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Mathematical application of time study model in a glass company

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Abstract

This paper presents a case study in the development and application of a time study model in a glass manufacturing plant. The organization engages in the production of different types of glass wares which are versatile and enigmatic. The motivation for this study was the need to intervene in frequent crises that normally arose between the employee-association and the management of the company regarding questions of productivity. The mathematical model was developed with the application of differential calculus to the elements of the production systems that have significant effect on the production output from the system. Our model incorporates some uncontrollable factors such as irregular supply of electricity, unavailability of raw materials as well as excessive machine breakdown due to old age. The study is however considered to be very beneficial to practicing managers in the industries and is therefore recommended for use.

Keywords: Time Study Model, Production System, Production Output, Uncontrollable Factors.

1.0 Introduction

Glass is an inorganic product of melting which has been cooled to a rigid state without crystallization, it is an amorphous solid usually formed by the solidification of a melt without crystallization. Melting is the sole large scale industrial production process of glass manufacturing and the company has realized that scientific approaches could be developed to aid dispute settlement between the employees' association of the company and management regarding issues of productivity.

Time study is one of the techniques used in solving productivity problems in such manufacturing companies. Time study is a technique of work measurement designed to establish the time for a qualified worker to carry out a specified job at a defined level of performance, (Oke, 2006 [7]). Research on time study incorporates a range of concerns, including its definition and management (Edo *et al.*, 2001 [4]; Worrall and Smith, 1985 [9]; Watson, 1988 [8]; Aft, 2000 [1]). Although research on work measurement has evolved in a scientific and rigorous fashion, based on early works of Gilbert and others, the quantitative mathematical modeling of production activities in terms of time study has not evolved in a similarly rigorous fashion (Barnes, 1980 [2]; Zandin, 2003 [10]; Doty, 1989 [3]; Karger and Bayha, 2003 [5]). In recent years, the manufacturing organization used as the case example in this work has realized that scientific approaches could be developed to aid dispute settlement between the employees' association of the company and management regarding issues of productivity. In order to achieve this, the company was motivated to approach a management consultant. This paper is an attempt to present the methodology used in solving productivity issues at the company, the Beta Glass PLC, Ughelli.

The manufacture of glass container is a continuous process running throughout the twenty four hours of the day. The production process is divided into various stages such as selection and weighing of raw materials, washing of raw materials, batch production, furnace operation, bottle making process, annealing

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and cold end operation. The main raw materials for the manufacture of glass wares include; silica sand (S_iO_2), soda ash (Na_2O), sodium carbonate (Na_2CO_3), limestone or calcium carbonate ($CaCO_3$). Silica is mainly required to form the glass while the soda ash and limestone are used as fluxes to lower the melting temperature of silica forming the glass (Odior, 2002 [6]).

2.0 Development of the time study model

The production of glass by company involves five major stages; washing and weighing of raw materials, batch production, furnace operation, bottle making process, annealing and cold end operation. The major activities were studied in order to have a background understanding of the problem, its formulation and solution. From the information obtained at the company floor, the production system is effective. It infers that no much losses or leakages in the production system. Thus, all the effort put into the production system would yield the desired results. The second type of information obtained from the production system is that the right calibre of production personnel is involved. A third information category is that there is a defined responsibility for each production worker. Thus, a production target is in place and could be monitored. The fourth information is that the machines are always available in a ready state. However, it is assumed that whenever a machine breaks down, it can always be repaired and restored in a negligible time frame.

The first mathematical expression for the model framework is as follows:

$$t = \sum_{i=1}^{n} t_i \tag{2.1}$$

where (t) represents the total time used in producing a unit of product. The variable (i) represents the various workstations of interests, (i.e. washing and weighing of raw materials, batch production, furnace operation, bottle making process, annealing and cold end operation,). With close observation of the various workstations, there are variations in the rate of working for both the individuals at the workstations and the machines doing the actual operation. Therefore, we introduce the rate of working for both the machines at the various workstations and the workers as differentials that are expressed mathematically. For instance, if machine i is represented as m_i where m_i may be m_1 for the machine that does the work such as washing and weighing of raw materials station, m_2 is the machine that does the work at batch production station, etc.). If the time taken by the 'in-process' product is time t, then mathematical expression becomes $\frac{dm_i}{dt}$. Also, if (w_i) represents the human worker at workstation (i), and this worker works for a period of time t units, then we can express the rate of working of this worker as $\frac{dw_i}{dt}$. Since in time study activities a provision of allowance is always very necessary, we now introduce a parameter 't_a' into the model. Therefore, the general mathematical expression for the production time t_i at each workstation is given as

$$t_i = \frac{dt}{dm_i} \cdot \frac{dt}{dw_i} f(x_i) + t_a \tag{2.2}$$

where x_i is a normalizing function which converts the expression into time units. Substituting Equation 2.2 into Equation 2.1 gives the following equation.

$$t = \sum_{i}^{n} \left(\frac{dt}{dm_i} \cdot \frac{dt}{dw_i} f(x_i) + t_a\right) \tag{2.3}$$

$$= \sum_{i}^{n} \left(\frac{dt}{dm_i} \cdot \frac{dt}{dw_i} \cdot f(x_i)\right) + \sum_{i}^{n} t_a \tag{2.4}$$

but $\sum_{i}^{n} t_a = nt_a$. Therefore

$$t = \sum_{i}^{n} \left(\frac{dt}{dm_i} \cdot \frac{dt}{dw_i} \cdot f(x_i)\right) + nt_a \tag{2.5}$$

We will assume that the rate at which machines are producing and the working rate of workers is constant.

