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Optimality of MRPP policies for inventory problem with stochastic demand

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Abstract

Material requirements and production planning (MRPP) is absolutely an essential activity which holds some form of detail lists in any production industry. This paper addresses a discrete time inventory problem when the demand for finished products is a random variable. Policies that make material requirements and production planning optimal are presented. Data from a typical plastic industry are analysed to evaluate the performance of MRPP, and quick response time is recommended.

Keywords: Inventory Models, Stochastic Demand, Material Requirements, Cost Control, Probability.

1.0 Introduction

Under the conditions of uncertain demand, the policy decision variables become "the determination of an optimal reorder point and order quantity" to minimize inventory related costs. The stochastic nature of demand can also produce the possibility of stockout of materials or surpluses which can also influence inventory related costs (Tito et al, 1999 [7]; Inderfurth, 1997) Material requirements and production planning is therefore a consensus for material management and production schedule system in a dependent demand environment. It is easy to produce anything as long as we have the materials and profitable if we have the materials at the right time (see Ehrhardt, 1997 [1]; Gotzel and Inderfurth, 2001 [4]). For optimal planning of material requirements and productions schedule when the decision environment is uncertain, this paper presents the policies that will sustain the system. For more details on material planning and production control, see Khang and Fujiwara (2001 [6]), and Feischmann and Kuik (1998).

2.0 The models

Let K denote the Kth period from the time of production. Therefore, $K \in$ Integer $(0,E)$, E being the probable life span of a material in the system (to be determined).

Let $t \in$ Integer $(0, \infty)$, denote the time, $t = 0$ being the time all items (materials) are new in the system

y = quantity of products produce

D = demand of product

h = holding cost per unit remaining at the end of Kth period

s = shortage cost per unit of unsatisfied demand

$f_D(z)$ = probability density function of D

$P_D(d)$ = probability distribution of D

w = initial stock level of products

$F(a)$ = cumulative distribution of D

To formulate a mathematical model for inventory problem with stochastic demand, we first assume that $P_n(d)$ is known for all values of $d(P[D = d])$ from past records.

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Let

$$f_n(z) = \begin{cases} \frac{1}{d}\lambda^{-z/d}, \text{ for } z \geq 0 \\ 0, \quad \text{otherwise} \end{cases} \tag{2.1}$$

and

$$F(a) = \int_o^a f_\Delta(Z)dz \tag{2.2}$$

for $w = 0$. Let a_1 and a_2 be any positive constants, also define $g(z, y)$ as

$$g(z,y) = \begin{cases} a_1(y-z), & \text{for} \quad y > z \\ a_2(z-y), & \text{for} \leq z \end{cases} \tag{2.3}$$

and

$$G(y) = \int_o^\infty g(z,y) f_D(z)dz + ay \tag{2.4}$$

where $a > 0$. then (2.4) is minimized at $y = y^*$ where y^* is the solution to

$$\phi(y^*) = \frac{a_2 - a}{a_2 + a_1} \tag{2.5}$$

See, for example, Hillier and Lieberman (1995 [5]).

Now, when $w > 0$, the decision to be made is about inventory replenishment by producing additional units so that $y = w + (y - w)$. Then, the objective function is given as

$$\underset{y \geq w}{Min}\, G(y) \tag{2.6}$$

where, $$G(y) = a(y-w)\int_y^\infty [s(z-y) + \gamma(z-y)]f_D(z)dz + \int_o^y [h(y-z) + \tau(y-z) f_D(z)dz] \tag{2.7}$$

where $\gamma(z-y$ and $\tau(y-z)$ are random elements with respect to γ and T. We assume that

$$E[\gamma(z-y)] = 0, E[\tau(y-z)] = 0 = E[\gamma(z-y),\tau(y-z)] = 0$$

and

$$\text{var } [\gamma(z-y),\tau(y-z)] = \sigma_I^2, I = \gamma,\tau \tag{2.8}$$

where the errors are identically and independently normally distributed.

Taking the first moment of (2.7) we have

$$E[G(y)] = a(y-w) + \int_y^\alpha s(z-y) f_D(z)dz + \int_o^y h(y-z) f_D(z)dz + E\left[\int_y^\alpha \gamma(z-y) f_D(z)dz + \int_o^y \tau(y-z) f_D(z)dz\right] \tag{2.9}$$

It therefore follows from Fubini's theorem (see Feller, 1966 [2]) that

$$E\int_y^\alpha \gamma(z-y) f_D(z)dz = \int_o^y \tau(y-z) f_D(z)dz = 0 \tag{2.10}$$

Then, $$E[G(y)] = a(y-w) + \int_y^\alpha s(z-y) f_D(z)dz + \int_o^y h(y-z) f_D(z)dz) \tag{2.11}$$

and (2.6) becomes

$$\min_{y \geq w} E[a(y-w)] + \int_y^{\alpha} s(z-y) f_D(z)dz \quad + \int_o^{y} h(y-z) f_D(z)dz \tag{2.12}$$

From (2.4), it follows that y* is the solution to $$\phi(y^*) = \frac{s-a^7}{s+h} \tag{2.13}$$

Then the optimal policy for production schedule is given as

$$OPT = \begin{cases} \text{Produce } y^*\text{-}w, \text{ if } w \geq y \\ \text{Do not produce, if } w \geq y \end{cases} \tag{2.14}$$

