Journal of the Nigerian Association of Mathematical Physics
Volume **13 (November, 2008), 405 - 410**
© J. of NAMP

Temperature and velocity profile on thermal ignition in a reactive variable viscosity poiseuille flow.

[1*]P. O. Olanrewaju, [2]A. Falohun, [3]D. A. Ajadi, and [1]R. O. Ayeni
[1]Department of Pure and Applied Mathematics,
[2]Department of Computer Science and Engineering,
[3]Department of Pure and Applied Physics
Ladoke Akintola University of Technology, Ogbomoso

Abstract

This study evaluates the physio-chemical properties that are associated with thermal ignition in a strong exothermic reaction of variable viscosity combustible material flowing through a channel with isothermal walls under Arrhenius kinetics, neglecting the construction of the material under physical and reasonable conditions to give further insight into the theory of combustion. It shows that these properties has greater influence on thermal ignition. Temperature and velocity profiles were also considered. The procedure reveals accurately the state thermal ignition criticality conditions for as well as their dependent on viscous heating parameter.The numerical method used can be used as an effective tool to investigate several other dependent nonlinear boundary – value problem.

1.0 Introduction

In petrochemical industries as well as petroleum refineries, the study of thermal ignition in a combustible reacting variable viscosity fluid is of great importance in order to ensure safety of life and properties ([3],[9]).

Thermal ignition occurs when the reactions produce heat too rapidly for a stable balance between heat production and heat loss to be preserved. Hence, it is important to know the critical values of the basic physical quantities, such as the ambient temperature, surface characteristics, the chemistry of the reacting combustible material and the physical storage geometry at which ignition occur ([1],[2],[4],[7],[8]).

Olarenwaju et al. ([11]) examine the existence and uniqueness result for a two-step reactive-diffusive equation with variables pre-exponential factor. They established the criteria and conditions for existence and uniqueness of solution. They further discovered that there are certain values for n,m,r and β that the problem can accommodates for the solution to be stable.Similarly, Frank-kameretskii parameters δ_1, δ_2 must not exceed certain values for the solution to exist and the same time stable.

Makinde [10] examine steady flow of a reactive variable viscosity fluid in a cylindrical pipe with an isothermal wall and the state thermal ignition criticality conditions and their dependent on both Frank-Kamenetskii and viscous heating parameters are accurately obtained.The results also revealed the rapid convergence of the approximation procedure with gradual increase in the number of series coefficients utilized in the approximants.

Olanrewaju [13] investigate the effect of Frank-Kamenetskii parameter and numerical exponents on temperature of two-step Arrhenius reactions and it was shown that it has greater influence on temperature of a given system.

In this paper we extend the work of Olanrewaju et al. [12] so as to make it extremely close to reality and applicable to real life problem

1* e-mail: Oladapo_anu@yahoo.ie

2.0 Mathematical formulation

The classical formulation of this type of problem was first introduced by Frank-Kameretskii ([4]). Neglecting the reactant consumption, the equation for the heat balance in the original variables together with the boundary conditions can be written as

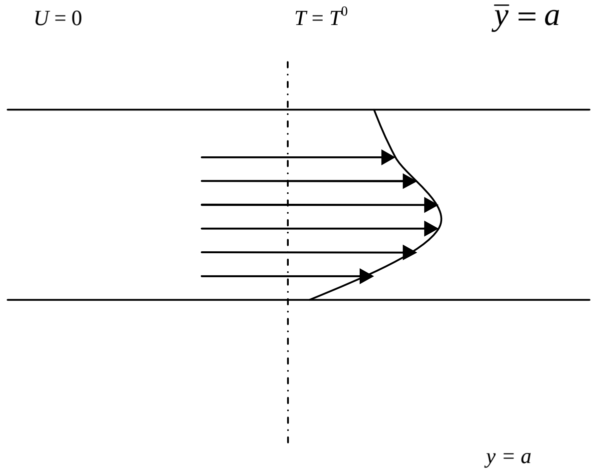

Figure 2.1: Geometry of the problem

$$\frac{d^2T}{dy^2} + Q\frac{C_0 A}{K}e^{\frac{-E}{RT}} + \mu/K\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2 = 0$$

