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The performance of FPE for the maximum entropy estimation method

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Abstract

The performance of FPEα, as defined by Bhansali and Downham* (1977) *is investigated here for α =* 1, 2, 3, 4 *and for the maximum entropy method of autoregression estimation. Here it is demonstrated using both artificial and real series that the optimum α is between* 2 *and* 4*, inclusive.

Keywords: Autoregressive modeling, FPE, maximum entropy method.

1.0 Introduction

A stationary time series $\{X_t\}$ is said to follow an autoregressive process of order p (designated $AR(p)$) if it satisfies the following difference equation

$$X_{t-1} - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \ldots - \alpha_p X_{t-p} = \varepsilon_t \qquad (1.1)$$

where $\{\varepsilon_t\}$ is a white noise process of variance σ^2 and the α_i's are constants such that

$$1 - \alpha_1 z - \alpha_2 z^2 - \ldots - \alpha_p z^p = 0, |z| > 1.$$

Using (1.1) to model a realization $X_1, X_2, \ldots, X_N$ of a time series involves in the first instance, the estimation of the order p and secondly, the estimation of the parameters α_i's.

To estimate p, Akaike (1969 [1]) proposed the FPE criterion defined by

$$FPE(p) = (1 + p/N)\,\hat{\sigma}^2_{p}, \quad p = 0, 1, 2, \ldots$$

where σ^2_p is the least squares estimate of σ^2. After specifying a maximum lag L, the estimate of p is the lag for which FPE is minimum.

Bhansali and Downham (1977 [[4]) generalized the FPE criterion as

$$FPE\alpha(p) = (1 + \alpha p/N)(1 - p/N)^{-1}\,\hat{\sigma}^2_{p}, \quad p = 0, 1, 2, \ldots$$

For the least squares method of autoregression estimation, Akaike (1973 [2]) has shown that $\alpha = 2$ is optimum whereas Bhansali and Downham (1977 [4]) have suggested that $2 \le \alpha \le 4$. Here, we are comparing α = 1, 2, 3, 4 for selection of full-order AR models for the maximum entropy method of estimation of parameters. We shall use artificial as well as real series.

2.0 The maximum entropy method of autoregression estimation.

This method proposed by Burg (1967 [7]), fits (1.1) to the realization $X_1, X_2, \ldots, X_N$ of $\{X_t\}$ by minimizing

$$\sum\{(X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \ldots - \alpha_p X_{t-p}) + (X_{t+p} - \alpha_1 X_{t+p-1} - \alpha_2 X_{t+p-2} - \ldots - \alpha_p X_{t-p})\}^2$$

with respect to $\alpha_1, \alpha_2, \ldots, \alpha_p$. Andersen (1974 [3]) has formalized this method by a recursive formula.

3.0 Simulation results.

We simulated four AR(2) series I, II, III and IV with (α_1, α_2) equal to (-1.68, 0.70), (-0.66, 0.10), (-1.08,0.77) and (-0.96, 0.08), respectively. Sixty realizations were generated for each time series: twenty of them 50-point, twenty 150-point and twenty 250-point. The white noise process of each simulation is a sequence of pseudorandom numbers obtained by the RAN function of FORTRAN 77 language and made standard Normal. Table 3.1 shows the results of comparing α = 2, α = 3 and α = 4.

It can be observed that for series I, FPEα tends to overestimate, the tendency increasing with decrease in α and increase in the sample size N. The efficiency in the selection of order 2 increases with increase in α and decreases with increase in N. For series II, FPEα tends to underestimate, the tendency increasing with α. The efficiency in order selection decreases with increase in α. Series III result is similar to series I result, and series IV result to that of series II. It is noteworthy that for series I and III, the partial autocorrelation is large; for II and IV it is small. Etuk (1987 [9]) has shown that on the overall, FPE4 does best.

Table 3.1: Frequency out of 20 of choice of order 2

Series	Size	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$
I	$N = 50$	0	0	0
		15	17	19
		2	3	1
	$N = 150$	0	0	0
		13	15	20
		7	5	0
	$N = 250$	0	0	0
		13	14	16
		7	6	4
II	N = 50	8	9	12
		6	7	6
		6	4	2
	$N = 150$	9	12	16
		4	4	3
		7	4	1
	$N = 250$	7	8	9
		11	11	11
		2	1	0

4.0 Real series results

In this section we shall explore their comparative performance by the use of well-analyzed real series. We shall use a maximum lag of 30. Our method includes comparison of our models with earlier ones. We shall also subject each model to the Box-Pierce (1970 [5]) portmanteau test with test-statistic R. We shall compare the parametric spectra with the raw one, for each series. To further help in model identification, we shall employ the inverse autocorrelation function (IACF) and the partial autocorrelation function (PACF) (See Etuk (1988 [10])).

