Journal of the Nigerian Association of Mathematical Physics
***Volume* 13 (November, 2008), 315 - 320**

Physical model of income distribution

***T. W. David, O. O. Odusote, and S. O. Kareem.**
Department of Physics,
Olabisi Onabanjo University, Ago-Iwoye, Nigeria

Abstract

In this study, a self-organized model was developed to demonstrate the inequality of income distribution. We assumed that income depends on two quantifiable parameters, namely, work and potential. Since every member of a population (or every nation) is free to decide how much work he/she will do, we model work by a random variable, called, the work index. The potential of each member of the population depends on biological (skill, intelligence), social and economic factors. For a state, the factors that affect the potential include the quality and quantity of a country's stock of the factors of production, technical knowledge, natural resources etc. In this model, a basic computer program was developed to compute the income distribution among population sample of 27. The results show that the wealth increases exponentially with the potential. Work however, has a linear relationship with potential; hence, the acquired resources also depend exponentially on work. There is no relationship between the initial potential, the final potential, and the final wealth. The model shows that although work is very important in the final potential and wealth, the time of work is more important to avoid lasting poverty.

1.0. Introduction

In recent times, some principles of physics have been applied to biological, chemical, social and economic systems. The branch of physics which addresses the complex problems in these fields is called synergetics.

One of the most fundamental problems in sociology, economics, mathematics/statistics and recently physics is the inequality of income distribution. Mandelbrot (1977 [3]) has however shown that the income distribution in America has a power-law tail, which means that the income distribution of rich people is fractal. Montroll and Shlesinger (1983 [4]) had plotted the distribution of income in the USA on log-normal graph paper for 1935-6. On such graph paper a cumulative log-normal distribution will be a straight line. This is the case for the first 98-9 percentiles; however, afterwards it follows a power-law (Shlesinger and Montroll, 1983 [5]). Bak and Chen (1989 [1]) have demonstrated that power-law fractals occur in nature due to self-organized critical dynamics.

The study of self-organized systems is a relatively new branch of physics which Haken (2007 [2]) called synergetics. It deals with complex systems which are made up of many components or subsystems. The study of how these subsystems cooperate among each other to bring about spatial, temporal and functional structures on macroscopic scale is called self-organization. Self-organization occurs in systems which are free to make choices under constraining forces.

Self-organized systems cut across all discipline: physics, chemistry, biology, ecology, economics, population dynamics, sociology etc. A special form of self-organized systems that has been extensively studied is the self-organized criticality in phase transition at critical points (e.g. water and ferromagnetisation).

At the critical point for continuous phase transition, the correlation function for the order parameter has power-law behaviour in both spatial and temporal domain. This power-law behaviour is the same as that of fractal objects. Bak and Chen (1989 [1]) have therefore described fractals as snapshots of systems operating at the self-organized critical state.

***Correspondence author 's e-m-mail: wemidavid@yahoo.com**

The occurrence of fractals in nature is widespread. . Many things which we used to think of as messy and having no structures are in fact characterized by well defined power-law spatial correlation functions. The statistical and mathematical theory of fractals is well developed. However, the physics of fractals are still being studied. Self organized criticality has been suggested as the physical mechanism responsible for the power-law fractal behaviour in the spatial and temporal domain of natural systems.
A system is said to be in a critical state if the order parameter that characterize the system is not uniquely determined but has a choice of two or more states. For example, at temperature of 373K and pressure of 1 atm, the density of water is not fixed but has the choice of a high value (water) or low value (steam).
In this study, the work dynamics as a function of available potential and the income derived would be investigated. The functional relationships would be analyzed by physical models.

2.0. Model description

2.1. Basic Assumption

The income of a state depends on several factors which are both quantitative and qualitative. For simplicity, we assumed that income depends on two quantifiable parameters, namely, work and potential. Work refers to any exercise embarked upon to earn an income in the form of wages, salaries, rentals, interests, profits etc. Since every member of a population (or every nation) is free to decide how much work he/she will do, we model work by a random variable, called, the work index. We believe that effort or work in productive activities by individuals and nations vary randomly.

