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# Application of mathematical models to educational planning: A review

***Virtue U. Ekhosuehi and †Augustine A. Osagiede**
**Department of Mathematics, University of Benin,**
**Benin City, Nigeria.**

***Abstract***

***This paper offers critical evaluation of previous works relevant to educational planning, school choice modelling and mode choice of student travel to school. In the review, the practical relevance of existing models is reported as well as the roles played by exploratory and normative models. The purpose of this paper is to give educational planners a realistic appraisal of the previous and current practice in educational planning, school choice modelling and mode choice of student travel and to provide pointers to areas where further research are needed.***

***Keywords*:** educational planning; log-normal model; manpower planning; mathematical model; multinomial logit model.

## 1.0 Introduction

Every society makes effort to identify the goods and services needed by its citizenry and develops the strategies to provide them. The educational system is one of the major strategic instruments for providing these needs. Higher education has the impetus to produce high level manpower needed to turn the economy around.

The planning and implementation of higher education take route from the identified jobs and functions to be performed in the society. This is sequel to the ability of people in the society to acquire their needs and requirements according to the desired quantities and qualities. Educational planning is a major factor in pursuing national development. Osezuah (1998 [32]) had earlier identified two basic factors influencing educational planning and implementation in Nigeria as manpower resource requirement and political influence.

Jhingan (2003 [16]) reported that "underdeveloped countries in their enthusiasm to spread higher education have been opening too many universities without trying to improve the standard of education. No restrictions are placed on higher education with the result that the proportion of failures at the higher secondary and university levels is very high. Mass failures and the general lowering of academic standards tend to lower the efficiency of undergraduates, and graduate employed both in the private and the public sectors do not promise well for the formation of a dynamic leadership for economic development. This leads to wastage of human resources".

Moreover, there has been little manpower planning in such economies, and no efforts are made to match the demand and supply of different types of critical skills in underdeveloped countries. Consequent upon this, few countries can go on absorbing poorly trained university graduates at a faster rate than their general economic growth. Sooner or later, with the present trend of educational expansion, many developing countries are likely to contend with the problems of discontentment and frustration, and that of graduate unemployment. Considering the high cost of education, the educated unemployed individuals are huge waste of human and materials resources.

Osagiede and Ekhosuehi (2007a [28]) had earlier developed a model which captured the manpower needs and human capital formation by the educational sector and apprentice training. The model can be used to match the demand and supply of different types of critical skills in developing counties.

***Corresponding author : e-mail: myvectorspace2000@yahoo.com,**

†e-mail: austaide2006@yahoo.com, Tel.: +234 803 381 4706

In brief, the success of educational planning is articulated in the job performance capacity and in providing the relevant needs of the society.

## 2.0 Manpower planning models

Models are simplified representation of real life systems. Basically, models are developed for the following reasons:

(a) to study, examine, and explain the working of a system;
(b) for forecasting and/or predicting future levels of the system; and
(c) to control the level and rate of change of the system.

The essential stages of model development are found in Mihram (1972 [21]). Edwards (1983 [8]) identified the features of a good model. Purkiss (1981 [34]) classified manpower planning models into two categories, namely: exploratory models and normative models.

### 2.1 Exploratory models

Exploratory models give insight into the way a system works and how it would respond to different stimuli. These models are also referred to as descriptive models because they are constructed to imitate the behaviour of the organization and using statistical techniques such as those based on Markov and renewal theory. Exploratory models can be sub-divided (Purkiss, 1981 [34]) to distinguish those which:

(a) do little more than display data, but in such a way that the analyst can identify more easily patterns in the data, quickly model and compare alternatives, and/or diagnose incipient problems in the organization;
(b) track and analyse aggregate movements of groups of people; and
(c) model progress of the individual in the organization.

