2\sqrt{\pi t \eta_y}} \tag{3.1}$$

If the width is bounded by impermeable boundaries, then Eq. 1 is now an infinite plane source in an infinite slab reservoir written as

$$s_2(y,t) = \frac{1}{y_e}\left[1 + 2\sum_{i=1}^{\infty} \exp\left(-\frac{\pi^2 l^2 \eta_y}{y_e}\right)\cos\frac{l\pi y}{y_e}\cos\frac{l\pi y_w}{y_e}\right]. \tag{3.2}$$

The source along the axis of the well length, $s(x,t)$, is obtained differently. The well may be assumed as located in the center of the reservoir. In this case, the source is an infinite slab in an infinite slab reservoir written for early time as

$$s_3(x,t) = \frac{1}{2}\left[erf\left(\frac{1+x}{2\sqrt{\tau}}\right) + rf\left(\frac{1-x}{2\sqrt{\tau}}\right)\right]. \tag{3.3}$$

and for long times as

$$s_4(x,t) = \frac{x_f}{x_e}\left[1 + \frac{4x_e}{\pi x_f}\sum_{i=1}^{\infty}\frac{1}{n}\exp\left(-\frac{\pi^2 n^2 \eta_x}{x_e}\right)\sin\frac{n\pi x_f}{2x_e}\cos\frac{n\pi x}{x_e}\cos\frac{n\pi x_w}{x_e}\right]. \tag{3.4}$$

If the well is located off-centre along the x-axis, then the source will produce the effects of an infinite plane source in an infinite slab reservoir, written as follows for long time

$$s_6(x,t)=\frac{1}{x_e}\left[1+2\sum_{n=1}^{\infty}\exp\left(-\frac{\pi^2(2n+1)^2\eta_x}{x_e}\right)\cos\frac{(2n+1)\pi x}{x_e}\cos\frac{(2n+1)\pi x_w}{x_e}\right]. \qquad (3.5)$$

or, at early times

$$s_6(x,t)=\frac{e^{-(x-x_w)^2/4\eta_x t}}{2\sqrt{\pi t\eta_x}}. \qquad (3.6)$$

Source functions for water influx boundaries (constant-pressure boundaries) are written according to the axes of influx into the wellbore. Along the x-axis, if influx occurs at the top/bottom only while the bottom/top is sealed then for centrally located well

$$s_7(x,t)=\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n+1}\exp\left(-\frac{(2n\pm1)^2\pi^2 t\eta_x}{4x_e^2}\right)\sin\frac{(2n\pm1)\pi x_f}{4x_e}\cos\frac{(2n+1)\pi x_w}{x_e}\cos\frac{(2n+1)\pi x}{x_e}. \qquad (3.7)$$

and for off-centrally located well

$$s_8(x,t)=\frac{2}{x_e}\sum_{n=1}^{\infty}\frac{1}{2n+1}\exp\left(-\frac{(2n\pm1)^2\pi^2 t\eta_x}{4x_e^2}\right)\cos\frac{(2n+1)\pi x_w}{x_e}\cos\frac{(2n+1)\pi x}{x_e} \qquad (3.8)$$

where (+) = top and (-) = bottom. If the influx occurs at both top and bottom, then for centrally-located well

$$s_9(x,t)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\pi^2 t\eta_x}{x_e^2}\right)\sin\frac{n\pi x_f}{2x_e}\sin\frac{n\pi x_w}{x_e}\sin\frac{n\pi x}{x_e} \qquad (3.9)$$

and off-centrally located well

$$s_{10}(x,t)=\frac{2}{x_e}\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\pi^2 t\eta_x}{x_e^2}\right)\sin\frac{n\pi x_w}{x_e}\sin\frac{n\pi x}{x_e} \qquad (3.10)$$

Influx arriving from the top/bottom of the y-axis is represented by

$$s_{11}(y,t)=\frac{2}{y_e}\sum_{n=1}^{\infty}\exp\left(-\frac{(2n\pm1)^2\pi^2 t\eta_y}{4y_e^2}\right)\cos\frac{(2n\pm1)\pi y_w}{y_e}\cos\frac{(2n\pm1)\pi y}{y_e} \qquad (3.11)$$

where (+) = top and (-) = bottom. But, if influx occurs at both ends of the well from the y-axis, then

$$s_{12}(y,t)=\frac{2}{y_e}\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\pi^2 t\eta_y}{y_e^2}\right)\sin\frac{n\pi y_w}{y_e}\sin\frac{n\pi y}{y_e} \qquad (3.12)$$