The potential of each member of the population depends on biological, social and economic factors. For example, the potential of a person to earn an income depends on his/her talent, skill, intelligence and emotional stability. Social and economic factors such as family background, education and cultural beliefs affect a person's ability to earn income. For a country, the factors that affect the potential to acquire wealth or resources include the quality and quantity of the available factors of production, technical knowledge, natural resources as well as political and social stability. In this model, we assumed a multi cascade distribution of the potential without considering the different factors that affect it. We further assumed that the potential is not static but dynamic. The potential of each member of a population depends on the quantity of resources or wealth he/she has acquired, and also on the quantity of resources acquired by other members of the population.

2.2. Model parameters and relationships

The four major parameters of the model are:

(i) *A*(*n*): Resource acquisition potentials of each member of a population. The potentials are taken as normalized functions. Hence, their initial values are determined by a multiplicative cascade model.

(ii) *C*(*n*): Given that the total annual available resources, *TR*, is fixed, *C*(*n*) is the proportion of the fixed resources each member of the population earns for work-done.

$$C(n) = A(n) \times TR \tag{2.1}$$

(iii) *B*(*n*){ – This is the total resources acquired by each member of the population at time *k*.}

$$B(n) = \sum_{i=1}^{k} C_i(n) \tag{2.2}$$

(iv) *WI*(*n*): This is the work-index which defines the total number of times each member of the population earns an income or acquires resources.

As the acquired resources of each member of the population changes, the potentials also change. To compute the new normalized potentials we introduce a dummy variable *D*(*n*) defined as:

$$D(n) = \begin{cases} A(n) & : 0 \le B(n) \le 1 \\ A(n) + G\,\log[\,B(n)] : B(n) \,\phi\, 1 \end{cases} \tag{2.3}$$

The functional form of this relation was taken for two reasons:

(i) We expect that as the wealth of any member of the population increases, the potential will also increase. We however, expect that the relative change in potential will decrease as the wealth increases.

(ii) The actual potential does not decrease with time as other members become wealthier.

The control parameter *G* determines the relative impact of the wealthy nations on the poor ones and the higher its value, the higher the impact of the rich on the poor. The new normalized potential is therefore defined as

$$A(n) = D(n) / \sum_n D(n) \tag{2.4}$$

2.3. Computational procedure

The first stage of the program involves the initialization of the model parameters. The population size was taken to be 27 and the multiplicative cascade subroutine was used to compute the initial potentials. As designed, the program can only take population size in multiples of 3. The initial values of B(n) and C(n) were zero, but B(n) could be varied. The program was designed to undergo twenty iterations which represent twenty-years. This number can also be changed.

As designed, each member of the population acquires a fraction of the available resources that is proportional to its potential when a subroutine "Ranz" for random number generation returns the number of any member of the population. The total number of times that each number is generated is the work-index (WI).

We note that non-randomness of the sequence of numbers generated by the subroutine Ranz may adversely bias the program. Therefore, before using the routine we first tested for randomness. For any random sequence, $x_i, x_2,..,x_n$, we define the following properties

mean
$$\overline{X} = \frac{1}{n}\sum_n X_n \tag{2.5}$$

standard deviation :
$$\sigma_x = [\frac{1}{n-1}\sum_n (x_n - \overline{x})^2]^{1/2} \quad . \tag{2.6}$$

Standardize random numbers:
$$Z_n = (x_n - \overline{x}) / \sigma_x \tag{2.7}$$

$$\overline{Z} = \frac{1}{n}\sum_n Z_n \tag{2.8}$$

$$\sigma_Z = [\frac{1}{n-1}\sum_n (z_n - \overline{z})^2]^{1/2} \tag{2.9}$$

If the series is random, we have $\overline{Z} = 0, \quad and \quad \sigma_z = 1$

