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# Asymptotic null controllability of nonlinear neutral volterra integrodifferential system with delays in control

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***Abstract***

***In this paper by using Larry-Schauder's fixed point theorem, we; obtained sufficient conditions for the asymptotic null controllability of nonlinear neutral volterra integrodifferential system.***

***Keywords:*** Asymptotic null controllability, volterra integral system, Larry-Schauder's Fixed point theorem.

## 1.0 Introduction

Neutral functional differential equation in Banach spaces has been studied by several authors [8,9,12] A neutral functional differential equations is one in which the derivatives of the past history or derivatives of functional of the past history are involved as well as the present state of the system. Neutral differential equations arise in many areas of applied mathematics and such equations have received much attention in recent years [4.5.7,8,11,13].

The concept of controllability plays a major role in finite-dimensional control theory, so it is natural to try to generalize this to infinite-dimension. The first step in the study of the problem of controllability is to determine if an object can be reached by some suitable control function. For continuous time invariant linear system in finite dimensional space the concept of controllability and reachability are equivalent and they are independent of time. But for infinite dimensional space, the situation is more complex and many different type of controllability and reachability has been studied in the literature. A weaker condition than exact controllability is the property of being able to steer all points exactly to the origin. This has important connections with the concept of stabilizability. Several authors have studied the null controllability of various kind of dynamic systems [1,7]. In [6], sufficient conditions for the controllability of semi-linear integrodifferential systems in a banach space are established using the asymptotic fixed point theorem for *k*-set contractions.

Rahima and Said [10], Jackfreece [4] established sufficient condtions for the relative controllability of the nonlinear neutral volterra integrodifferential system with distributed delays in the control of the form (1.1) using Schauder's fixed point theorem.

In this paper, we used the Larry-Schauder's fixed point theorem to develop conditions for the nonlinear neutral volterra integrodifferential system with distributed delays in the control of the form (1.1) to be asymptotically null controllable. Specifically, system (1.1) is defined by the nonlinear neutral volterra integrodifferential system of the form

$$\frac{d}{dt}\left[x(t)-\int_0^t C(t,s)x(s)ds-g(t)\right]=A(t)x(t)+\int_0^t G(t,s)x(s)ds$$
$$+\int_{-h}^0 d_\theta H(t,\theta,x(t),u(t))u(t+\theta)+f(t.,x(t),u(t)) \qquad (1.1)$$
$$x(0)=x_0$$

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where $x \in R^n$, $u \in R^p$, $t \in J = [0, \infty)$ and $H(t, \theta\, x(t), u(t)$ is an $n \times p$ dimensional matrix, continuous in $(t,x(t), u(t))$ for fixed $\theta$ and of bounded variation in $\theta$ on $[-h,0]$ for each $(t,x,u) \in J \times R^{n\times p}$, the $n \times n$ matrix $A(t)$, $C(t,s)$ and $G(t,s)$ are continuous in their various variables. The $n$-vector functions $f$ and $g$ are respectively continuous and absolutely continuous. The integral is in the Lebesgue-Stieltjes sense.

## 2.0 Preliminary results

Let $D$ denote the Banach space of continuous $R^n \times R^p$-valued function defined on $J = [0,\infty)$ with the sup norm $\|(x,u)\| = \|x\| + \|u\|$ where $\|x\| = \sup\{|x(t)| : t \in J\}$ and $\|u\| = \sup\{|u(t)| : t \in J\}$.

Consider the linear system

$$\frac{d}{dt}\left[x(t) - \int_0^t C(t,s)x(s)ds - g(t)\right] = A(t)x(t) + \int_0^t G(t,s)x(s)ds + \int_{-h}^0 d_\theta H(t,\theta,x(t),u(t))u(t+\theta) \qquad (2.1)$$

