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A new analytical solution to the optimal control problem for the control of higher-order non-dispersive wave

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Abstract

***The explicit expressions of the functions: z(x,y,t) and u(x,y,t) were obtained in this paper. These two functions are known as the state and the control variables for the optimal control model of higher order nondispersive wave equation. They are indispensable in the implementation of the model using the extended conjugate gradient method* [2].**

Keywords: optimal control, maximum principle, state variable, control variable

1.0 Introduction

Solutions of optimal control models of partial differential equations arising from mathematical physics have been of interest in many scholarly works. For example the optimal control of the Navier-Stoke's equation was discussed in [7] and the control of the Energized wave equation in [8], and so on. We present the analytical solution of the higher order nondispersive wave in this work.

The optimal control problem for higher order nondispersive wave, [6], is given by:

$$\min J(z,u) = \min \int_0^1\int_0^1\int_0^1 (z^2(x,y,t) + u^2(x,y,t))\, dxdydt$$

such that

$$\frac{\partial^2}{\partial t^2} z(x,y,t) = -c_0 \frac{\partial^2}{\partial x^2} z(x,y,t) - c_0 \frac{\partial^2}{\partial y^2} z(x,y,t \quad + \quad u(x,y,t)$$

$$z(0,y,t) = z(1,y,t) = z(x,0,t) = z(x,1,t) = 0,\; z(x,y,0) = z_0(x,y),\; z(x,y,1) = z_1(x,y) \tag{1.1}$$

We shall convert equation (1.1) into an unconstrained problem using the penalty function μ, [2] and [3]. Thus we have:

$$\min J(z,u) = \min \int_0^1\int_0^1\int_0^1 \Big\{\big(z^2(x,y,t) + u^2(x,y,t)\big) + \mu \left\| \frac{\partial^2 z(x,y,t)}{\partial t^2} + c_0 \frac{\partial^2 z(x,y,t)}{\partial x^2} + c_0 \frac{\partial^2 z(x,y,t)}{\partial t^2} - u(x,y,t) \right\|^2 \Big\} dxdydt \tag{1.2}$$

2.0 The Hamiltonian of the system

From [5] we can set up the Hamiltonian of the system in equation (1.1) as:

$$H(z(x,y,t),\, u(x,y,t),\, \lambda(t)) = z^2(x,y,t) + u^2(x,y,t) + \lambda(t)(u(x,y,t) - c_0 \frac{\partial^2}{\partial x^2} z(x,y,t) - c_0 \frac{\partial^2}{\partial y^2} z(x,y,t)) \tag{2.1}$$

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For simplicity, we adopt the following substitutions. Let

$$z = z(x,y,t),\; u = u(x,y,t),\; z_{xx} = \frac{\partial^2}{\partial x^2} Z(x,y,t),\; z_{yy} = \frac{\partial^2}{\partial y^2} z(x,y,t) \text{ and } \lambda = \lambda(t)$$

Applying the maximum principle, [5], on equation (1.2), we have:

$$\frac{\partial H}{\partial u} = 0 \text{ and } \lambda_t = -\frac{\partial H}{\partial z} \tag{2.2}$$

Hence applying the condition stated in equation (2.2) on equation (1.2) we have:

$$\frac{\partial H}{\partial u} = 2u + \lambda = 0$$

and

$$\frac{\partial H}{\partial z} = 2z \tag{2.3}$$

Therefore from equation (2.3) we have:

$$2u + \lambda = 0$$

$$\Rightarrow \quad \lambda = -2u \tag{2.4}$$

Differentiating equation (2.4) with respect to t, we have:

$$\lambda_t = -2\frac{\partial u}{\partial t} \tag{2.5}$$

But from equation (2.2) we also have that: $\lambda_t = -2\frac{\partial H}{\partial z}$

Combining equation (2.3), equation (2.4) and equation (2.5) we have:

$$\frac{\partial u}{\partial t} = z \tag{2.6}$$

We shall apply the result of equation (3.1) on the constrain of equation (1.1). Thus we have:

$$\frac{\partial^3 u}{\partial t^3} = u - c_0 z_{xx} - c_0 z_{yy} \tag{2.7}$$

Equation (2.7) is of interest in this work.