3.0. Results and discussion

3.1. Analysis of potential, work and resources

Table 3.1 gives the variation of the potential and resources for each member of the population with time. It is seen that where the initial resources are to zero for all the population, the final resources vary greatly from 7.5×10^{-6} for population number 2, to 67.402 for population number 6. Whereas, the initial potential varied from 0.036265 to 0.037483, the final potential varied from 0 to 0.085965 (figure 3.1). One can deduce that the resources have a relationship with the potential but the total resource acquired within the first few iterations is very important in addition to the work index. Figure 3.2 shows that there is a linear relationship between the work-index and the final potential, provided the potential is greater than 0.02. This is to be expected since the work index determines the resources and the resources determine the potential. The resources however have an exponential relationship with the work-index (figure 3.3), provided the resource is greater than a threshold value. The final resources also have an exponential relation with the final potential (figure 3.4). This is also to be expected since the work-index varies linearly with potential and exponentially with the final resources.

Since the potential determines the resources, we are interested in the relationships, between the initial potential and the final potential (figure 3.6) as well as that between the initial potential and the final resources (figure 3.7). From the two graphs, we see that there is no relationship between the initial and the final potential.

To test the validity of our result we assumed that the Human Development Index (HDI) defined by the United Nations Development Programme and computed for all nations of the world is related to the resource acquisition potential defined in this study. The Gross Domestic Product Per Capita (GDP) is also taken as an indication of the wealth or resources of each nation; hence we plotted the HDI against the GDP for 175 nations in

Figure 3.5. There is clear exponential dependence of the GDP with the HDI in agreement with the model result. Note that the exponential dependence is not an intrinsic property of the model. It results from the interaction between the population and the rate of doing work.