$$\frac{d}{d\bar{y}}\left(\mu\frac{d\bar{u}}{d\bar{y}}\right) = -G \tag{2.1}$$

$$u = 0, T = T_0, \; on \; \bar{y} = a \tag{2.2}$$

$$\frac{dT}{dy} = \frac{d\bar{u}}{d\bar{y}} = 0 \quad \text{on} \quad \bar{y} = 0 \tag{2.3}$$

where T is the absolute temperature
G is the constant for axial pressure gradient
T_0 is the wall reference temperature
K is the thermal conductivity of the material
Q is the heat of reaction
A is the rate constant
E is the activation energy
R is the universal gas constant
C_0 is the initial concentration of the reactant species
A is the chemical characteristics half width.
$(\bar{x}, \bar{y})$ is the distance measured in the axial and normal directions respectively.

Following Gutlamann et al, ([6]), we defined the dynamic viscosity of the combustible material as

$$\mu = \mu_0 e^{E/RT} \tag{2.4}$$

where μ_0 is the combustible material references viscosity. We use the following dimensionless variables in equations (2.1) – (2.3)

$$\begin{aligned}
&\theta = \frac{E}{RT^2{}_0}(T - T_0), \quad \varepsilon = \frac{RT_0}{E}, \quad y = \frac{\bar{y}}{a} \\
&\lambda = \frac{QEAa^2 c_o}{T_0{}^2 Rk} e^{-E/RT_0}, \quad W = \frac{\mu_0 \bar{u}}{Ga^2} e^{E/RT_0} \\
&\beta = \frac{G^2 a^2}{Qc_0 A\mu_0}
\end{aligned} \tag{2.5}$$

and obtain the dimensionless governing equation together with the corresponding boundary conditions as

$$\frac{dw}{dy} = -y\exp(\theta/_{1+\varepsilon\theta}), \quad \frac{d^2\theta}{dy^2} + \lambda(1 + \beta y^2)\exp(\theta/_{1+\varepsilon\theta}) = 0 \tag{2.6}$$

$$\frac{d\theta(0)}{dy} = 0, \quad \theta(\pm 1) = 0, \quad w(1) = 0 \tag{2.7}$$

where $\lambda, \varepsilon, \beta$ represent the Frank-Kameretskii parameter, activation energy parameter and the viscous heating parameter respectively.

In the next section, equation (2.6) and (2.7) are solved by using shooting techniques and Runge kutta instead of both perturbation and multivariate series summation techniques that take care of some terms of the problem ([10] [7] [8] [10] [12]

3.0 Method of solution

We let

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} y \\ w \\ \theta \\ \theta' \end{pmatrix} \tag{3.1}$$

and

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{pmatrix} = \begin{pmatrix} 1 \\ -y_1\, e^{y_3/_{1+\varepsilon y_3}} \\ y_4 \\ -\lambda(1 + \beta y_1{}^2)\, e^{y_3/_{1+\varepsilon y_3}} \end{pmatrix} \tag{3.2}$$

Satisfying

$$\begin{pmatrix} y_1(-1) \\ y_2(-1) \\ y_3(-1) \\ y_4(-1) \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \\ a \end{pmatrix} \tag{3.3}$$

4.0 Analysis of result and conclusion

The results obtained are shown in the tables below :

Table 4.1: Computations showing the procedure rapid convergence for λ = 1,β = 0, ε = 1, α = 1.369106