Deterministic aggregate models form part of exploratory models. In these models, flows such as promotions or transfers between states in a manpower system and between it and the labour market outside may be one of two kinds. A ‘pull’ flow which is generated by the need to fill vacancies (e.g. promotion to meet an increase in the need for managers, relocation to fill a vacancy) and a ‘push’ flow which is as a result of pressure at the source of the flow rather than at its destination (e.g. automatic promotion on completing training).

### 2.2 Normative models

Normative models prescribe policies to follow. Purkiss (1981 [34]) reported that normative models are more efficient in situations where goals can be identified and optimality specified in terms of some objective function. Such models include dynamic programming, linear programming, goal programming and probabilistic programming models. The description of these models and their application to decision making are found in Al-Harbi (2000 [2]), Li and Li (2000 [17]) Hillier and Lieberman (2005 [14]), Price (1978 [33]), Rao (1990 [36]) and Taha (2002 [38]).

## 3.0 The place of models in educational planning

The use of mathematical models for educational planning has gained prominence in recent times as a means to better quantitative planning in education. This section focuses on the models constructed by previous authors for the educational system.

### 3.1 Wastage Analysis

Adeyemi (1998 [1]) analysed the wastage rate of primary schools in a local government area in Nigeria. The analysis was based on the projection rate given as;

$$K_g^t = \frac{E_g^{t+1}}{E_g^t} \; x \; 100 \qquad (3.1)$$

where $K^t_g$ is progression rate in grade g, year t; $E_g^{t+1}$ is enrolment in grade ‘g’ in year ‘t + 1’; and g = 1, 2, …, n, depending on the level of education. Adeyemi (1998 [1]) attributed wastage in schools to the inadequacies of school production variables such as: the learning environment, teacher-pupil ratio, teacher motivation, school organizational climate and the pattern of system’s goal implementation.

Chu and Lin (1994 [6]) applied log-normal model to estimate the wastage rate of staff in a tertiary institution. The study was based on cohort analysis (i.e. the wastage pattern for entrants into the system in the same period of time) for the teaching staff. The model for the estimation of wastage rate W with respect to length of service t is

$$W(t) = \frac{1}{\sigma t \sqrt{2\pi}} \exp\left\{ -\frac{1}{2}\left(\frac{Int - w}{\sigma}\right)^2 \right\}, \ t \geq 0 \qquad (3.2)$$

where $W(t)$ is the wastage rate when the length of service is $t$, $w$ is the mean of Int and $\sigma$ is the standard deviation of Int. The cumulative wastage rate from the start of service to time t is given by

$$\int_0^t W(\xi)\, d\xi = \phi \left(\frac{Int - w}{\sigma}\right) \qquad (3.3)$$

where $\phi(\bullet)$ stands for the cumulative distribution function of a standard normal random variable.

Of note is that the model in Chu and Lin (1994 [6]) dealt with wastage and retention rate only for a cohort for the purpose of long-term planning. In the works of Adeyemi (1998 [1]) and Chu and Lin (1994 [6]), performance incentives and condition of the learning environment, which are factors that affect wastage and retention, were not taken into consideration.

Feuer and Schinnar (1984 [13]) had earlier examined the impact of changes in grade size targets, hiring policy and attrition rates on promotion opportunities for staff in a large university. This work also did not incorporate the effects of performance incentives and condition of the learning environment.

### 3.2 Determination of school fees

The need to understand how private schools survive as government does not give out grants or subvention to them has motivated researchers to study educational financing. Olubor (1999 [24]) posited that the government spends less per student in public secondary schools, while the proprietors of private schools spend more per student. On the contrary, some people advocate that the government is spending more because of salary of staff paid with less efficiency of output. This opinion is strengthened by one of the findings of Awopegba (2002 [4]) that more funding of the educational sector in less developed countries does not necessarily imply better standards and better quality of education due to inadequate allocation, misappropriation and internal inefficiencies in the sector. Considering the survival of private schools, Olubor (1999 [24]) reported that they (private schools) are able to cope with school fees that are being charged.

