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A measure of confidence for examination process

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Abstract

The zeal of candidates to pass examinations at all costs in order to progress in their educational pursuit or gain undue advantage over others has resulted to examination malpractice. To curb this, new strategies for examination have evolved over the years. In this study, we derive mathematical models to measure the level of confidence on the effectiveness of new strategies that are being introduced to remedy examination malpractice. The model is extended to the case where the previous strategies are claimed to be effective. The methods presented in this paper have the advantage of summarizing in one figure the confidence level on examination process.

***Keywords*:** beta function; confidence level; gamma function; examination; estimate.

1.0 Introduction

Education is the cornerstone for rapid economic growth and development. Awopegba (2002 [2]) examined the contribution of education to national growth and development. Through education, higher level manpower needed to turn the economy around is produced. Out-turn of higher level manpower from educational institutions is facilitated by adequate educational planning. Osezuah (1998 [13]) identified factors influencing educational planning an implementation in Nigeria.

In recent times, models are being extensively used for analyzing the educational systems. Uche (2000 [15]) proposed the use of Markov chain model for estimating future enrolment in schools in developing countries. Adeyemi (1998 [1]) analysed the wastage rate in, and determined the internal efficiency of, public primary schools in a local government area in Nigeria. Omosigho (2000 [9]) modelled students' population in Nigerian universities. The methods for estimating new intake into the first grade of an educational system were proposed by Osagiede and Omosigho (2004 [12]). Osagiede and Ekhosuehi (2006 [10]) developed new and better method for estimating the growth rate in the educational system while the new methods were applied by Ekhosuehi and Osagiede (2006 [10]). Osagiede and Ekhosuehi (2007 [11]) employed mathematical model to determine the optimum fees to be charged in Nigerian private schools.

In spite of the elaborate research in educational planning and the emerging policies for the smooth functioning of the system, and the massive out-turn from educational institutions in Nigeria the educational sector is yet to transform all spheres of the Nigerian economy. Its failure in this regard can be attributed to the systemic cankerworm referred to as examination malpractice. Examination malpractice is any acts contrary to the rules and regulations guiding the successful conduct of examination (Imolorhe, 1998). Such acts encompass: cheating, stealing of question papers, personating, collusion with persons with the intent to cheat or secure unfair advantages for self or for another, disturbance at examination, failure to obey lawful orders of supervisors, invigilators, or agents of the examination body, forgery of result, breach of duty, to mention but a few.

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Imolorhe (1998 [7]) had earlier reported that demand for paper qualification, rapid population growth, rising cost of education fees, accommodation, unpreparedness on the part of the school, absenteeism of teachers and students' unpreparedness are the causes of examination malpractice in Nigeria. Daniel (1998 [3]) in an investigation into the factors that motivate students to indulge in examination malpractice found that the strong emphasis on the use and value of certificate stands out as a strong compelling factor to cheating on examination. Undoubtedly, examination malpractice has deleterious effects on the educational system and the nation at large. Examination malpractice defames a nation's image as out-turn from educational institutions are valued as "half-baked graduates", and it leads to lack of qualitative value for certificates awarded in the international labour market. Additionally, cheats from the educational institution cannot contribute meaningfully to the development of the economy and worse still, they contribute to the further decay of academic standards when granted the privilege to teach. Teachers who connive with students to perpetuate examination malpractice abuse their office and set bad examples for the students.