The solution of (2.2) is given as in [12] by

$$x(t) = Z(t,0)[x(0) - g(0)] + g(t) + \int_0^t \dot{Z}(t,s)g(s)ds + \int_0^t Z(t.s)\left[\int_{-h}^0 d_\theta H(s,\theta,x(s),u(s))u(t+\theta)\right]ds \qquad (2.2)$$

where $Z(t,s)$ *and* $\dot{Z}(t,s)$ are continuous matrices satisfying

$$\dot{Z}(t,s) - \int_0^t \dot{Z}(t,\tau)C(\tau,s)d\tau + C(t,s) = -Z(t,s)A(s) - \int_0^t Z(t,s)G(t,s)d\tau \qquad (2.3)$$

and where $Z(t,s) = I\ (identity\ matrix)$ and the solution of (1.1) is given by

$$x(t) = Z(t,0)[x(0) - g(0)] + g(t) - \int_0^t \dot{Z}(t,s)g(s)ds + \int_0^t Z(t,s)\left[\int_{-h}^0 d_\theta H(s,\theta,x(s),u(s))u(t+\theta) + f(s,x(s),u(s))\right]ds \qquad (2.4)$$

Let us assume the following limit exists

$\lim_{t\to\infty} Z(t) = Z \neq 0$, $\lim_{t\to\infty} \dot{Z}(t) = \overline{Z}$, $\lim_{t\to\infty} k\,(cons\ \ \tan\ t)$, $\lim_{t\to\infty} W(t) = W$

where

$$W(t) = \int_0^t S(t,s)S^T(t,s) \qquad (2.5)$$

$W(t)$ is the controllability matrix where $T$ denotes the matrix transpose and

$$S(t,s) = \int_{-h}^0 Z(t,s-\theta)d_\theta \overline{H}(s-\theta,\theta,x,u)$$

where $\overline{H}(s,\theta,x,u) = \begin{cases} H(s,\theta,x,u) & for \quad s \le t \\ 0 & for \quad s > t \end{cases}$

**Definition 2.1**

*The system* (1.1) is *said to be asymptotically null controllable if for every* $x_0 \in R^n$ *there exists a control u defined in J such that* $x(t) = x_0$ *and* $\lim_{t\to\infty} x(t) = 0$.

***Theorem* 2.2**

*The linear system* (2.1) *is asymptotically null controllable if and only if W is non-singular.*

***Proof***

Assume that W is non-singular, then for each $x_0 \in R^n$, $x_0 \neq 0$, define the control $u$ in $J$ as

$$u(t) = -\left(S^T(t,s)\right)W^{-1}\left[Z(x(0) - g(0)) + k + \int_0^\infty \overline{Z}g(s)ds\right]$$

clearly $x(0) = x_0$ *and* $\lim_{t\to\infty} x(t) = 0$ and so system (2.1) is asymptotically null controllable. Conversely assume that $W$ is singular. Then there exists a vector $v \neq 0$ such that $v^T W v = 0$. It follows that

$$\int_0^{\infty} v^T S(t,s)\left(v^T S^T(t,s)\right)^T ds = 0,$$

therefore $v^T S(t,s) = 0$ for $s$ Є $J$. Since the solution is asymptotically null controllable, there exists a control $u(.)$ such that

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} \Big\{ Z(t,0)[x(0) - g(0)] + g(t) + \int_0^t \dot{Z}(t,s)g(s)ds$$
$$+ \int_0^t Z(t,s)\left(\int_{-h}^0 d_\theta H(s,\theta,x(s),u(s))u(t+\theta)\right)ds \quad \Big\} = 0$$

Letting $g = 0$ we have $Z(t,s)x_0 + \int_0^\infty S(t,s)u(t+\theta)ds = 0$, so that $v^T Z(t,s)x_0 + \int_0^\infty v^T S(t,s)u(t+\theta)ds = 0$ which implies that $v^T Z(t,s)x_0 = 0$, so $v^T = 0$ which is a contradiction to the fact that $v \neq 0$, hence $W$ must be non-singular. For the asymptotic null controllability of the non-linear system (1.1), we define an operator $T: C^n \times C^m \to C^n \times C^m,\ T(x,u) = (z,w)$ for which any fixed point $(x,u)$ Є $C^n \times C^m$ will satisfy (1.1) with $x(0) = x_0$ *and* $\lim_{t\to\infty} x(t) = 0$, Now for each $(x,u)$ Є $C^n \times C^m$ we define

$$z(t) = Z(t,0)[x(0) - g(0)] + g(t) + \int_o^t \dot{Z}(t,s)g(s)ds$$
$$+ \int_0^t Z(t,s)\left[\int_{-h}^0 d_\theta H(s.\theta,x(s),u(s))u(s+\theta) + f(s,x(s),u(s))\right]ds$$
$$w(t) = \left(S^T(t,s)\right)W^{-1}\left[-Z(x(0)-g(0)) - k - \int_0^\infty \bar{Z}g(s)ds - \int_0^\infty Zf(s,x(s))u(s)ds\right]$$