Finally, only the *z*-axis is now remaining. In terms of concept, no influx occurs through the z-axis in an edge water drive reservoir. Furthermore, thin reservoirs, in terms of thickness, are most attractive for horizontal or lateral wells. Therefore, considering the ends of the z-axis sealed, the only possible source function is that of an infinite plane in an infinite slab reservoir, given as follows at long time

$$s_{13}(z,t)=\frac{1}{h}\left[1+2\sum_{i=1}^{\infty}\exp\left(-\frac{\pi^2 n^2\eta_z}{h}\right)\cos\frac{n\pi z}{h}\cos\frac{n\pi z_w}{h}\right]. \qquad (3.13)$$

At early time,

$$s_{14}(z,t)=\frac{e^{-(z-z_w)^2/4\eta_{zy}t}}{2\sqrt{\pi t\eta_z}} \qquad (3.14)$$

With the above background information, it is now possible to write down relevant instantaneous source functions for all possible influx patterns for edge water movement.

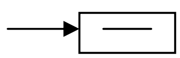
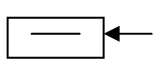
4.0 Results and discussion

Table 4.1 shows thirty-six (36) different instantaneous source functions from eleven (11) different influx patterns identified. For each pattern, all the possible instantaneous source functions are written down. Influx patterns for edge water movement at both ends of an axis yield fewer instantaneous functions because apart from the mandatory infinite-acting period, flow is dominated by steady state flow occasioned by irreversible arrival of water under a singular transient regime. Except for areal anisotropy, steady-state sets in

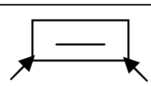
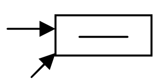
rapidly for high production rates and vice versa. For axes with mixed boundaries, such as serial numbers 1, 2, 3, 4 and 7, it is suggested that infinite activity would prevail if (1) well perforations are significantly far from either boundary, (2) flow rate is low and (3) there is large anisotropy. This is irrespective of well location. For central well location on the x-axis (well length axis), however, the source is a slab in a slab reservoir. If the well is located off-centre, source functions are written as emanating from an infinite plane and according to the boundary likely to be felt first. If a no-flow boundary is felt first, a prescribed flux source is suggested. But if the constant-pressure boundary is felt first, then the prescribed flux source may never be felt and a prescribed pressure source is suggested.

For ease of reference, the arguments of the individual sources are dropped in Table 4.1.

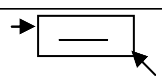
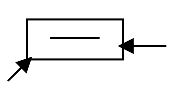
Table 4.1: Possible 3D Instantaneous Source Functions for Edge Water Drive Reservoir

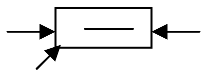
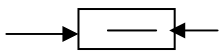
S/N	Model Diagram	Description of boundaries along the axes			Instantaneous source function
		x-axis	y-axis	z-axis	
1.		Aquifer at bottom, top bounded	Infinite extents	Bounded top and bottom	$S_3. S_1. S_{13}.$ $S_7.S_1.S_{13}$ $S_8.S_1.S_{13}$
		Aquifer at bottom, top bounded	Bounded at top and bottom	Bounded top and bottom	$S_3. S_1. S_{13}.$ $S_7.S_2.S_{13}$ $S_8.S_1.S_{13}$ $S_8.S_2.S_{13}$
		Aquifer at bottom, top infinite	Infinite extents	Bounded top and bottom	$S_3. S_1. S_{13}.$ $S_7.S_2.S_{13}$ $S_8.S_1.S_{13}$ $S_5.S_2.S_{13}$ $S_4.S_1.S_{13}$ $S_6.S_2.S_{13}$
		Aquifer at bottom, top infinite	Bounded extents	Bounded top and bottom	$S_3.S_1.S_{13}.$ $S_7.S_1.S_{13}$ $S_8.S_1.S_{13}$ $S_5.S_1.S_{13}$ $S_4.S_1.S_{13}$ $S_6.S_1.S_{13}$ $S_3.S_2.S_{13}$ $S_4.S_2.S_{13}$ $S_5.S_2.S_{13}$ $S_6.S_2.S_{13}$ $S_7.S_2.S_{13}$ $S_8.S_2.S_{13}$
2		Aquifer at top, bottom bounded	Infinite extents	Bounded top and bottom	$S_3.S_1.S_{13}.$ $S_7.S_1.S_{13}$ $S_8.S_1.S_{13}$ $S_6.S_1.S_{13}$

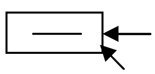
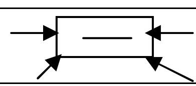
		Aquifer at top, bottom infinite	Infinite extents	Bounded top and bottom	$S_3. S_1. S_{13}.$ $S_7.S_2.S_{13}$ $S_8.S_1.S_{13}$ $S_5.S_2.S_{13}$ $S_4.S_1.S_{13}$ $S_6.S_2.S_{13}$
		Aquifer at top, bottom bounded	Bounded top and bottom	Bounded top and bottom	$S_3. S_1. S_{13}.$ $S_7.S_2.S_{13}$ $S_8.S_1.S_{13}$ $S_8.S_2.S_{13}$