With a view to curbing examination malpractice in Nigeria, penalties for cases of irregularities have been clearly stated in the regulations and syllabuses of the examination bodies such as that of JAMB (2004 [8]). The examination bodies in Nigeria include: Joint Admissions and Matriculation Board (JAMB), the West African Examinations Council (WAEC), National Business and Technical Examinations Board (NABTEB), National Examination Council (NECO), etc. The penalties for examination malpractice encapsulate: withheld, pending and cancellation of results, arrest and imprisonment of culprits, and de-recognition of centres involved in mass cheating. Discouragement of undue emphasis on paper qualification, reinforcement of desirable moral values, upgrade and regular payment of teachers salaries, provision of modern educational facilities and well spaced examination halls have been suggested in literature as remedies to examination malpractice in Nigeria (Daniel, 1998 [3]; Imolorhe, 1998 [7]). The need to discourage students from examination malpractice has led to the introduction of Post Universities Matriculation Examination (PUME) and Post Direct Entry examinations in some universities like the University of Benin, Benin City, Nigeria. Recently, the use of options for objective test and online registration for examinations were introduced by the examination bodies as new strategies to further reduce the chances of examination malpractice in Nigeria. But the problem is "To what extent do we rely on the examination process". In this study, we attempt to proffer solutions to the problem by deriving mathematical models to determine the level of confidence on the effectiveness of the new strategies to remedy examination malpractice. One of the models is made up of an incomplete beta function and it is extended to the case where the previous strategies are claimed to be effective.

2.0 The model

Let $N_j(t)$ be the number of candidates registered for examination j in period t, prior to the introduction of the new strategies, $t = 1,2,\ldots,T$. The subscript j is coded to capture any of the recognised examinations. Let $n_j(t)$ be the number of candidates caught for examination malpractice in examination j, period t, prior to the introduction of the new strategies. Let $p_j(t)$ denote the proportion of candidates caught for examination malpractice in examination j, period t, prior to the introduction of the new strategies. That is

$$p_j(t) = \frac{n_j(t)}{N_j(t)} \qquad (2.1)$$

The pooled estimate of $p_j(t)$ is given as

$$p_j = \frac{\sum_{t=1}^{T} n_j(t)}{\sum_{t=1}^{T} N_j(t)} \qquad (2.2)$$

Suppose that new strategies to curb examination malpractice are introduced in period $T+1$. Let θ_j represent the pooled estimate of proportion of candidates caught for examination malpractice after the new strategies are introduced. Then, the new strategies are considered effective if $\theta_j < p_j$. Suppose a random sample of n schools taking examination j, denoted by X_{ij}, $i = 1, 2, \Lambda, n,$ are selected. Then two possibilities arise, namely: either cases of examination malpractice are detected in school X_{ij}, or no case of examination malpractice is detected in school $X_{ij}\cdot$ Usually, when new strategies are introduced in a system, a target is set. Let the target in this regard be to reduce the cases of examination malpractice to a minimum, c. Thus, number of cases of examination malpractice in n schools under inspection can be $0, 1, 2, \Lambda, or\ c$ for the new strategies to be considered effective. Let Y_j represent the number of cases of examination malpractice detected in examination j in n schools under inspection i.e.

$$Y_j = \sum_{i=1}^{n} X_{ij} \tag{2.3}$$

Then, the effectiveness of the new strategies is indicated by $Y_j \le c\cdot$ In the light of the foregoing, we derive a model which incorporates an incomplete beta function, from the binomial probability function to measure the confidence level of the examination process. The model is given as

$$P\left(Y = 0, 1, 2, \Lambda, or\ c / \theta_j < p_j\right) = 1 - \frac{\Gamma(n+1)}{\Gamma(c+1)\Gamma(n-c)} \int_0^{\theta_j} t^c (1-t)^{n-c-1} dt \tag{2.4}$$

The integral in equation (2.4) is an incomplete beta function, and $\Gamma(\bullet)$ is the gamma function. Now, suppose we claim that the previous strategies were effective and that the new strategies are introduced to complement the previous ones. Then $p_j \to 0$. Since effectiveness of the new strategies requires that $\theta_j < p_j$, then $\theta_j \to 0 \quad as \quad p_j \to 0$. Using a large number of schools for the study, the confidence point estimate for the effectiveness of the new strategies can be measured by taking the limit as $n \to \infty$ of the system in equation (2.4), and setting $n\theta_j = \alpha_j$, where α_j is a constant. We therefore obtain

$$\lim_{n\to\infty} P\left(Y = 0, 1, 2, \Lambda, or\ c / \theta_j < p_j\right) = 1 - \int_0^{\alpha_j} \frac{1}{2^{c+1}\Gamma(c+1)} t^c e^{-t/2} dt \tag{2.5}$$

The integral in equation (2.5) is the area of the chi-square distribution from 0 to α_j with $2(c+1)$ degrees of freedom. We shall now prove the results in equations (2.4) and (2.5).