Note that if $(x,u)$ Є $C^n \times C^m$ is a fixed point of $T$ we have

$$x(t) = Z(t,0)[x(0) - g(0)] + g(t) + \int_0^t \dot{Z}(t,s)g(s)ds$$
$$+ \int_0^t Z(t,s)\left[\int_{-h}^0 d_\theta H(s,\theta,x(s),u(s))u(s+\theta) + f(s,x(s),u(s))\right]ds$$
$$u(t) = \left(S^T(t,s)\right)W^{-1}\left[-Z(x(0)-g(0)) - k - \int_0^t \bar{Z}g(s)ds - \int_0^\infty Zf(s,x(s),u(s))ds\right]$$

Thus $x(t)$ is the solution of (1) corresponding to the control $u(t)$, with $x(0) = x_0$ *and* $\lim_{t\to\infty} x(t) = 0$. To find such a fixed point we introduce a parameter $\mu \in [0,1]$ into problem (1.1) as follows

$$\frac{d}{dt}\left[x(t) - \int_0^t C(t,s)x(s)ds - g(t)\right] = A(t)x(t) + \int_0^t G(t,s)x(s)ds$$
$$+ \mu\left[\int_{-h}^0 d_\theta H(t,\theta,x(t),u(t))u(t+\theta) + f(t,x(t),u(t))\right]$$

Consider the operator $T(x,u,\mu) = (z,w)$, where

$$z(t) = \mu\Big\{Z(t,0)[x(0) - g(0)] + g(t) + \int_0^t \dot{Z}(t,s)g(s)ds$$
$$+ \int_0^t Z(t,s)\left[\int_{-h}^o d_\theta H(s,\theta,x(s),u(s))u(t+\theta) + f(s,x(s),u(s))\right]ds\Big\}$$
$$w(t) = \mu\Big\{\left(S^T(t,s)\right)W^{-1}\left[-Z(x(0)-g(0)) - k - \int_0^\infty \bar{Z}g(s)ds - \int_0^\infty Zf(s,x(s))u(s)ds\right]\Big\}$$

We need to show that in an appropriate Banach space D there exists a pair $(x,u) \in D\ with T(x,u,1) = (x,u)$. To show this we need the following Larry-Schauder's fixed point theorem as used in [2,3]. Note that $T(x,u,\mu)$ is completely continuous in $(x,u)$ for each μ = [0,1] and maps every bounded subset of $D$ into a relatively compact set.

## 3.0 Main result

We state and prove sufficient conditions for the system (1.1) to be asymptotically null controllable. For this result we denote $\sigma = \|W^{-1}\|$ and for any bounded $J$ we let $C(J, R^{n+m})$ be the Banach space of all continuous $R^n$-valued function on $J$ with the sup norm.

***Theorem* 3.1**

*Assume the following to hold for system* (1.1) *and system (2.1)*

*(i)* *System* (2.1) *is asymptotically null controllable*

*(ii)* *The fundamental matrix solution Z(t) is such that* $\|Z(t)\| \le m_1$ *and* $\|g(t)\| \le m_2$ *for* $t \in J$

*where* $m_1$ *and* $m_2$ *are some positive constants.*

*(iii)* *The* $n \times p$ *continuous matrix* $H(t, \theta, x(t), u(t))$ *is bounded on J with*

$$\max \sup \left\| \int_{-h}^{0} d_\theta H(s, \theta, x(s), u(s)) ds \right\| \le \beta \text{ for } \beta > 0, s \in J$$

*(iv)* *There exist constants P,R,N,M such that for* $s \in J$ *we have*

$$\lim_{t\to\infty} \sup \left\| \int_0^t \dot{Z}(t, s) g(s) ds \right\| \le P$$

$$\lim_{t\to\infty} \sup \left\| \int_0^t Z(t, s) \left( \int_{-h}^{0} d_\theta H(s, \theta, x(s), u(s)) u(s+\theta) \right) ds \right\| \le M$$

$$\lim_{t\to\infty} \sup \left\| \int_0^t Z(t, s) f(s, x(s), u(s)) ds \right\| \le R(\|x\| + \|u\|) + N$$