		Aquifer at top, bottom infinite	Bounded top and bottom	Bounded top and bottom	$S_3.S_1.S_{13}.$ $S_7.S_1.S_{13}$ $S_8.S_1.S_{13}$ $S_5.S_1.S_{13}$ $S_4.S_1.S_{13}$ $S_6.S_1.S_{13}$ $S_3.S_2.S_{13}$ $S_4.S_2.S_{13}$ $S_5.S_2.S_{13}$ $S_6.S_2.S_{13}$ $S_7.S_2.S_{13}$ $S_8.S_2.S_{13}$
3.		Bounded top and bottom	Aquifer at bottom and top	Bounded top and bottom	$S_3. S_{12}. S_{13}.$ $S_5.S_{12}.S_{13}$ $S_4.S_{12}.S_{13}$ $S_6.S_{12}.S_{13}$
		Bounded top and infinite bottom	Aquifer at bottom and top	Bounded top and bottom	$S_3. S_{12}. S_{13}.$ $S_5.S_{12}.S_{13}$ $S_4.S_{12}.S_{13}$ $S_6.S_{12}.S_{13}$
		Infinite extents	Aquifer at bottom and top	Bounded top and bottom	$S_3. S_{12}. S_{13}.$ $S_6.S_{12}.S_{13}$
4.		Aquifer at bottom top bounded	Aquifer at bottom, top bounded	Bounded top and bottom	$S_3. S_{11}. S_{13}.$ $S_7.S_{11}.S_{13}$ $S_8.S_{11}.S_{13}$
		Aquifer at bottom, top bounded	Aquifer at bottom, top infinite	Bounded top and bottom	$S_3. S_{11}. S_{13}.$ $S_7.S_{11}.S_{13}$ $S_8.S_{11}.S_{13}$ $S_3. S_1. S_{13}.$ $S_7. S_1. S_{13}.$ $S_8. S_1. S_{13}.$
		Aquifer at bottom, top infinite	Aquifer at bottom, top bounded	Bounded top and bottom	$S_3. S_{11}. S_{13}.$ $S_4.S_{11}.S_{13}$ $S_5.S_{11}.S_{13}$ $S_6. S_{11}. S_{13}.$ $S_7. S_{11}. S_{13}.$ $S_8. S_{11}. S_{13}.$

		Aquifer at bottom, top infinite	Aquifer at bottom, top bounded	Bounded top and bottom	s3. s1. s13. s4.s1.s13 s5.s1.s13 s6. s1. s13. s7. s1. s13. s8. s1. s13. s3. s11. s13. s4. s11. s13. s5. s11. s13. s6. s11. s13. s7. s11. s13. s8. s11. s13.
		Infinite ends	Aquifer at bottom, top bounded	Bounded top and bottom	s3. s11. s13. s6. s11. s13.
		Bounded ends	Aquifer at bottom, top bounded	Bounded top and bottom	s4. s11. s13. s5. s11. s13. s6. s11. s13
		Infinite ends	Aquifer at bottom, top infinite	Bounded top and bottom	s3. s11. s13 s6. s11. s13 s3. s1. s13 s6. s1. s13

		Bounded ends	Aquifer at bottom, top infinite	Bounded top and bottom	$s_3. s_1. s_{13}.$ $s_4.s_1.s_{13}$ $s_5.s_1.s_{13}$ $s_6. s_1. s_{13}.$ $s_3. s_{11}. s_{13}.$ $s_4.s_{11}.s_{13}$ $s_5.s_{11}.s_{13}$ $s_6. s_{11}. s_{13}.$
6.		Aquifer at bottom, top bounded	Infinite bottom, aquifer at top	Bounded top and bottom	$s_7. s_1. s_{13}.$ $s_8. s_1. s_{13}.$ $s_7. s_{11}. s_{13}.$ $s_8. s_{11}. s_{13}.$
		Aquifer at bottom, top bounded	Bounded bottom, aquifer at top	Bounded top and bottom	$s_7. s_{11}. s_{13}.$ $s_8. s_{11}. s_{13}.$
		Aquifer at bottom, top infinite	Infinite bottom, aquifer at top	Bounded top and bottom	$s_3. s_1. s_{13}.$ $s_7. s_1. s_{13}.$ $s_8. s_1. s_{13}.$ $s_3. s_{11}. s_{13}.$ $s_7. s_{11}. s_{13}.$ $s_8. s_{11}. s_{13}.$
		Aquifer at bottom, top infinite	Bounded bottom, aquifer at top	Bounded top and bottom	$s_3. s_{11}. s_{13}.$ $s_7. s_{11}. s_{13}.$ $s_8. s_{11}. s_{13}.$
7.		Bottom bounded, aquifer at top	Bounded top, aquifer at bottom	Bounded top and bottom	$s_3. s_{11}. s_{13}.$ $s_7. s_{11}. s_{13}.$ $s_8. s_{11}. s_{13}.$
		Bottom bounded, aquifer at top	Infinite top, aquifer at bottom	Bounded top and bottom	$s_3. s_{11}. s_{13}.$ $s_7. s_{11}. s_{13}.$ $s_8. s_{11}. s_{13}.$ $s_3. s_1. s_{13}.$ $s_7. s_1. s_{13}.$ $s_8. s_1. s_{13}.$