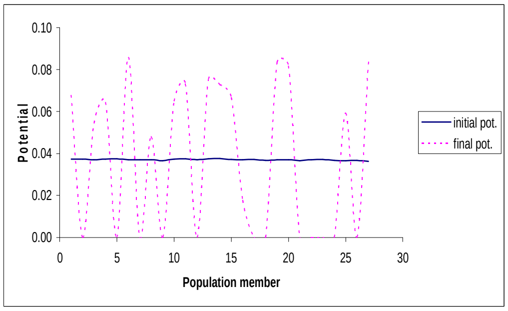

Figure 3.1: Variation in initial and final potential

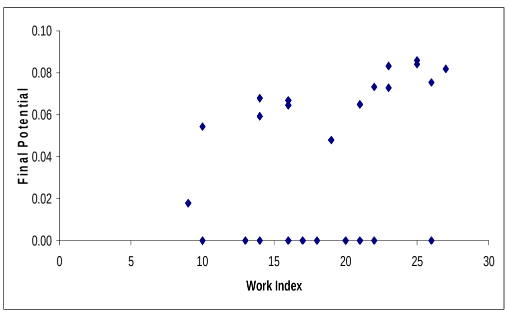

Figure 3.2: Final potential versus work index

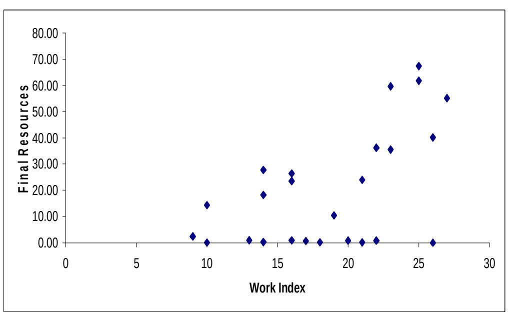

Figure 3.3: Final resources versus work index

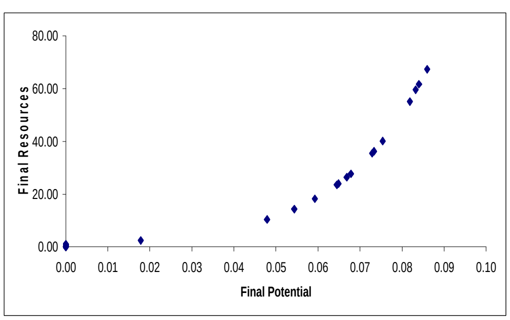

Figure 3.4: Final resources versus final potential

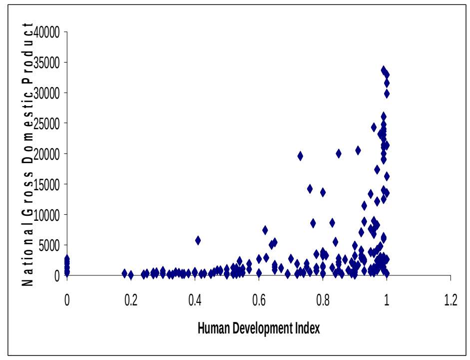

Figure 3.5: National gross domestic product versus human development index

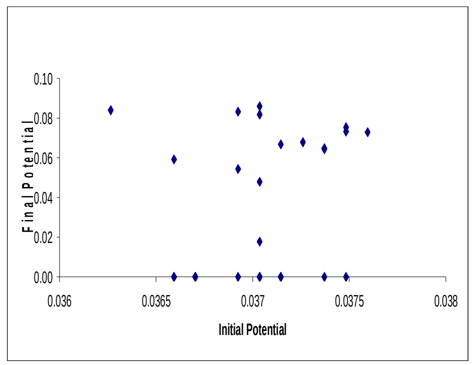

Figure 3.6: Final potential versus initial potential

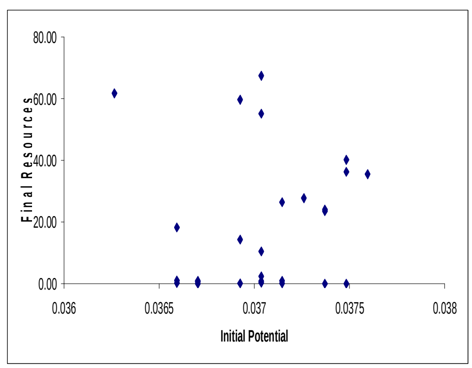

Figure 3.7: Final resources versus initial potential

3.2. Distribution of resources

The distribution of the per capita GDP for 175 countries listed in the World Development Report (1997) of the United Nation's Development Programme are as shown in Table 3.2 with the distribution obtained from the model. Using the student t-test, we observed that the two distributions are the same with 80% significance.

Note that both the real GDP per capita and the model result are not fractal. Hence, we have not actually been able to explain the power-tail of income distribution in the U.S.A. The present model did not give the study population any free choice. This is a basic requirement for self-organized power-law behaviour.

Table 3.1: Variation in potential and resources with time. The first set is the variation in potential while the other set is the variation of resources'.