Thus Equation 2.5 becomes; $(\frac{dt}{dm_i} \cdot \frac{dt}{dw_i})\sum_i^n f(x_i))$ $+ nt_a$. We generalize the model by taking $f(x_i)$ as

$f(x)$, $\frac{dt}{dm_i}$ as $\frac{dt}{dm}$ and $\frac{dt}{dw_i}$ as $\frac{dt}{dw}$. Thus,

$$t = \frac{dt}{dm} \cdot \frac{dt}{dw}\int_1^n f(x)dx + nt_a \tag{2.6}$$

Assuming that the total number of products produced is denoted by symbol (y), while T is the total time spent for all the products, Equation 6 above becomes,

$$T = yt = y(\frac{dt}{dm} \cdot \frac{dt}{dw}\int_1^n f(x)dx + nt_a) \tag{2.7}$$

Equation 2.7 is the general formula for the total time spent in producing y products.

2.1 Raw materials and electricity supply.

The issue of unavailability of raw materials and irregular electricity supply is to be considered and assuming that $f(x_i)$ is a function of these two parameters of indices such that we have $f(x_i)$ and $f(x, z)$. It should be noted that $f(x)$ and $f(x, z)$ could be any mathematical function that defines (x) and (x and z) respectively. If we assume the electricity supply index (x) is a linear function, the equation that describes it is displayed as $f(x) = ax + b$, where a and b are constants. Also, if the electricity supply index (x) is quadratic, the equation that describes its state is displayed as $f(x) = cx^2 + jx + k$, where c, j and k are constants. Electricity supply index (x) could also describe a state of exponential condition in which case the equation that described it is displayed as $f(x) = e^{px}$, where p is a constant.

If we take the raw materials availability index as (z) (it should be noted that the electricity supply index is taken as x), then relating the two indices under one function, gives us f(x, z) which describes a state of linear function where the equation is displayed as shown as $f(x, z) = qxz + r$, where q and r are constants and (x) and (z) as described above.

The case where $f(x, z)$ describes the state of quadratic function, the equation would be quadratic in (x, z) and is displayed as $f(x, z) = a^{/}(xz)^2 + b^{/}xz + c^{/}$, where $a^{/}$, $b^{/}$ and $c^{/}$ are constants

Therefore Equation 2.7 can now be expressed as follows:

$$T = yt = y\left(\frac{dt}{dm} \cdot \frac{dt}{dw}\int_1^n f(x,z)dxdz + \text{nta}\right) \tag{2.8}$$

This equation gives the real general formula for the total time spent in producing y products with n number of workstations.

To modify the mathematical model, let us consider a situation where the rates at which the machines and workers are operating are the same. Equation (2.2) is now modified as follows:

$$t_i = (\frac{dm_i}{dt})^{-1} \cdot \frac{dt}{dw_i} \cdot f(x_i) + t_a \tag{2.9}$$

Now, let $\frac{dm_i}{dt}$ be represented by

g(x), $\frac{dt}{dw_i}$ by $h(x)$ and $f(x_i)$ by $f(x)$,

where (x) could be any particular parameter. Then by substituting these values into Equation 2.6 gives. Thus,

$$t = \int_1^n \frac{f(x)}{g(x)h(x)}dx + \text{nt}_a \tag{2.10}$$

The total number of products produced now becomes

$$T = yt = y\left(\int_1^n \frac{f(x)}{g(x)h(x)}dx + \text{nta}\right) \tag{2.11}$$

However, for a situation where $f(x_i)$ is defined by $f(x, z)$, the equation becomes:

$$T = yt = y\left(\left(\int_1^a\int_1^n \frac{f(x)}{g(x)h(x)}\right)dxdz + nt_a\right) \tag{2.12}$$