		Bottom infinite , aquifer at top	Bounded top, aquifer at bottom	Bounded top and bottom	$S_3 . S_2 . S_{13}$. $S_7 . S_2 . S_{13}$. $S_8 . S_2 . S_{13}$. $S_3 . S_{11} . S_{13}$. $S_7 . S_{11} . S_{13}$. $S_8 . S_{11} . S_{13}$.
		Bottom infinite, aquifer at top	Infinite top, aquifer at bottom	Bounded top and bottom	$S_3 . S_2 . S_{13}$. $S_7 . S_2 . S_{13}$. $S_8 . S_2 . S_{13}$. $S_3 . S_{11} . S_{13}$. $S_7 . S_{11} . S_{13}$. $S_8 . S_{11} . S_{13}$.
8.		Aquifer at bottom and top	Aquifer bottom, bounded top	Bounded top and bottom	$S_9 . S_{11} . S_{13}$. $S_{10} . S_{11} . S_{13}$.
		Aquifer at bottom and top	Aquifer at bottom, infinite top	Bounded top and bottom	$S_9 . S_{11} . S_{13}$. $S_{10} . S_{11} . S_{13}$. $S_9 . S_1 . S_{13}$. $S_{10} . S_1 . S_{13}$.
9.		Aquifer at bottom and top	Bounded bottom and top	Bounded top and bottom	$S_9 . S_1 . S_{13}$. $S_{10} . S_1 . S_{13}$. $S_9 . S_2 . S_{13}$. $S_{10} . S_2 . S_{13}$.
		Aquifer at bottom and top	Infinite bottom and top	Bounded top and bottom	$S_9 . S_1 . S_{13}$. $S_{10} . S_1 . S_{13}$.

10.		Aquifer at top, bounded bottom	Aquifer at top, bounded bottom	Bounded top and bottom	$S_3 . S_{11} . S_{13}$. $S_7 . S_{11} . S_{13}$. $S_8 . S_{11} . S_{13}$.
		Aquifer at top, bounded bottom	Aquifer at top, infinite bottom	Bounded top and bottom	$S_3 . S_{1.} S_{13}$. $S_7 . S_1 . S_{13}$. $S_8 . S_1 . S_{13}$. $S_3 . S_{11} . S_{13}$. $S_7 . S_{11} . S_{13}$. $S_8 . S_{11} . S_{13}$.
		Infinite bottom, aquifer at top	Aquifer at top, bounded bottom	Bounded top and bottom	$S_3 . S_{1.} S_{13}$. $S_7 . S_1 . S_{13}$. $S_8 . S_1 . S_{13}$. $S_3 . S_{11} . S_{13}$. $S_7 . S_{11} . S_{13}$. $S_8 . S_{11} . S_{13}$.
		Infinite bottom, aquifer at top	Aquifer at top, infinite bottom	Bounded top and bottom	$S_3 . S_{1.} S_{13}$. $S_7 . S_1 . S_{13}$. $S_8 . S_1 . S_{13}$. $S_3 . S_{11} . S_{13}$. $S_7 . S_{11} . S_{13}$. $S_8 . S_{11} . S_{13}$.
11.		Aquifer at top and bottom	Aquifer at top and bottom	Bounded top and bottom	$S_9 . S_{12} . S_{13}$. $S_{10} . S_{12} . S_{13}$.

5.0 Conclusion

One major problem in fluid flow modeling in horizontal or lateral wells is the selection of instantaneous source functions contributed from the flow axes. This work is yet another effort at solving this problem. Only edge water influx patterns were considered. Attempts were made for cases where they govern flow character in a transient period. Other flow character may however be observed during a well test. Such can be easily modeled if a mastery of writing down relevant instantaneous source function is acquired. Thirty-six (36) instantaneous source functions derived from eleven (11) different edge water influx patterns have been tabulated for quick reference.

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