S/N	*y*	*w*	*θ*	*θ*¹
0	-1	0.0000	0.0000	1.3691
1	-0.9000	-0.1008	0.1319	1.2627
2	-0.8000	-0.2004	0.2526	1.1451
3	-0.7000	-0.2951	0.3610	1.0186
4	-0.6000	-0.3819	0.4563	0.8848
5	-0.5000	-0.4586	0.5379	0.7452
6	-0.4000	-0.5233	0.6054	0.6012
7	-0.3000	-0.5748	0.6582	0.4537
8	-0.2000	-0.6122	0.6961	0.3038
9	-0.1000	-0.6349	0.7190	0.1523
10	0.0000	-0.6425	0.7266	-0.0000
11	0.1000	-0.6349	0.7190	-0.1524
12	0.2000	-0.6122	0.6961	-0.3039
13	0.3000	-0.5747	0.6582	-0.4538
14	0.4000	-0.5232	0.6054	-0.6013
15	0.5000	-0.4584	0.5380	-0.7454
16	0.6000	-0.3818	0.4563	-0.8850
17	0.7000	-0.2950	0.3610	-1.0188
18	0.8000	-0.2002	0.2526	-1.1454
19	0.9000	-0.1004	0.1320	-1.2630
20	1.0000	0.0004	0.0000	-1.3695

Table 4.2: Computations showing the procedure rapid convergence for λ = 1 α = 2.394968 ε = 1 β = 2

S/N	*y*	*w*	*θ*	*θ*¹
0	-1	0.0000	0.0000	2.3950
1	-0.9000	-0.1046	0.2245	2.0860
2	-0.8000	-0.2128	0.4174	1.7745
3	-0.7000	-0.3173	0.5795	1.4777
4	-0.6000	-0.4135	0.7130	1.2038
5	-0.5000	-0.4984	0.8203	0.9552
6	-0.4000	-0.5699	0.9041	0.7311
7	-0.3000	-0.6266	0.9666	0.5282
8	-0.2000	-0.6677	1.0097	0.3423
9	-0.1000	-0.6926	1.0351	0.1681
10	0.0000	-0.7009	1.0434	-0.0002
11	0.1000	-0.6925	1.0350	-0.1685
12	0.2000	-0.6677	1.0097	-0.3428
13	0.3000	-0.6266	0.9665	-0.5287
14	0.4000	-0.5698	0.9040	-0.7315
15	0.5000	-0.4983	0.8202	-0.9557
16	0.6000	-0.4134	0.7129	-1.2043
17	0.7000	-0.3171	0.5794	-1.4783
18	0.8000	-0.2125	0.4173	-1.7753
19	0.9000	-0.1042	0.2245	-2.0873
20	1.0000	0.0006	0.0000	-2.3970

Table 4.3: Computations Showing the Procedure rapid convergence for α = 1.162605
λ = 0.878, ε = 1 β = 0

S/N	*y*	*w*	*θ*	*θ*¹
0	-1	0.0000	0.0000	1.1626
1	-0.9000	-0.0999	0.1119	1.0700
2	-0.8000	-0.1976	0.2140	0.9689
3	-0.7000	-0.2896	0.3057	0.8609
4	-0.6000	-0.3736	0.3862	0.7472
5	-0.5000	-0.4475	0.4551	0.6290
6	-0.4000	-0.5099	0.5120	0.5073

7	-0.3000	-0.5594	0.5566	0.3828
8	-0.2000	-0.5954	0.5886	0.2562
9	-0.1000	-0.6172	0.6079	0.1284
10	0.0000	-0.6245	0.6143	-0.0000
11	0.1000	-0.6172	0.6079	-0.1285
12	0.2000	-0.5954	0.5886	-0.2563
13	0.3000	-0.5594	0.5566	-0.3828
14	0.4000	-0.5098	0.5121	-0.5074
15	0.5000	-0.4474	0.4552	-0.6292
16	0.6000	-0.3735	0.3863	-0.7474
17	0.7000	-0.2895	0.3057	-0.8610
18	0.8000	-0.1973	0.2141	-0.9691
19	0.9000	-0.0997	0.1119	-1.0702
20	1.0000	0.0003	0.0000	-1.1629

Table 4.4: Computations showing the procedure rapid convergence for $\lambda = 1$. $\varepsilon = 1$, $\beta = 1$, $\alpha = 1.866365$