3.0 Application of the mathematical model.

For the application of the model to our study the electricity unavailability index and the unavailability of raw materials are defined by functions $f(x)$ and $f(z)$ then $f(x_i)$ is given as a function of (x) and (z). And so,

$$(x_i) = f(x, z) \tag{3.1}$$

Assuming that the electricity supply index (x) obeys a linear function such as $2x + 5$, then the expression is now $f(x) = 2x + 5$, where 2 and 5 are constants. From the above equations, we know that (n) is the number of workstations while (t_a) is the time allowance. From the actual production observation, the mathematical model that fits the time problem in terms of number of machines is:

$$t = mx^3 + m^2x^2 + x \tag{3.2}$$

Differentiating Equation 3.2 gives:

$$\frac{dt}{dm} = x^3 + 2mx^2 \tag{3.3}$$

Also, the mathematical expression that represents time with respect to the number of workers is:

$$t = wx^3 + w^2x^2 + x \tag{3.4}$$

Differentiating Equation 3.4 gives:

$$\frac{dt}{dw} = x^3 + 2wx^2. \tag{3.5}$$

Note that (n) has been stated earlier as the number of workstations, and (t_a), the time allowance. If 12,000 products are produced by the company for 0.5 second per unit product, then $t_a =$ 12,000 x 0.5 seconds. Therefore $t_a = 6000$ seconds. From Equation 2.6, we have

$$t = t_i = \frac{dt}{dm}\cdot\frac{dt}{dw}\int_1^n f(x)dx + \text{nt}_a.$$

But $$\frac{dt}{dm} = x^3 + 2mx^2 \text{ and } \frac{dt}{dw} = x^3 + 2wx^2.$$

There are 6 workstations for the production processes, hence $n = 6$. and $t_a = 12{,}000$ seconds. The average period electricity fails in a day is 45 minutes, while the average daily working time is 9 hours.

Note that x is the ratio of the period when electricity fails in a day to that of the working hours for that same day. Thus,

$$x = \frac{45\,\text{minutes}}{9x60\,\text{minutes}} = \frac{45}{540} = 0.0833.$$

This gives an index value of 0.0833.

Note that the number of machines $m = 5$, number of workers $w = 120$. Then since $f(x) = 2x + 5$, we now evaluate the function by substituting into Equation 6 as follows:

$$t = \frac{dt}{dm}\cdot\frac{dt}{dw}\int_1^n f(2x+5)dx + nt_a \tag{3.6}$$

so $$t = \frac{dt}{dm}\cdot\frac{dt}{dw}(x^2 + 5x + k') + nt_a \tag{3.7}$$

were k' is the production constant. Note that at the start of production process, all the factors are zero since no product has been produced. This gives the production constant k' to be zero.

$$\therefore \quad t = \frac{dt}{dm}\cdot\frac{dt}{dw}(x^2 + 5) + nt_a \tag{3.8}$$

Now substituting the required values into the equation gives:

$$t = t_i = (x^3 + 2mx^2)(x^3 + 2wx^2)(x^2 + 5x) + nt_a \tag{3.9}$$

Therefore $t = \{(0.0833^3 + 2 \times 5 \times 0.0833^2)(0.0833^3 + 2 \times 120 \text{ x } 0.0833^2)(0.0833^2 + 5 \times 0.0833)\}$ $+ (6x \times 12000)$ seconds = 72,000.04936 seconds.

$\therefore$ t = 20.00001371 hours = 20 hours

Note that t_i = 0.5 second per unit product, therefore the total products produced in 20 hours

$$\frac{20\,hours}{0\cdot 5\ \text{sec}onds\ per unit\ \ product}$$

$$\frac{20\times 36000\text{sec}onds}{0\cdot 5\ \text{sec}onds\ per unit\ \ product} = 144{,}000 \text{ units of product.}$$

That is 144,000 units of product would be produced in 20 hours. In conclusion, we have therefore be able to apply a time study mathematical model in calculating the time required for operational activities in the production processes for the manufacture of glass products it is seen that 144,000 units of glass product could be produced in twenty hours.

4.0 Observations:

The impact of setting standards in the achievement of production targets in glass manufacturing company has not been given a thorough consideration until this current study. The company however realized that one of the approaches in achieving this aim is the application of time study models in the monitoring and control of employees on the production floor. It was observed that the current model is slightly different from previous models in the sense that it incorporates some uncontrollable factors such as irregular supply of electricity, unavailability of raw materials, excessive and frequent machine breakdown due to old age, etc. All of these factors have been considered to have a positive impact on the model.

5.0 Conclusion

The production of glass wares by the company studied has been thoroughly examined. It has been observed that the setting of standards in the achievement of production targets is very important and one of the techniques for achieving this aim is the application of time study models in the monitoring and control of employees on the production floor. The mathematical model was developed with the application of differential calculus to the elements of the production systems that have significant effect on the production output from the system.

In this paper therefore, the time study concept in the production process of glass wares by beta glass company, Ughelli was studied and it was discovered that though there is a wide variety of glasses ranging from hard, soft, ductile and brittle types of glasses, they all have the same process of production. About 84.5% of the raw materials used in glass production are obtained locally, while the remaining raw materials are imported.

The mathematical model was developed with the application of differential calculus to the elements of the production systems that have significant effect on the production output from the system. Our model incorporates some uncontrollable factors such as irregular supply of electricity, unavailability of raw materials as well as excessive machine breakdown due to old age. The study is however considered to be very beneficial to practicing managers in the industries and is therefore recommended for use.

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