1	0.03726	0.075415	0.127836	0.126936	0.098548	0.084324	0.077661	0.076468	0.077787	0.077065	0.073131	0.072609	0.070489	0.072497	0.070884	0.068616	0.68605	0.068896	0.068791	0.069327	0.067829
2	0.037371	0.021406	0.003332	0.000257	2.00E-06	2.40E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0.036925	0.10192	0.146954	0.102873	0.077078	0.659297	0.059235	0.53482	0.50449	0.05385	0.05669	0.06184	0.060956	0.062295	0.062295	0.058238	0.056717	0.057801	0.056341	0.05555	0.054349
4	0.037371	0.021406	0.091173	0.109311	0.08608	0.736663	0.066186	0.074491	0.071589	0.075757	0.075654	0.075398	0.075177	0.073103	0.073103	0.068319	0.067584	0.066818	0.066838	0.65911	0.064487
5	0.037483	0.02147	0.003342	0.000258	2.00E-06	2.40E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0.037036	0.039002	0.090022	0.153868	0.145226	0.139366	1.131934	0.122474	0.118204	0.109251	0.106387	0.10183	0.101227	0.099005	0.099005	0.093605	0.091449	0.089778	0.088333	0.087755	0.08596
7	0.036925	0.02115	0.003292	0.000254	1.90E-06	2.30E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0.037036	0.021213	0.003302	0.001376	0.001578	0.002827	0.002654	0.002398	0.002262	0.00426	0.013448	0.029295	0.028724	0.030452	0.030452	0.039915	0.038949	0.041115	0.042865	0.043276	0.047887
9	0.036593	0.02096	0.003263	0.000252	1.90E-06	2.30E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0.037371	0.021406	0.012358	0.020165	0.017772	0.152251	0.019876	0.020439	0.019293	0.032248	0.031131	0.035942	0.04053	0.052813	0.052813	0.062922	0.068773	0.067796	0.067262	0.066318	0.064884
11	0.037483	0.023815	0.025554	0.044087	3.60E-02	3.08E-01	5.94E-02	6.62E-02	7.16E-01	7.54E-02	7.29E-02	6.92E-02	7.02E-02	6.78E-02	6.78E-02	7.60E-02	0.075753	0.074622	0.074732	0.074918	0.0733
12	0.037036	0.021213	0.003302	0.000255	2.00E-06	2.30E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0.037483	0.027427	0.007617	0.002925	7.80E-03	3.48E-02	4.39E-02	5.73E-02	6.91E-02	7.83E-02	8.72E-02	8.83E-02	8.55E-02	8.25E-02	8.25E-02	8.04E-02	0.07973	0.077735	0.076992	0.077064	0.075398
14	0.037595	0.030771	0.008986	0.009989	4.27E-02	5.78E-01	6.15E-02	7.43E-02	7.63E-02	7.08E-02	7.55E-02	7.17E-02	7.02E-02	6.99E-02	6.99E-02	7.20E-02	0.073735	0.073453	0.07159	0.073245	0.072871
15	0.037146	0.044427	0.083837	0.09167	8.93E-02	8.54E-02	7.97E-02	8.01E-02	8.04E-02	7.90E-02	7.50E-02	7.12E-02	7.19E-02	6.95E-02	6.95E-02	6.73E-02	0.065566	0.065905	0.069222	0.068303	0.066827
16	0.037036	0.02632	0.006756	0.002378	1.57E-03	1.34E-03	1.20E-03	1.09E-03	1.02E-03	1.73E-03	1.95E-03	2.67E-03	4.18E-03	6.29E-03	6.29E-03	8.82E-03	0.008591	0.008375	0.011244	0.0111	0.017829
17	0.037146	0.021277	0.003312	0.000256	2.00E-06	2.30E-08	1.40E-09	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0.036703	0.021023	0.003273	0.000253	1.90E-06	2.30E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0.036925	0.114479	0.120744	0.130314	1.21E-01	1.11E-01	1.04E-01	0.105186	0.010222	0.10004	0.097768	0.096606	0.095177	0.092327	0.09143	0.088506	0.086196	0.085043	0.083898	0.083631	0.08325
20	0.037036	0.024429	0.083131	0.030673	9.84E-02	9.76E-02	9.50E-02	0.086705	0.084592	0.080198	0.079502	0.075658	0.080816	0.080359	0.079416	0.076914	0.076778	0.077347	0.077754	0.080694	0.081844
21	0.036593	0.02096	0.003263	0.000252	1.90E-06	2.30E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0.037036	0.021213	0.003302	0.000255	2.00E-06	2.30E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0.037146	0.021277	0.003312	0.000256	2.00E-06	2.30E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0.036703	0.021023	0.003273	0.000253	1.90E-06	2.30E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0.036593	0.02299	0.022025	0.018917	5.94E-02	8.14E-01	7.51E-02	0.067801	0.063957	0.059104	0.056084	0.053261	0.05152	0.04975	0.048031	0.05014	0.054821	0.058724	0.059434	0.058601	0.059255
26	0.036703	0.021023	0.003273	0.000253	1.90E-06	2.30E-08	4.00E-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0.036265	0.150988	0.130163	0.101471	1.18E-01	1.19E-01	1.23E-01	1.12E-01	1.11E-01	1.03E-01	9.77E-02	9.45E-02	9.34E-02	9.14E-02	8.99E-02	8.83E-02	8.71E-02	8.66E-02	8.47E-02	8.43E-02	8.40E-02