*(v)* *The matrix W is nonsingular*

*The system (1.1) is asymptotically null controllable if we can choose the constants such that*

$$[(M + R) + \beta m_1 \sigma R] < 1$$

***Proof***

Let $J_1 = [0,1]$, $d = n + p$and let the sup norm of $C(J_1,R^d)$ be $\|.\|_1$. Assume that $\omega = (x,u) \in C(J_1,R^n)$ and consider the function $\overline{\omega}(.)$ defined by $\overline{\omega}(t) = \omega(t)$ $t \in J_1$, $\overline{\omega}(t) = \omega(1)$ $t \in [1, \infty)$. Clearly the set of all $\overline{\omega}$ is a Banach space which we designate by $C_1$ with norm $\|\overline{\omega}\|_2 = \|\omega\|_1$. Here $C_1 = D_1 \times E_1$ where $D_1$ is defined with elements $x \in C(J_1,R^n)$ and $E_1$ defined with elements $u \in C(J_1,E^p)$ so $\|\overline{\omega}\|_2 = \|\overline{x}\|_{D_1} + \|\overline{u}\|_{E_1}$ where $\overline{\omega} = (\overline{x}, \overline{u}) \in C_1$. Consider the operator $T: C_1 \to C_1$defined by $T(\overline{x}, \overline{u}, \mu) = (\overline{z}, \overline{w})$, where

$$\overline{z}(t) = \mu\{Z(t,0)[x(0) - g(0)] + g(t) + \int_0^t \dot{Z}(t, s) g(s) ds$$

$$+ \int_0^t Z(t, s) \left[ \int_{-h}^{0} d_\theta H(s, \theta, x(s), u(s)) u(s+\theta) + f(s, x(s), u(s)) \right] ds \quad \}$$

$$\overline{w}(t) = \mu\left\{ (S^T(t, s)) W^{-1} \left[ -Z(x(0) - g(0)) - k - \int_0^\infty \overline{Z} g(s) ds - \int_0^\infty Z f(s, x(s), u(s)) ds \right] \right\}$$

Here we want to show that $T(\overline{x}, \overline{u}, 1)$ has a fixed point. First we shall prove that $T(\overline{x}, \overline{u}, \mu)$ is continuous in $\mu$. To see this let $\mu_1, \mu_2 \in [0,1]$ *and* $(\overline{x}, \overline{u}) \in C_1$. Then we have

$$\left|T(\bar{x},\bar{u},\mu_1)(t)-T(\bar{x},\bar{u},\mu_2)(t)\right| \le |\mu_1-\mu_2|\Big\{\left|Z(t)[x(0)-g(0)]\right|+|g(t)|+\left|\int_0^t \dot{Z}(t,s)g(s)ds\right|$$

$$+\left|\int_0^t Z(t,s)\left[\int_{-h}^0 d_\theta H(s,\theta,\bar{x}(s),\bar{u}(s))u(s+\theta)+f(\bar{x}(s),\bar{u}(s))\right]ds\right| \Big\}$$

$$+|\mu_1-\mu_2|\left\|(S^T(t,s))W^{-1}[Z(x0)-g(0)]\right\|-k-\int_9^\infty \bar{Z}g(s)ds$$

$$-\int_0^\infty Zf(s.\bar{x}(s),\bar{u}(s))ds\quad )|$$

$$\le |\mu_1-\mu_2|m_1|x_0-g(0)|+m_2+p+M\|\bar{u}\|+R(\|\bar{x}\|+\|\bar{u}\|)+N$$

$$+|\mu_1-\mu_1|\beta m_1\sigma\{m_1|x_0-g(0)|+k+P+R(\|\bar{u}\|+\|\bar{x}\|)+N\}$$

Since $\| T(\bar{x},\bar{u},\mu_1)-T(\bar{x},\bar{u},\mu_2) \|_2=\sup_{t\in J_1}\left|T(\bar{x},\bar{u},\mu_1)(t)-T(\bar{x},\bar{u},\mu_2)(t)\right|$. It follows that the operator $T(\bar{x},\bar{u},\mu)$ is continuous in $\mu$ uniformly in any bounded subset of $C_1$. Let

$$\varpi=(\bar{x},\bar{u})\qquad \varpi_n=(\bar{x}_n,\bar{u}_n),\ \omega,\omega_n\in C_1\quad n=1,2,\Lambda$$

$$\bar{y}_n=(\bar{z}_n,\varpi_n)=T(\bar{x}_n,\bar{u}_n,\mu),\ \bar{y}=(\bar{z},\varpi)=T(\bar{x},\bar{u},\mu)$$