Again, let θ = total number of materials (items) in stock, p = probability that an item will get out of stock before the first period of lead time, q = probability that an item will get out of stock before the second period of lead time and $g(k)$ = expected number of stock outs at the end of Kth period, then, the expected number of stock outs can be got by

$$g(k) = \frac{\theta\left[-(-q)^{k+1}\right]}{1+q} \tag{2.15}$$

To show this, let θ_1 be the expected number of stock-out at the end of the ith period ($i = 0, \Lambda\ , n$),
for i = 0,

$$\theta_0 = \theta$$

for i = 1

$$\theta_1 = \theta_0 p = \theta(1-q)$$

for i = 2

$$\theta_2 = \theta_0 q + \theta_1 p = \theta(1-q+q^2)$$

for i = 3

$$\theta_0 q + \theta_1 q + \theta_{2o} p = \theta(1-q+q^2-q^3)$$

for i = n

$$\theta_n = \theta\left[1-q+q^2-q^3+\Lambda\ +(-q)^n\right] \text{ and}$$

for i = n + 1

$$\theta_{n+1} + \theta_{n-1} q + \theta_n p = \theta\left[1-q+q^2+\Lambda\ +(-q)^{n+1}\right] \tag{2.16}$$

Thus,

$$g(k) = \theta\left[1-q+q^2+\mathrm{K}\ +(-q)^k\right] = \frac{\theta\left[1-(-q)^{k+1}\right]}{1+q} \tag{2.17}$$

and we can also take the limit of (2.17) as

$$\lim_{k\to\infty} g(k) = \lim_{k\to\infty}\left[\frac{\theta\left[1-(-q)^{k+1}\right]}{1+q}\right] \tag{2.18}$$

$$= \frac{\theta}{1+q}, \text{ for } q\ \pi\ 1 \text{ as } k \to \infty \tag{2.19}$$

where, $$1+q = p+2q \tag{2.20}$$

The average period of stock outs.

Since time is a factor in the planning process, we may be able to observe the evolution of the process for some periods and use the data to make predictions about the future. Thus, let

α = unit cost of purchasing orders for group items

β = unit cost of purchasing orders for individual items

$p(t)$ = extinction probability of each item until the end of time t.
α^* = average cost of purchasing orders for group within period K
β^* = average cost of purchasing orders for individual within period k
Then, from (2.19),

$$\beta^* = \frac{\theta\beta}{1+q} \tag{2.21}$$

and

$$\alpha^* = \theta\alpha + \theta(1-q)\beta \tag{2.22}$$

The optimal policy for material requirements is given therefore as,

$$OPT = \begin{cases} \text{Produce } g\text{roup items at the end of period where } \alpha * \beta * \\ \text{Purchase individually, otherwise} \end{cases} \tag{2.23}$$

3.0 Numerical example

A typical plastic manufacturing company in Nigeria has three production lines (plates and cups, chairs, and safety tanks). Data four periods more collected as summarized below (all data represent averages while all costs are in million naira):

Initial material stock level: 5000 = θ
Initial inventory level for products: 13767 = ω
Demand status: 3394665 = D
Holding Cost: 0.30 = h
Shortage Cost: 2.88 = s
Cost of purchasing orders for individual: 5000 = β
Cost of purchasing order for group: 4500 = α
Percentage rate of stockouts: 10, 30, 55 and 100 respectively.

With the above information we have

Table 3.1: Expected Number of Stockouts and Related cost of purchasing Orders

Period	P(t)	θi	E	β*	α*
1	0.10	500	0.10	2.5	25
2	0.20	1050	0.40	5.25	13.87
3	0.25	1455	0.75	7.27	9.92
4	0.45	2731	1.80	13.65	9.03

Again, for w = 13767, we have

$$y^* = 1724606 \text{ and } y^* - \omega = 1710839 \tag{3.1}$$

Thus, the optimal policies for MRPP are:

(i) Since β^* at the end of period four is greater than α^* at the end of period three, group purchases is more profitable if orders are place every end of period three.

(ii) For every four periods, an average production of 1710839 will produce a safety production stock level and minimize related inventory costs.

4.0 Concluding remarks

Although the data, as was supplied by the company, may not be 100% accurate record of events in the company (the company may have their reasons for this), they are sufficient for experimental purposes. Stochastic demand was replaced by deterministic forecasts and simple deterministic cost comparisons were applied for selecting the optimal reorder point for stockout items (materials).

On the whole the performance of MRPP was evaluated with the available data, and therefore, a quick response time (i.e the time taken to implement the MRPP policies and update the transactional data) is strongly recommended.

References

[1] Ehrhardt, R. (1997): A Model of JIT Mark – to-Stock Inventory with Stochastic Demand. Operational Research Society, 48:1013 – 1021.

[2] Feller, W. (1966): An Introduction to Probability Theory and its Applications. John Wiley and Sons Inc., Vol. 2 (2nd Ed). New York.

[2] Fleischmann, M. and Kuik, R. (1998): On Optimal Inventory Control with Stochastic item Returns. Management Report 21, Erasmus University, Rotter Dam.

[4] Gotzel, C. and Inderforth, K. (2001): Performance of MRP In Product Recovery Systems With Demand, Return and Leadtime Uncertainties. Preprint 6 (FWW), University of Magdeburg, Germany.

[5] Hillier, S. F. and Lieberman, G. J. (1995): Introduction to Operations Research, (6th Ed). Mc Graw-Hill Inc., New York.

[6] Khang, D. B. and Fujiwara, O. (2001): Optimality of Myopic Ordering Policies for Inventory Model with Stochastic Supply. Operations Research, 48(1): 181-184.

[7] Tito, H., Alexander, S. and Mark, L. (1999): Finding Optimal Material Release Times Using Simulation-Based Optimization. Management Science, 45(1): 86-102.