3.0 Fourier series representation of higher order non-dispersive wave

We assume a Fourier series representation of the state, $z(x,y,t)$ and the control, $u(x,y,t)$ variables of the optimal control problem, equation (1.1). Thus we adopt the following substitutions:

$$z(x,y,t) = \sum_{i=1}^{\infty} z_i(t)\sin(i\pi x)\sin(i\pi y)$$

$$u(x,y,t) = \sum_{i=1}^{\infty} u_i(t)\sin(i\pi x)\sin(i\pi y) \tag{3.1}$$

Hence

$$u_{ttt} = \sum_{i=1}^{\infty} u_{ittt}(t)\sin \pi ix \sin \pi iy$$

$$z_{xx} = -\sum i^2\pi^2 z_i(t)(\sin(i\pi x)\sin(i\pi y))$$

$$Z_{yy} = -\sum i^2\pi^2 z_i(t)\sin(i\pi x)\sin(i\pi y) \tag{3.2}$$

Using equation (3.1) and equation (3.2) in equation (3.1), we have:

$$\sum_{i=1}^{\infty} z_{ttti} \sin(i\pi x)\sin(i\pi y) = \sum_{i=1}^{\infty} u_i(t)\sin(i\pi x)\sin(i\pi y)$$
$$+ c_0 \sum i^2\pi^2 z_i(t)\sin(i\pi x)\sin(i\pi y) + c_0 \sum_{i=1}^{\infty} z_i(t) i^2\pi^2 \sin(i\pi)\sin(i\pi y) \qquad (3.3)$$

Equation (3.3) is another form of the constraint on equation (2.7). Therefore the two equations are equivalent. The n^{th} term of the series of equation (3.3) is:

$$u_{nttt}(t) - 2c_0 n^2\pi^2 u_{nt}(t) - u_n(t) = 0 \qquad (3.4)$$

Using equation (3.4) and the result of [6], we rewrite the constraint problem of equation (1.1) as:

$$\min\left\{\int_0^1 \left[u_{1t}^2 + u_{2t}^2 + \quad \ldots \quad + u_{it}^2 \right]dt + \left\{\int_0^1 \left[u_{2t}^2 + u_2^2 + \quad \ldots \quad + u_i^2 \right]dt\right.\right.$$
$$\left\{\int_0^1 \left[u_{1t}^2 + u_{2t}^2 + \quad \ldots \quad + u_{it}^2 \right]dt + \left\{\int_0^1 \left[u_1^2 + u_2^2 + \quad \ldots \quad + u_i^2 \right]dt\right.\right.$$
$$\left\{\int_0^1 \left[u_{1t}^2 + u_{2t}^2 + \quad \ldots \quad + u_{it}^2 \right]dt + \left\{\int_0^1 \left[u_1^2 + u_2^2 + \quad \ldots \quad + u_i^2 \right]dt\right.\right.$$

such that

$$u_{1ttt}(t) = 2i^2\pi^2 u_{1it}(t) + u_1(t)$$
$$u_{2ttt}(t) = 2.2^2\pi^2 u_{2t}(t) + u_2(t)$$
$$u_{3ttt}(t) = 2.3^2\pi^2 u_{3t}(t) + u_3(t)$$

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$$u_{nttt}(t) = 2n^2\pi^2 u_{nt}(t) + u_n(t) \qquad (3.5)$$

Using [6], we convert the unconstrained problem, equation (1.2), into a constraint problem. Thus we have :

$$\min\left\{\int_0^1 \left[u_{1t}^2 + u_{2t}^2 + \Lambda + u_{it}^2 \right]dt + \int_0^1 \left[u_1^2 + u_2^2 + \Lambda + u_i^2 \right]dt\right.$$
$$\mu\left[\left\| u_{1ttt}(t) - 2i^2\pi^2 u_{1t}(t) - u_1(t) \right\|^2 + \left\| u_{2ttt}(t) - 2i^2\pi^2 u_{2t}(t) - u_2(t) \right\|^2 + \ldots +\right.$$
$$\left.\left.\left\| u_{nttt}(t) - 2n^2\pi^2 u_{nt}(t) - u_n(t) \right\|^2 \right]dt\right\} \qquad (3.6)$$