Osagiede and Ekhosuehi (2007b [29]) developed a mathematical model for the determination of school fees in private schools. The model is given as:

$$f_i = \sum_{j=1}^{k} \left( \alpha_{ij} \ w_j + w_j^{'} \sum_{i=1}^{m} \alpha_{ij} \ E(n_i \ (t)) \right) \qquad (3.4)$$

In equation (2.4), $f_i$ denotes the school fee charged per student in class $i; E(n_i \ (t))$ is the expected number of students in session $t$; $w_j$ is the average cost of input $j$; $w_j$ is the derivative of $w_j$; and $\alpha_{ij}$ is the technical coefficient.

The limitation in the use of the model in equation (3.4) is that $w_j$ is assumed to be continuous, while in practice it is a discrete variable. There is need therefore for a refinement of the model in Osagiede and Ekhosuehi (2007b [29]).

### 3.3 Enrolment projection process

Mathematical models have been developed to analyse student's population in the educational system. Omosigho (2000 [26]) developed unlimited growth model to examine the implication of the National Universities Commission's policy on students' enrolment. The model is given as

$$P(t) = P(t-1) + I\ (0)\ [1 - \exp(-\alpha k)] \exp\ [\alpha\ (t-1)] \qquad (3.5)$$

where $P(t)$ is the total population of students at epoch $t$, $I(0)$ is the initial population of the intake, α is the growth rate, t is the epoch in time and k is the duration of the degree programme at epoch t.

The theory of Markov chain (Coleman, 1974 [7]; Feller, 1968 [12]; and Srinivasan and Mehata, 1978 [37]) employed in manpower planning (McClean, 1991 [18]; McClean and Gribbin, 1987 [19]; McClean et al, 1992 [20]; Raghavendra, 1991 [35]; and Zanakis and Maret, 1980 [41]) is now widely used in school enrolment projection process. Uche (2000 [39]) considered the use of Markov models for estimating the future enrolment level for schools in developing countries. The problem in the work of Uche (2000 [39]) is that of finding a suitable estimator for the rate of new intake into the educational system.

Osagiede and Omosigho (2004 [31]) attempted a solution to the method of estimating the number of new intake into the first grade of the educational system using spreadsheet under scanty data. The use of spreadsheet system to build a simple Markovian manpower planning model had earlier been described in Anthony and Wilson (1990). The snag in the method of Osagiede and Omosigho (2004 [31]) is the arbitrary value for the constant rate of increase in the number of new intake.

Osagiede and Omosigho (2002 [30]) proposed the use of non-homogeneous Markov chain to model the educational system. The reason for the proposal was that school enrolment varies from time to time. In estimating the transition probabilities for the homogeneous Markov chain, Osagiede and Omosigho (2002 [30]) applied the method of Adeyemi (1998 [1]). The method of Adeyemi (1998 [1]) is defective in that there is the possibility of getting probability estimates greater than one. This anomaly is a drawback in the use of the method of Osagiede and Omosigho (2002 [30]).

In Osagiede and Ekhosuehi (2006 [27]), the limitations in the works of previous authors were rectified. Osagiede and Ekhosuehi (2006 [27]) derived an estimator for the constant rate of increase in new intake as

$$\ln(1+\beta_i)=\frac{12\sum_{t=1}^{n} t\ln N_t^i - 6(n+1)\ln\prod_{t=1}^{n} N_t^i}{n(n^2-1)} \tag{3.6}$$

where $\beta_i$ is the rate of increase in new entrants into grade $i$; and $N_t^i$ is the number of new intake into level $i$, year $t$, $t = 1,2, \ldots, n.$