Proof

Consider the integral

$$\int_0^{\theta_j} t^c (1-t)^{n-c-1}\, dt \tag{2.6}$$

By method of integration by parts, the integral in equation (2.6) is evaluated as

$$\int_0^{\theta_j} t^c (1-t)^{n-c-1}\, dt = -\frac{\Gamma(c+1)}{(n-c)(n-(c-1))(n-(c-2))..n} \left(\sum_{y=0}^{c} \binom{n}{y} \theta_j^y (1-\theta_j)^{n-y} - 1 \right) \tag{2.7}$$

Further simplification of the systems in equation (2.7) yields

$$1 - \frac{\Gamma(n+1)}{\Gamma(c+1)\Gamma(n-c)} \int_0^{\theta_j} t^c (1-t)^{n-c-1}\, dt = \sum_{y=0}^{c} \binom{n}{y} \theta_j^y (1-\theta_j)^{n-y}$$

$$= P\left(Y = 0, 1, 2, \Lambda, or\ c / \theta_j < p_j\right), \tag{2.8}$$

which is the result in equation (2.4). Setting $n\theta_j = \theta_j$ and taking limit as $n \to \infty$, equation (2.8) becomes

$$\lim_{n\to\infty} P\left(Y=0,\ 1,\ 2,\Lambda\ , or\ \ c/\theta_j < p_j\right) = \lim_{n\to\infty} \sum_{y=0}^{c} \binom{n}{y} \left(\frac{\alpha_j}{n}\right)^y \left(1-\frac{\alpha_j}{n}\right)^{n-y} \qquad (2.9)$$

After some simplifications and applying Stirling's approximation (Stephenson, 1973 [14]) given as

$$n! \ = \ n^n e^{-n} \sqrt{(2\pi n)} \ \text{, when } n \text{ is large} \qquad (2.10)$$

equation (2.9) becomes

$$\lim_{n\to\infty} P\left(Y=0,\ 1,\ 2,\Lambda\ ,\ or\ \ c/\theta_j < p_j\right) = \ e^{-\alpha_j} \ + \ \alpha_j e^{-\alpha_j} + \frac{\alpha_j^2\ e^{-\alpha_j}}{2!} \ +\Lambda + \frac{\alpha_j^c\ e^{-\alpha_j}}{c!} \qquad (2.11)$$

Equation (2.11) is equivalent to writing

$$\lim_{n\to\infty} P\left(Y=\ 0,\ 1,\ 2,\ \Lambda\ ,\ or\ \ c/\theta_j < p_j\right) \ = \ 1-\frac{1}{\Gamma(c+1)}\int_0^{\alpha_j} e^{-t}\ \ t^c \ \ dt \qquad (2.12)$$

Setting $t=\frac{t}{2}$ so that $dt\ =\frac{1}{2}\ dt$, we have

$$\lim_{n\to\infty} P\left(Y\ =0,\ 1,\ 2,\Lambda\ ,\ or\ \ c/\theta_j\ <\ p_j\right) \ = \ 1-\int_0^{\alpha_j} \frac{1}{2^{c+1}\Gamma(c+1)}\ t^c e^{-\frac{1}{2}}\ dt \qquad (2.13)$$

which is the result in equation (2.5). This completes the proof.