Consider the general form of the constraint in equation (3.5). That is:

$$u_{nttt} - 2c_0 n^2\pi^2 u_{nt} - u_n = 0 \qquad (3.7)$$

The auxiliary system of equation (3.7)) is:

$$(m^3 - 2c_0 n^2\pi^2 \; m - 1)u_n = 0 \qquad (3.8)$$

Since $u(x,y,t)$ is not zero, we have:

$$m^3 - 2c_0 n^2\pi^2 \; m - 1 = 0 \qquad (3.9)$$

With MATLAB the roots of equation (2.9) are:

$$m_1 = \left(\frac{\frac{1}{6}P + 2k}{P} \right) \tag{3.10}$$

$$m_2 = \frac{(-\tfrac{1}{2}P - k)(P - i\frac{\sqrt{3}}{2}P + 2k)}{(P - \frac{2k}{P})^2 + \frac{3}{4}P^2} \tag{3.11}$$

$$m_3 = \frac{(-\tfrac{1}{2}P - k)(P + i\frac{\sqrt{3}}{2}P - 2k)}{(P - 2k/P)^2 + \frac{3}{4}P^2} \tag{3.12}$$

where

$k = -2c_0 n^2\pi^2$ and $P = 108 + 12\sqrt{(-12k^3 + 81)^{1/3}}$

Hence the solution:

$$u_n(t) = Ae^{m_1 t} + Be^{m_2 t} + Ce^{m_3 t}. \tag{3.13}$$

Substitute equation (3.10), equation (3.11) and equation (3.12) into equation (3.13) we have:

$$\therefore u_n(t) = A\exp((p+2k)/6p)t + B\exp((-\frac{1}{2}p-k)(p-i\frac{\sqrt{3}}{2}+2k)/((p-\frac{2k}{p})^2+\frac{3}{4}p^2))t \; + \; C\exp((-\frac{1}{2}p-k)(p+i\frac{\sqrt{3}}{2}+2k)/((p-\frac{2k}{p})^2+\frac{3}{4}p^2))t \tag{3.14}$$

Simplifying equation (2.14), noting that m_2 and m_3 are complex conjugates, we have:

$$u_n(t) = A_1\exp((p+2k)/6p)t + A_2\exp(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))t \; \cos(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t \; + \; A_3\exp(2p(-p-4k(p+k)))t/4(p^2-2k)^2+3p^2))\sin(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t \tag{3.15}$$

where $A_1 = A,\ A_2 = (B+C)$ *and* $A_3 = i(B-C)$

3.1 Determination of the explicit expressions for the State and the control variables

From equation (3.15) we can express the control variable, $u(x,y,t)$ as:

$$u(x,y,t) = u_n(t) = \sum u_i(t)\sin i\pi x \sin i\pi y \tag{3.16}$$

From [8], equation (3.16) becomes:

$$u(.\,.,t) = u_n(t) = A_1\exp((p+2k)/6p)t + A_2\exp(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))t \; \cos(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t + A_3\exp(2p(-p-4k(p+k)))t/4(p^2-2k)^2+3p^2)) \; \sin(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t = \sum u_i(t)\sin i\pi x \sin i\pi y \tag{3.17}$$

Putting $t = 0$ on equation (3.17), we have:

$$u\ (.,.,0) = A_1 + A_3 = \sum u_i(0)\sin i\pi x \sin i\pi y \tag{3.18}$$

Further, differentiating equation (3.17) with respect to t yields:

$$u_t(.,.,t) = A_1((p+2k)/6p)\exp((p+2k)/6p) + A_2\exp(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))t((2p(-p-4k(p+k))/4(p^2-2k)^2+3p^2)$$
$$\cos(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t - (p\sqrt{3}(p+2k)/(4(p^2-2k)+3p^2)\sin(p\sqrt{3}(p+2k)/(4(p^2-2k)^2)$$

$$+ A_3\exp(2p(-p-4k(p+k)))t/4(p^2-2k)^2+3p^2))t)(2p(-p-4k(p+k))/4(p^2-2k)^2+3p^2)$$
$$\sin(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t + p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2)\cos(p\sqrt{3}(p+2k)/4(p^2-2k)+3p^2)$$

$$= \sum u_{it}(t)\sin i\pi x \sin i\pi y \tag{3.19}$$

Similarly, putting $t = 0$ in equation (3.19) we have:

$$u_t(x,y,0) = A_1((p+2k)/6p) + A_2(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)) + A_3(2p(-p-4k(p+k))/4(p^2-2k)^2+3p^2)$$
$$= \sum u(0)\sin i\pi x\sin i\pi y \tag{3.20}$$