4.1 Canadian Lynx numbers (1821 – 1934) (Campbell and Walker, 1977, pp. 430 [8]).

We used the logarithmic transformation. With $\alpha = 1$, the chosen order is 24. With $\alpha = 2$, the model

$$X_t - 1.127X_{t-1} + 0.521X_{t-2} - 0.288X_{t-3} + 0.325X_{t-4} - 0.178X_{t-5} + 0.180X_{t-6} - 0.093X_{t-7} + 0.088X_{t-8} - 0.179X_{t-9} - 0.145X_{t-10} - 0.191X_{t-11} + 0.135X_{t-12} = \varepsilon_t \quad (4.1)$$

is chosen. With $\alpha = 3$ or 4, the AR(11)

$$X_t - 1.174X_{t-1} + 0.551X_{t-2} - 0.269X_{t-3} + 0.319X_{t-4} - 0.168X_{t-5} + 0.158X_{t-6} - 0.070X_{t-7} + 0.045X_{t-8} - 0.143X_{t-9} - 0.219X_{t-10} + 0.349X_{t-11} = \varepsilon_t, R = 18.831 \quad (4.2)$$

is selected. Etuk (1988 [10]) has shown that the PACF suggests an order of 11 and the IACF of one. Etuk (1987 [9]), using the least squares estimation procedure found the order of 11 best. Also Haggan and Oyetunji (1984 [11]) using their subset modeling algorithm fitted an AR (11) to the data. Figure 4.1 is a superimposition of the spectrum of (4.2) on a raw one. The agreement between them is close confirming the adequacy of (4.2). Also the R-value of 18.831 for (4.2) is non-significant. Therefore, $\alpha = 3$ or 4 performs best.

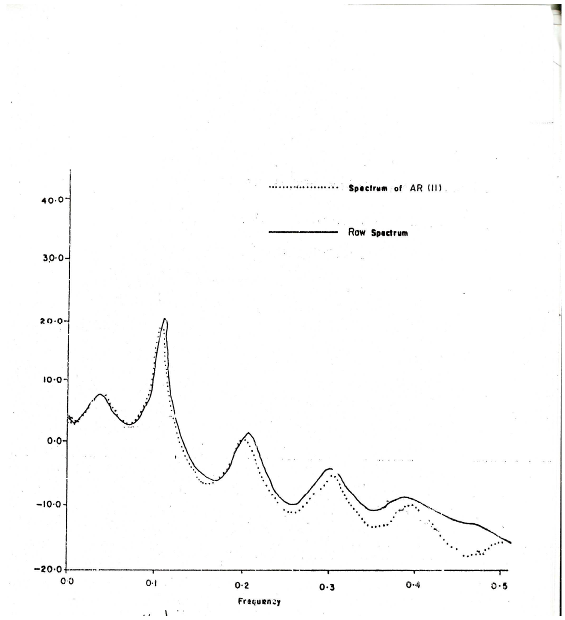

Figure 4.1: Some log lynx spectra (in db)

4.2 Wolfer's Sunspot numbers (1700 – 1955).

The annual sunspot numbers are available from 1700 onwards (Waldmeier, 1961 [14]). We used the 256 values from 1700 to 1955.

With $\alpha = 1$ the AR(29) is chosen and with $\alpha = 2$ an order of 18 is chosen. With $\alpha = 3$ or 4, the AR(9)

$$X_t - 1.184X_{t-1} + 0.407X_{t-2} + 0.197X_{t-3} - 0.223\,X_{t-4} - 0.178X_{t-5} - 0.061X_{t-6} + 0.053X_{t-7} - 0.083X_{t-8} - 0.123X_{t-9} = \varepsilon_t,\ \sigma^2 = 196.10,\ R = 25.16 \qquad (4.3)$$

is chosen. Etuk (1987 [9]) has shown that model (4.3) is adequate; the *R*-test is not significant and its spectrum agrees closely with a non-parametric one. He, moreover fitted an AR(9) using the algorithm of Haggan and Oyetunji (1984 [11]). Incidentally, Morris (1977 [12]), by the use of forward and backward stepwise regression, selected an order of 9. Thus $\alpha = 3$ or 4 is best.

4.3 Series A (Box and Jenkins, 1976, pp.526 [5])

Putting $\alpha = 1$ gives $p = 30$, $\alpha = 2$ or 3 gives $p = 7$ and $\alpha = 4$, $p = 2$. The AR(7) model chosen is

$$X_t - 0.356X_{t-1} - 0.187X_{t-2} - 0.020X_{t-3} - 0.024X_{t-4} + 0.024X_{t-5} - 0.072X_{t-6} - 0.188X_{t-7} = \varepsilon_t,\ \sigma^2 = 0.0926,\ R = 21.91 \qquad (4.4)$$

and the AR(2) is

$$X_t - 0.426X_{t-1} - 0.254X_{t-2} = \varepsilon_t,\ \sigma^2 = 0.0999,\ R = 99.80 \qquad (4.5)$$

Etuk (1987 [9]) found (4.4) adequate and (4.5) an under-parameterization. Ozaki (1977 [13]) fitted an AR(7). The IACF and PACF both recommend an AR(7). Hence, $\alpha = 4$ underestimates the order but $\alpha = 2$ or 3 chooses the correct order.

4.4. Series E (Box and Jenkins, 1976; pp. 530 [5])

FPE1 chooses an order of 25, FPE2 and FPE3 an order of 8 and FPE4 an order of 3. Box and Jenkins (1976) also fitted an AR(3). Here $\alpha = 4$ seems best.

From our study it is apparent that $\alpha = 1$ is invariably too small; $\alpha = 2$ can be optimal if the partial correlation is small. $\alpha = 4$ can be optimal if the partial correlation is large. Clearly there is need for further exploration for optimum α with a wider variety of models.

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