S/N	*y*	*w*	θ	θ^1
0	-1	0.0000	0.0000	1.8664
1	-0.9000	-0.1027	0.1766	1.6605
2	-0.8000	-0.2066	0.3322	1.4495
3	-0.7000	-0.3063	0.4666	1.2413
4	-0.6000	-0.3979	0.5805	1.0403
5	-0.5000	-0.4787	0.6747	0.8483
6	-0.4000	-0.5469	0.7502	0.6655
7	-0.3000	-0.6011	0.8078	0.4910
8	-0.2000	-0.6404	0.8484	0.3233
9	-0.1000	-0.6642	0.8725	0.1604
10	0.0000	-0.6722	0.8805	-0.0001
11	0.1000	-0.6642	0.8725	-0.1606
12	0.2000	-0.6404	0.8484	-0.3236
13	0.3000	-0.6011	0.8078	-0.4913
14	0.4000	-0.5468	0.7502	-0.6658
15	0.5000	-0.4786	0.6747	-0.8486
16	0.6000	-0.3977	0.5805	-1.0407
17	0.7000	-0.3061	0.4666	-1.2417
18	0.8000	-0.2064	0.3322	-1.4500
19	0.9000	-0.1023	0.1766	-1.6611
20	1.0000	0.0005	0.0000	-1.8673

The results above were used to plot the following graphs:
Figure 2.1 shows the geometry of the problem. Figure 4.1 shows the curve of temperature against position y for various values of β and fixed values of $\lambda = 1$ and $\varepsilon = 1$.It is shown that as β increases the temperature also increases. The fluid temperature increases with an increase in the viscous heating parameter.

Figure 4.2 shows the graph of temperature against position y for various values of λ and fixed values of $\beta = 0$ and $\varepsilon = 1$.The fluid temperature increases with increase in frank-Kamenetskii parameter which also agree with the literature.

Figure4.3 shows the graph of W against position y for various values of β and fixed values of $\lambda = 1$ and $\varepsilon = 1$.It is shown that as β increases the temperature decreases. The fluid temperature increases with an increase in the viscous heating parameter along negative direction.

.

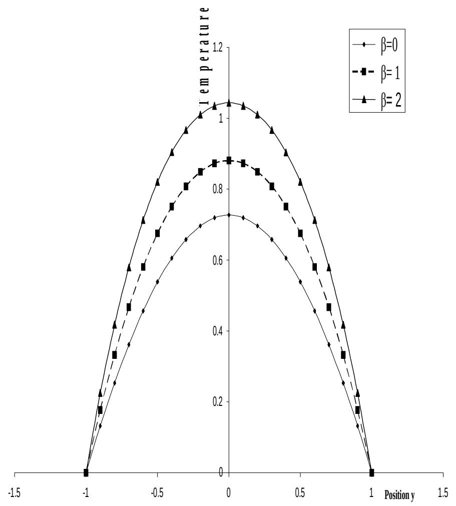

Figure 4.1: Graph of temperature against position y for various values of β and fixed values of λ = 1 and ε = 1

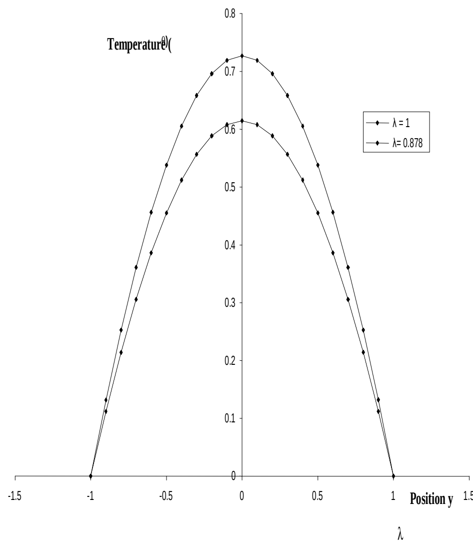

Figure 4.2: Graph of temperature against position y for various values of λ and fixed values of β = 1 and ε = 1