Equation (2.6) removes the arbitrary choice of the constant rate of increase in Osagiede and Omosigho (2004 [31]). The transition probabilities of the system were obtained via the maximum likelihood method. Thus, Osagiede and Ekhosuehi (2006 [27]) derived the projection matrix as

$$Q^T = \begin{bmatrix} (P_{10})(P_{01}) & (P_{20})(P_{01}) & (P_{30})(P_{01}) & (P_{40})(P_{01}) & (P_{50})(P_{01}) & (P_{60})(P_{01}) \\ (P_{10})(P_{02})+P_{12} & (P_{20})(P_{02}) & (P_{30})(P_{02}) & (P_{40})(P_{02}) & (P_{50})(P_{02}) & (P_{60})(P_{02}) \\ 0 & P_{23} & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{34} & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{56} & P_{66} \end{bmatrix} \text{w}$$

here $P_{0j}$ is the transition probability of a student in stage i moving to stage j and

$$\sum_{j=1}^{6}\left[P_{ij}+(P_{i0})(P_{0j})\right]=1 \tag{3.7}$$

Using the equation of projection

$$(\hat{n}_i\ (t)) = (Q^T)^t\ \ (\bar{n}_i\ \ (0)) + \sum_{c=1}^{t}(Q^T)^{t-c}\ \ \Delta^*\ \ N_{(c)}^j \tag{3.8}$$

Ekhosuehi and Osagiede (2006 [9]) applied the methods of Osagiede and Ekhosuehi (2006 [27]) to a course of study in a university in Nigeria. In equation (3.8), $(\hat{n}_i\ (t))$ is a 6 x 1 vector showing the estimated number of students in level $i$ at the beginning of session $t$; $(\bar{n}_i\,(0))$ is a 6 x 1 vector of enrolment levels at the base year; and $\Delta^*\ N_{(c)}^j$ is the growth size for level *j*.

It is worthy of note that the models of Osagiede and Ekhosuehi (2006 [27]) and Osagiede and Omosigho (2002 [30]) apply only to a particular educational system. In these works, the assumption of no demotion in the system is not generally practice as the possibilities of suspension and rustication as penalties to indiscipline exist in a typical educational system. For this reason, there is need to develop a generic educational planning model.

It is perhaps appropriate to mention here the worthy efforts of researchers in educational planning and the models they have developed to give a fair idea of what is involved in school enrolment projection. However, they neglect a critical aspect: the influence of the household in school choice and the subsequent enrolment. Consequent upon this, school choice modelling becomes of imperative concern.

## 3.4 School choice modelling and mode choice of student travel

### 3.4.1 School choice modelling

The term 'school choice' is seen as programmes which enable the household to choose a school among all existing schools for their children. The goals of school choice are to increase the quality and quantity of skills, knowledge and attitude of the child if he/she attends a particular school, and to create competition among schools, thereby giving schools an incentive to perform better. The direct contributions to school choice modelling are found in Belfield (2004 [5]). Belfield (2004 [5]) generalized Neal's choice model. Neal as cited by Belfield (2004) specified a utility function for household i where;

$$U_i = U\ (Y_i,\ EC_i,\ M_i) \tag{3.9}$$

*Y* denotes the educational outcomes from schooling; EC is the unobserved consumption goods from schooling (e.g. religiosity, dutifulness to parents); and *M* is a composite commodity with a price normalized to one. Belfield (2004 [5]) generalized the model in equation (3.9) to include public schooling, private – independent schooling, private-religious schooling and home-schooling. The educational outcomes were determined across each of the choice as;

$$Y_{ip} = X_i\,\beta_p + V_i \tag{3.10}$$

$$Y_{id} = X_i\,\beta_d + \in_{id} + Y_d + V_i \tag{3.11}$$

$$Y_{ir} = X_i\,\beta_r + \in_{ir} + Y_r + V_i \tag{3.12}$$

$$Y_{ih} = X_i\,\beta_h + \in_{ih} + Y_h + V_i \tag{3.13}$$

where *p* is public schooling, d is private – independent schooling, r is private – religious schooling, and h is home-schooling. The vector X denotes input and control variables, $Y_j$ parameters represent the mean outcome effect for school type j relative to public schooling, $\in ij$ parameters identify the match between household i and the selected school type j and the $V_i$ term is a household effect (error term). Belfield (2004 [5]) therefore estimated a multinomial logit model of the form:

$$P_r\ (\text{choice } j = 1,\ldots, 4) = f\ (\text{household, child, mother/father, community}) \tag{3.14}$$