Suppose that $\lim_{n\to\infty}\|\varpi_n-\varpi\|_2=\lim_{n\to\infty}\|\omega_n-\omega\|_1=0$. That is, $\lim_{n\to\infty}\|\bar{x}_n-\bar{x}\|=0$ and $\lim_{n\to\infty}$ $\|\bar{u}_n-\bar{u}\|=0$

Then we have

$$\|\bar{y}_n-\bar{y}\|=\sup_{t\in J_1}|y_n(t)-y(t)|=\sup_{t\in J_1}\left[|z_n(t)-z(t)|+|w_n(t)-w(t)|\right]$$

Now

$$\sup_{t\in J_1}|z_n(t)-z(t)|\le\int_0^\infty\Big\|Z(s)\Big[\int_{-h}^0 d_\theta H(s,\theta,\bar{x}(s),(\bar{u}_n(s)-\bar{u}(s)))(\bar{u}_n(s+\theta)-\bar{u}(s+\theta))$$

$$+f(s,\bar{x}(s),\varpi_n(s))-f(s,\bar{x}(s),\varpi(s))\ \Big]\Big\|ds$$

And the integral tends to zero as $n\to\infty$. Thus it follows that from the lebesgue dominated convergence theorem that $\lim_{n\to\infty}\|\bar{z}_n-\bar{z}\|=0$. Similarly

$$\lim_{n\to\infty}\|\varpi_n-\varpi\|=0\text{, therefore }\lim_{n\to\infty}\|\bar{y}_n-\bar{y}\|=0.$$

Thus this proves that $T(\varpi,\mu)$ with respect to $\varpi$ is continuous. Let $K$ be a bounded subset of $C_1$ with bound $b_K$. We now show that the family of functions $\bar{y}=T(\varpi,\mu),\ \varpi\in K,\ \mu\in[0,1]$ are equicontinuous. Let $t_1,t_2\in[0,1]$, then,

$$|\bar{y}(t_1)-\bar{y}(t_2)|=|\bar{z}(t_1)-\bar{z}(t_2)|+|\bar{w}(t_1)-\bar{w}(t_2)|$$

Now

$$\left|z(t)-\bar{z}(t)\right| \le \left\|Z(t_1)-Z(t_2)\right\|\left\|x_0-g(0)\right|+\left|g(t_1)-g(t_2)\right|+\left\|\int_0^{t_1} Z(t_1,s)g(s)ds-\int_0^{t_2} Z(t_2,s)g(s)ds\right\|$$

$$+\left\|\int_0^{t_1} Z(t_1,s)\left[\int_{-h}^0 d_\theta H(s,\theta,\bar{x}(s),\bar{u}(s))\bar{u}(s+\theta)+f(s,\bar{x}(s),\bar{u}(s))\right]ds\right.$$

$$\left.-\int_0^{t_Z} Z(t_2,s)\left[\int_{-h}^0 d_\theta H(s,\theta,\bar{x}(s)\bar{u}(s))\bar{u}(s+\theta)+f(s,\bar{x}(s),\bar{u}(s))\right]ds\right.$$

$$\le \left\|Z(t_1)-Z(t_2)\right\|\left\|x_0-g(0)\right|+\left\|g(t_1)-g(t_2)\right\|+\int_0^{t_1}\left\| Z(t_1,s)-Z(t_2,s)\right\|\left\|g(s)\right\|ds-\int_{t_1}^{t_2}\left\|Z(t_2,s)\right\|g(s)ds$$

$$+\int_0^{t_1}\left\|Z(t_1,s)-Z(t_2,s)\right\|\left[\int_{-h}^9 d_\theta H(s,\theta,\bar{x}(s),\bar{u}(s))\bar{u}(s+\theta)+f(s,\bar{x}(s),\bar{u}(s))\right]ds$$