1	0	2.019283	4.195778	7.775185	7.78E+00	7.78E+00	9.91E+00	9.91E+00	1.20E+01	1.42E+01	1.42E+01	1.61E+01	1.61E+01	2.01E+01	2.01E+01	2.01E+01	2.20E+01	2.30E+01	2.58E+01	2.77E+01	2.77E+01
2	0	0	0	0	0.00E+00	7.20E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06	7.50E-06
3	0	2.116853	4.97062	4.97062	4.97E+00	4.97E+00	4.97E+00	4.97E+00	4.97E+00	6.37E+00	7.85E+00	1.11E+01	1.11E+01	1.28E+01	1.28E+01	1.28E+01	1.28E+01	1.43E+01	1.43E+01	1.43E+01	1.43E+01
4	0	0.966091	5.99981	5.99981	6.00E+00	6.00E+00	6.00E+00	9.73E+00	9.73E+00	1.36E+01	1.57E+01	1.78E+01	1.98E+01	1.98E+01	1.98E+01	1.98E+01	2.17E+01	2.17E+01	2.35E+01	2.35E+01	2.35E+01
5	0	0	0	0	6.96E-03	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04	7.51E-04
6	0	1.775388	5.773833	15.81838	2.41E+01	3.20E+01	3.59E+01	3.94E+01	4.28E+01	4.28E+01	4.87E+01	4.87E+01	5.44E+01	5.72E+01	5.99E+01	5.99E+01	6.25E+01	6.25E+01	6.50E+01	6.74E+01	6.74E+01
7	0	0	0	0.081072	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02	8.26E-02
8	0	0	0.945612	1.033492	1.03E+00	1.07E+00	1.07E+00	1.07E+00	1.07E+00	1.24E+00	1.64E+00	3.11E+00	3.11E+00	3.89E+00	3.89E+00	5.75E+00	5.75E+00	6.82E+00	7.96E+00	7.96E+00	1.04E+01
9	0	0.704554	0.963664	0.963664	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01	9.64E-01
10	0	0.805917	1.146955	1.448188	1.45E+00	1.45E+00	1.85E+00	1.85E+00	1.85E+00	3.09E+00	3.09E+00	3.95E+00	4.95E+00	8.68E+00	1.33E+01	1.67E+01	2.21E+01	2.40E+01	2.40E+01	2.40E+01	2.40E+01
11	0	1.020331	1.405388	2.116221	2.12E+00	2.12E+00	6.39E+00	8.15E+00	8.15E+00	1.40E+01	1.40E+01	1.40E+01	1.59E+01	1.59E+01	1.98E+01	2.79E+01	3.00E+01	3.21E+01	3.42E+01	3.62E+01	3.62E+01
12	0	0	0.240316	0.240316	2.41E-01	2.41E-01	2.41E-01	2.41E-01	2.41E-01	2.41E-01	2.41E-01	2.41E-01	2.41E+05	2.41E-01	2.41E-01	2.41E-01	2.41E-01	2.41E-01	2.41E-01	2.41E-01	2.41E-01
13	0	1.041792	0.104179	1.041792	3.30E+00	3.30E+00	3.30E+00	5.76E+00	5.76E+00	1.52E+01	2.43E+01	2.92E+01	2.92E+01	2.92E+01	3.14E+01	3.36E+01	3.59E+01	3.59E+01	3.80E+01	4.02E+01	4.02E+01
14	0	1.05267	1.05267	1.205751	4.17E+00	4.17E+00	5.71E+00	9.38E+00	9.38E+00	1.14E+01	1.55E+01	1.55E+01	1.74E+01	1.74E+01	2.13E+01	2.33E+01	2.74E+01	2.94E+01	2.94E+01	3.35E+01	3.55E+01
15	0	2.562392	2.562392	6.440405	6.44E+00	8.66E+00	8.68E+00	1.29E+01	1.29E+01	1.51E+01	1.51E+01	1.51E+01	1.71E+01	1.71E+01	1.90E+01	1.90E+01	1.90E+01	2.08E+01	2.64E+01	2.64E+01	2.64E+01
16	0	1.03306	1.03306	1.03306	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	1.06E+00	1.11E+00	1.11E+00	1.18E+00	1.30E+00	1.47E+00	1.47E+00	1.47E+00	1.47E+00	1.70E+00	1.70E+00	2.40E+00
17	0	0.937351	0.937351	0.937351	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01	9.37E-01
18	0	0.648282	0.648282	0.669044	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01	6.69E-01
19	0	2.118743	7.873602	11.48486	1.48E+01	1.48E+01	1.77E+01	2.35E+01	2.35E+01	3.19E+01	3.47E+01	4.01E+01	4.28E+01	4.53E+01	4.79E+01	4.79E+01	4.79E+01	5.03E+01	5.27E+01	5.50E+01	5.96E+01
20	0	1.024295	3.672647	3.672647	8.11E+00	1.08E+01	1.35E+01	1.35E+01	1.35E+01	1.58E+01	1.80E+01	1.80E+01	2.44E+01	2.67E+01	2.89E+01	2.89E+01	3.10E+01	3.53E+01	3.96E+01	4.83E+01	5.51E+01
21	0	0	0.161761	0.161761	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01	1.68E-01
22	0	0.828332	0.828332	0.828332	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01	8.30E-01
23	0	0	0	0.103313	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01	1.03E-01
24	0	0.877312	0.877312	0.877312	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01
25	0	1.014633	1.371989	1.351989	3.92E+00	7.64E+00	7.64E+00	7.64E+00	7.64E+00	7.64E+00	7.64E+00	7.64E+00	7.64E+00	7.64E+00	7.64E+00	8.96E+00	1.18E+01	1.49E+01	1.66E+01	1.66E+06	1.82E+01
26	0	0	0	0	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03	9.06E-03
27	0	3.686598	3.686598	8.814666	1.17E+01	1.82E+01	2.83E+01	2.83E+01	3.46E+01	3.46E+01	3.46E+01	3.72E+01	3.98E+01	4.24E+01	4.49E+01	4.74E+01	4.99E+01	5.47E+01	5.47E+01	5.71E+01	6.17E+01