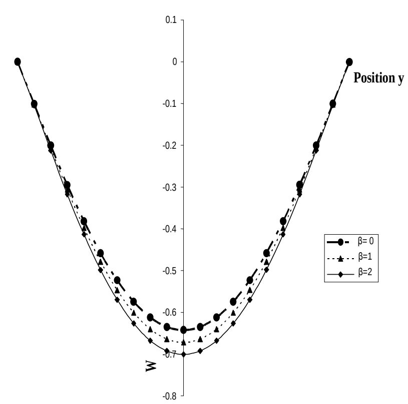

Figure 4.3: Graph of *W* against position y for various values of β and fixed values of λ = 1 and ε = 1

5.0 Conclusions

This study evaluates the physio-chemical properties that are associated with thermal ignition in a strong exothermic reaction of variable viscosity combustible material flowing through a channel with isothermal walls under Arrhenius kinetics, neglecting the construction of the material under physical and reasonable conditions to

Give further insight into the theory of combustion. It shows that these properties has greater influence on thermal ignition. Temperature and velocity profiles were also considered. The procedure reveals accurately the state thermal ignition criticality conditions for as well as their dependent on viscous heating parameter.The numerical method used can be used as an effective tool to investigate several other dependent nonlinear boundary – value problem. It is shown that as β increases the temperature also increases. The fluid temperature increases with an increase in the viscous heating parameter. .The fluid temperature increases with increase in frank-Kamenetskii parameter which also agree with the literature. The fluid temperature increases with an increase in the viscous heating parameter along negative direction for the velocity profile.

References

[1] Balakrishnan, E., Swift, A., Wake, G .C., 1996. Critical values for some non-class A geometries in thermal ignition theory, Mathl. Comput. Modelling 24(8), 1-10.

[2] Bebernes, J., Eberly, D., 1989. Mathematical problems from combustion theory.Springer-Verlag, New York.

[3] Bowes, P. C.,1984. Self-heating: Evaluating and Controlling the Hazard. Amsterdam:Elsevier.

[4] Frank Kamenetskii, D.A.,1969. Diffusion and Heat Transfer in chemical kinetics. Plenum Press, New York.

[5] Gbolagade, W. and Makinde, O. D.,2005. Thermal ignition in a reactive variable Viscosity Poiseuille flow. Journal of the Nigerian Association of Mathematical Physics.Vol. 9.573-578.

[6] Guttamann, A. J.,1989. Asymptotic analysis of power – series expansions, Phase Transitions and Critical Phenomena,C. Domb and J. K. Lebowitz,eds.Academic Press, New York, 1-234.

[7] Makinde, O. D. ,2004.Exothermic explosions in a Slab: A case study of series summation technique. International Communications in Heat and Mass Transfer, Vol.31,No.8,1227-1231.

[8] Makinde, O. D.,2005. Strong Exothermic explosions in a cylindrical pipe:A case Study of series summation technique. Mechanics Research Communications Vol.32,191-195.

[9] Makinde, O. D., Osulusi, E.,2005. Second lae analysis of Laminar flow in a Channel filled with saturated porous media. Entropy, Vol.7, No.2, 148-160.

[10] Makinde, O.D. ,2007. On steady flow of reactive variable viscosity fluid in A cylindrical pipe with an isothermal wall International journal of Numerical methods for Heat and Fluid flow.Vol.17, No.2,187-194.

[11] Olanrewaju, P. O., Ayeni, R. O., Ajala, A. O., Adebimpe, O. and Ajayi, A. O., 2006. On the existence and uniqueness result for a two-step reactive-diffusive Equation with variable pre-exponential factor. Vol.10,403-408.

[12] Olanrewaju,P.O., Ademola,A.K. and Hammed,O. S. (accepted). Research journal of applied sciences (RJAS).

[13] Olanrewaju,P.O.,2007.On the influence of Frank-Kamenetskii parameter and Numerical exponents on temperature field of two-step Arrhenius reactions. Journal of Engineering and Applied Sciences 2(7):1144-1146.