3.0 Discussion

The system in equations (2.4) and (2.5) give a quantitative measure of the confidence point estimate for the effectiveness of new strategies to curb examination malpractice in examination *j*. To use the models, enrolment figures for examination *j*. before and after the introduction of the new strategies, and the number of cheats caught for the same period should be given. A stipulated target, c, for the new strategies, and the sample size, *n*, must be stated. Minimum cases of examination malpractice are allowable because an examination devoid of examination malpractice may be too expensive to conduct. However, the minimum allowable cases of examination malpractice, c, must not be too large, otherwise the new strategies are rejected. It is important for the sample size to be sufficiently large so much so that it is representative of the number of schools taking the particular examination *j*. The *n* schools are selected at random so that personal bias of the researcher does not affect the result obtained. The value of $n\theta_j$ is assumed fixed because if $n\theta_j \to\infty$ as *n* becomes very large and $\theta_j \to 0$ would imply infinitely many school taking examination *j*. When the new strategies completely eradicate examination malpractice, $\theta_j \to 0$. The measure of its effectiveness from equation (2.4) gives

$$P\left(Y=0,\ 1,\ 2,\Lambda\ ,\ or\, c/\theta_j =0\right) = 1-\frac{\Gamma(n+1)}{\Gamma(c+1)\ \Gamma(n-c)}\ \int_0^{\theta_j=0} t^c\left(1-t\right)^{n-c-1}\ dt = \ 1.$$

Similarly, equation (2.5) for $\theta_j\ =0$ gives

$$\lim_{n\to\infty} P\left(Y=\ 0,\ 1,\ 2,\ \Lambda\ ,\ or\ \ c/\alpha_j\ =0\right) = 1-\int_0^{\alpha_j=0} \frac{1}{2^{c+1}\Gamma(c+1)}\ t^c e^{-\frac{1}{2}}\ dt\ =\ 1.$$

The results imply that we have no reason to reject the effectiveness of the new strategies i.e. we are 100% confident that the new strategies are effective and that the examination process is reliable. Conversely, if the new strategies fail to remedy examination malpractice, then $\theta_j\ \to 1$. From equation (2.4), we have

$$\lim_{\theta_j\to 1} \ P\left(Y=0,\ 1,\ 2,\Lambda\ ,\ or\ \ c/\theta_j\right) = \ 1-\frac{\Gamma(n+1)}{\Gamma(c+1)\,\Gamma(n-c)}\int_0^1 t^c\left(1-t\right)^{n-c-1}\ \ dt \ = 0 \ \text{because}$$

$$\int_0^1 t^c (1-t)^{n-c-1} \ dt = \frac{\Gamma(c+1)\ \Gamma(n-c)}{\Gamma(n+1)}$$

(Gupta, 1993 [6]).. Now, considering equation (2.5), $\alpha_j = n$ as $\theta_j = 1$. As $n \to \infty$, $\alpha_j \to \infty$ also. So, equation (2.5) becomes

$$\lim_{\alpha_j \to \infty} \lim_{n \to \infty} P\left(Y = 0, 1, 2, \Lambda\ , or\ c / \frac{\alpha_j}{n} \right) = 1 - \int_0^\infty \frac{1}{2^{c+1}\Gamma(c+1)}\ t^c e^{-t/2}\ dt = 1 - \frac{1}{2^{c+1}\Gamma(c+1)} \cdot 2^{c+1}\Gamma(c+1) = 0 \cdot 1$$

This means that no confidence is placed on the new strategies. Hence the examination process should be rejected. In either case therefore $P(Y = 0, 1, 2, \Lambda\ , or\ \ c/\theta_j < p_j)$ decreases as θ_j increases. For this reason, greater confidence is placed on the effectiveness of the new strategies if the value of $P(Y = 0, 1, 2, \Lambda\ , or\ \ c/\theta_j < p_j)$ is high. The models in equations (2.4) and (2.5) are easy to use as their values are found in statistical tables (see Devore, 1991 [4]).

4.0 Conclusion

In this paper, we have successfully modelled the effectiveness of introducing new strategies to curb examination malpractice and demonstrated rigorously how to obtain the confidence point estimate by probability measure (see equations 2.4 and 2.5). The model in equation (2.5) is an extension of that of equation (2.4) to capture the situation where the previous strategies are claimed to be effective. The methods presented in this paper have the advantage of summarizing in one figure the level of confidence on examination process. Further, this work utilizes incomplete beta function and chi-square function to estimate confidence level, rather than using confidence intervals. In all, we discuss the models in their raw form prior to its application in subsequent study.

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