Table 3.2: Statistics of the GDP per capita and the model data

Class Interval	Model		Real Data	
	Actual	Percentage (%)	Actual	Percentage (%)

0 – 10	14	51.85	143	75.66
10 – 20	2	7.41	15	7.94
20 – 30	4	14.81	8	4.23
30 – 40	3	11.11	7	3.70
40 – 50	-	-	11	5.82
50 – 60	2	7.41	2	1.06
60 – 70	2	7.41	3	1.59

3.3 Conclusion

We have attempted to model the global income distribution pattern of nations in terms of works, potential and resources. Both real data and the model results show that the wealth of a nation increases exponentially with the potentials.

The richest countries are also the ones that are more likely to acquire more resources. Work has a linear relationship with potential and hence the acquired resources also depend exponentially on work. However, we observe that temporal variation of work has significant effect on the final potential and the final resources. The richest members of model are the ones with good start at the first two years, while the poorest are the ones that do not earn any income in the first one to four years. With this initial poor start, it became very difficult for these poor members to earn income though they have high work index.

The study has also shown that there is no relationship between the initial potential and the final potential. The implication of this is that every nation has the ability to be successful. A sincere global effort to re-distribute potential (not resources) may bridge the wide disparity in wealth across nations.

References

[1] Bak, P. and Chen, K. (1989): *The Physics of fractals*, Elsevier science publisher, Amsterdam, pp 5-12

[2] Haken, H. (2007) : *Synergetics,* Schorlarpedia, **2(1)**, pg1400

[3] Mandelbrot B. B. (1977): *Fractals: Form, chance and dimension*, Journal of the American Statistical Association, **73**, 438

[4] Montroll E. W. and Shlesinger M. F. (1983): *Power-law behaviours in high income distribution*, Journal of Statistical Physics, pg 344-348[5]

[5] Shlesinger M. F. and Montroll E. W. (1983) : *The Mathematics and Physics of disordered media,* edited by B. D. Hughes and B. W. Ninham, Springer Verlag, Berlin, pg 109

[6] World development report (1977): *The United Nation's Development Programme*