$$+\int_{t_1}^{t_2}\left\|Z(t_2,s)\left[\int d_\theta H(s,\theta,\bar{x}(s),\bar{u}(s))\bar{u}(s+\theta)+f(s,\bar{x}(s),\bar{u}(s))\right]\right\|ds$$

$$+\int_0^{t_1}\left\|Z(t_1,s)-Z(t_2,s)\right\|\left[\int_{-h}^9 d_\theta H(s,\theta,\bar{x}(s),\bar{u}(s))\bar{u}(s+\theta)+f(s,\bar{x}(s),\bar{u}(s))\right]ds$$

$$+\int_{t_1}^{t_2}\left\|Z(t_2,s)\left[\int d_\theta H(s,\theta,\bar{x}(s),\bar{u}(s))\bar{u}(s+\theta)+f(s,\bar{x}(s),\bar{u}(s))\right]\right\|ds$$

and $\left|\bar{w}(t_1)-\bar{w}(t_2)\right| \le \left\|S^T(t_1,s)-S^T(t_2,s)\right\|\sigma\left[m_1\left|x_0-g(0)\right|+k+P+R\left\|\bar{\omega}\right\|+N\right]$

These estimates show that the given family of functions is equicontinuous. To show that the family of functions is uniformly bounded follows from the following argument, $\left\|\bar{y}\right\|_2=\left\|\bar{z}\right\|_{D_1}+\left\|\bar{w}\right\|_{E_1}$

where

$$\left\|\bar{z}\right\|_{D_1} \le m_1\left|x_9-g(0)\right|+m_2+P+M\left(\left\|\bar{u}\right\|_{E_1}+\left\|\bar{x}\right\|_{D_1}\right)+N$$
$$\left\|\bar{w}\right\|_{E_1} \le \beta m_1\sigma\left[m_1\left|x_0-g(0)\right|+k+P+R\left(\left\|\bar{u}\right\|_{E_1}+\left\|\bar{x}\right\|_{D_1}\right)+N\right]$$

Thus the given family of functions is uniformly bounded and equicontinuous. Hence, the equicontinuity and uniform boundedness of the family of functions implies that $T(k,\mu)$ is relatively compact in $C_1$ for each $\mu\in[0,1]$. Now assume that the equation

$$T(\bar{\omega},\mu)-\bar{\omega}=0 \tag{3.1}$$

Has a solution $\bar{\omega}\, in\, C_1$ for fixed $\mu\in[0,1]$, then,

$$\left|\bar{\omega}(t)\right|=\left|\bar{x}(t)+\left|\bar{u}(t)\right|\right| \le m_1\left|x_0-g(0)\right|+m_2+P+M\left\|\bar{u}\right\|+R\left\|\bar{\omega}\right\|+N$$
$$+\beta m_1\sigma\left[m_1\left|x_0-g(0)\right|+k+P+R\left\|\bar{\omega}\right\|+N\right]\le \delta_1+\delta_2\left\|\bar{\omega}\right\| \quad t\in[0,1]$$

where $\delta_1=(1+\beta m_1\sigma)\left[m_1\left|x_0-g(0)\right|+P+N\right]+m_2+\beta m_1\sigma,\ \delta_2=(M+R)+\beta m_1\sigma R$

which implies $$\left\|\bar{\omega}\right\|_{C_1}=\left\|\omega\right\|\le\delta_1(1-\delta_2)^{-1}\equiv\delta \tag{3.2}$$

By the assumption $\left[(M+R)+\beta m_1\sigma R\right]<1$. It follows that the solution of (3.1) is uniformly bounded with respect to $\mu[0,1]$. Hence the conditions of Larry-Schauder's theorem are satisfied for the interval $J_1$ = [0,1], so that $T(\bar{\omega},1)$ has a fixed point $\bar{y}=T(\bar{\omega},1)=\omega\in C_1$ and so equation (1.1) is asymptotically null controllable in the interval $J_1$. The above analysis can be similarly carried to further intervals and indeed the entire interval $J=(t_0,\infty)$ by considering intervals $J_k=[t_0,t_k]$, $1\le k<\infty$. Hence theorem is proved.

## 4.0 Conclusion

We proved the asymptotic null controllability of a nonlinear neutral volterra integrodifferentail system using the Larry-Schauder's theorem. The method involves the development of sufficient conditions that guarantees steering the solution of the control system to the zero in the long run.

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