From the extensive review of literature by Belfield (2004 [5]), it is clear that the problem associated with school choice became pronounced in 1994. A significant contribution of the work of Belfield (2004 [5]) is that it gives an insight into the way choices are made among school types and how such choices would respond to different stimuli. However, the work fails to give a clear-cut approach to enable the household choose a school in a more efficient manner. In addition, no attempt was made to estimate the maximum benefits the household is expected to have if the child attends a particular school and to determine how much input components the school should provide to capture the taste and preferences of the household. More so, the work of Belfield (2004 [5]) is characterized by statistical sophistication. This makes the model difficult for it to be used by the lay man. Edwards (1983 [8]) had earlier reported that the most common opinion by respondents in a survey was that there is at present no need for any more sophisticated models than those already described in literature. It is appropriate to mention that one of the limitations to the use of multinomial logit models (MLMs) in developing countries is the dearth of the statistical packages and the difficulty in accessing the available ones. These are therefore drawbacks to the estimation of the model in Belfield (2004 [5]) in developing countries.

### 3.4.2 Mode choice of student travel

Of recent, research into the relationship between mode of travel to school and the full range of factors that might affect mode choice became that of great concern. Ewing et al (2004 [10]) is credited for being the first to examine this relationship.

From the work of Ewing et al (2004 [10]), it is reported that children are more likely to walk or bike to small schools in walkable neighbourhoods than to large schools in remote locations. Ewing et al (2004) employed the multinomial model to examine school mode choice. The model is of the form:

$$P_k = \frac{\exp(U_k)}{\sum_{IEK} \exp(U_I)} \qquad (3.15)$$

where $P_k$ is the probability of choosing mode K for a school trip and $U_k$ is the utility function for mode *K* defined as:

$$U_k = \alpha_k + \beta T_k^{ij} + \gamma SE^m + \theta SC^n + \delta BE^i + wBE^j + \in_k \qquad (3.16)$$

In equation (3.16), $\alpha_k =$ vector of constants; $T_k^{ij}$ and β = trip characteristics and corresponding parameter vectors for trips from *i* to *j* by mode *k*, including travel time; $SE^m$ and γ = socioeconomic characteristics and corresponding parameter vectors for a student from household m characteristics such as income and automobile ownership; $SC^n$ and θ = school characteristics such as enrolment and corresponding parameter vectors for school *n*; $BE^i$ δ = built environmental characteristics and corresponding parameter vectors for origin *i* with *i* being a neighbourhood, census tract, traffic analysis zone (TAZ), or other small area (the vector may include measures of density, land use mix, walking quality and site design); $BE^j$ and *w* = built environmental characteristics and corresponding parameter vectors for destination *j;* and $\in_k$ = an extreme value error specific to mode *k*.

Ewing et al (2004 [10]) found that distance from home to school and sidewalk coverage were significant in determining the school mode choice, while neighbourhood population density, street tree coverage in the vicinity of school, age of schools and student enrolment were not significant. Furthermore, walking and biking were less likely when a household had more licensed drivers to provide rides and in a poor walking environment.

Since school mode choice modelling is a novel research area, more investigation into the factors affecting school mode choice is required particularly in developing counties where the pedestrian routes are not well developed.

## 4.0 The cause-effect scenario in the educational system

From the literature reviewed so far a network of causal relationship can be drawn to give a better understanding of the educational system. In doing this, exogenous factors such as the population of school age children, political influence and degree of uncertainty of educational planning models are taken into consideration. The information about the effect of each factor in the cause-effect diagram in fig. 2 below is relayed by '+' and '-' signs. The '+' sign indicates that the factor at the tail of the arrow and the item at the head of the arrow change in the same direction, while the '-' sign indicates that the item at the head of he arrow change in the opposite direction. The schematic representation of the cause-effect scenario for the educational system is

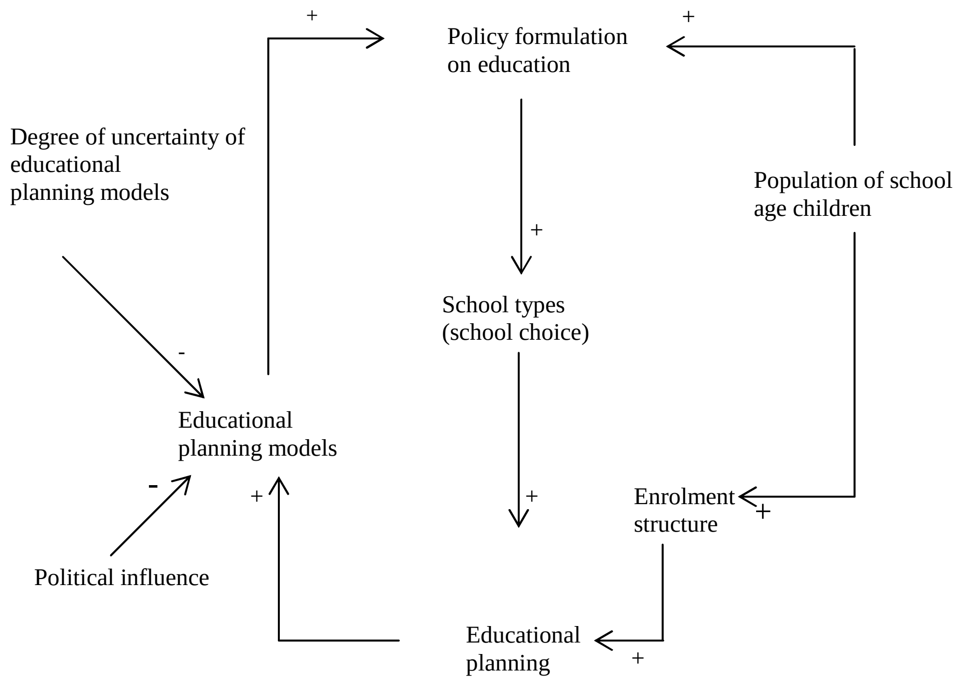


**Figure 4.1** Cause-Effect Scenario in the Educational System

Interestingly, considerable works abound in literature on population of school age children (Nwagwu, 1999 [21]; Omosigho, 2000 [25]), school choice (Belfield, 2004 [5]), enrolment structure (Adeyemi, 1998 [1]), educational planning (Fadipe, 1998 [11]; Osezuah, 1998 [31]), educational planning models (Osagiede and Ekhosuehi, 2006 [26]; Osagiede and Omosigho, 2002 [29]; Uche, 2000 [38]), political influence (Osezuah, 1998 [31]) and policy formulation on education (Imarhiagbe, 1998 [15]; and Olaniyi, 1998 [22]). However, no article was found directly on the uncertainty of educational planning models. This implies that the uncertainty of educational planning models is a grey area that demands further study.

## 5.0 Conclusion

In this paper, outstanding works and future direction of research are presented. A common feature of the models developed by previous authors is that the models operate on the basis of aggregation rather than on specific individuals. The models are also descriptive and they are mathematically and statistically sophisticated. Of note is that there is no best model. Each model is appropriate to a given circumstance. Moreover, none of the works by previous authors in the field of educational planning accounts for the heterogeneity of wastage and enrolment. The methods discussed in Ugwuowo and McClean (2000 [39]) can serve as reference tools in modelling heterogeneity in wastage and enrolment projection for the educational system. One of the contributions of this study is the development of a network of causal relationship for the educational system. In brief, the materials reviewed suggest the need to measure uncertainty of educational planning models and to develop a simple model to assist the household in choosing a school for the school age child among all potentially competitive schools in an efficient manner.

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