By differentiating equation (3.19) with respect to t and applying the initial condition again we also have:

$$u_{it}(.,.,0) = A_1(((p+2k)/6p)^2) + A_2((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)$$
$$-((p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2) + A_3(4(-p-4k(p+k))/$$
$$4(p^2-2k)^2+3p^2)(p\sqrt{3}(p+2k)/4(p^2-2k)+3p^2)$$
$$= \sum u_{itt}(0)\sin i\pi x\sin i\pi y \tag{3.21}$$

The summary of the results from equation (3.18) to equation (3.21) are as follows:

$$A + A_3 = \sum u_t(0)\sin nx\pi\sin n\pi y \tag{3.22}$$

$$A_1((p+2k)/6p) + A_2(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)) + + A_3(2p(-p-4k(p+k))/4(p^2-2k)^2+3p^2)$$
$$= \sum u_{it}(0)\sin i\pi x\sin i\pi y \tag{3.23}$$

$$A_1(((p+2k)/6p)^2) + A_2((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))^2 - ((p\sqrt{3}(p+2k)/4(p-2k)^2+3p^2)$$
$$+ A_3(4p(-p-4k(p+k))/4(p^2-2k)^2+3p^2)(p\sqrt{3}(p+2k)/4(p^2-2k)+3p^3)$$
$$= \sum u_{itt}(0)\sin i\pi x\sin i\pi y \tag{3.24}$$

Solving equation (3.22), equation (3.23) and equation (3.24) simultaneously for the constants A_1, A_2 and A_3 we obtain:

$$A_1 = \{(\sum u_{it}(0)\sin i\pi x\sin i\pi y - [2(\sum u_{itt}(0)\sin i\pi x\sin i\pi y - ((P+12k)/6p)\sum u_i(0)\sin i\pi x\sin i\pi y)(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)) -$$
$$(\sum u_{itt}(0)\sin i\pi x\sin i\pi y - ((P+12k)/6p)^2\sum u_i(0)\sin i\pi x\sin i\pi y)]/2(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)$$
$$[(4p(-p-4(k(p+k))/(4(p^2-2k)+3p^2)-(p+12k)/6p)]-[((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)^2)-$$
$$((p+12k)/6p)^2 - (p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2))^2]\} \tag{3.25}$$

$$A_2 = [2(\sum u_{itt}(0)\sin i\pi x \sin i\pi y \; -((P+12k)/6p)\sum u_i(0)\sin i\pi x \sin i\pi y)(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)) -$$
$$(\sum u_{itt}(0)\sin i\pi x \sin i\pi y - ((P+12k)/6p)^2 \sum u_i(0)\sin i\pi x \sin i\pi y)]/2(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)$$
$$[(4p(-p-4(k(p+k))/(4(p^2-2k)+3p^2)-(p+12k)/6p)] - [((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)^2)-$$
$$((p+12k)/6p)^2 - (p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2))^2]$$

(3.26)

$$A_3 = [(\sum u_{itt}(0)\sin i\pi x \sin i\pi y - (((p+12k)/6p)^2)\sum u_i(0)\sin i\pi x \sin i\pi y)[2(-p-4k(p+k)/4(p^2-2k)^2+3p^2)-((p+12k)/6p)] -$$
$$[(\sum u_{it}(0)\sin i\pi x \sin i\pi y - ((p+12k)/6p)\sum u_i(0)\sin i\pi x \sin i\pi y)$$
$$[((-p-4k((p+k))/4(p^2-2k)^2+3p^2)^2)-((p+2k)/6p)^2-((p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2)^2))]/$$
$$p\sqrt{3}((p+k)))/4(p^2-2k)^2+3p^2)(2p(-p-4k(p+k))/(4(p^2-2k)^2+3p^2)[(4p(-p-4(k(p+k))/(4(p^2-2k)+3p^2)-(p+12k)/6p)]$$

(3.27)

Substituting equation (3.25), equation (3.26) and equation (3.27) in equation (3.15) we obtain an explicit expression for the control, $u(x,y,t)$. Thus:

$$\{(\sum u_{it}(0)\sin i\pi x \sin i\pi y \; - (2(\sum u_{itt}(0)\sin i\pi x \sin i\pi y \; -((p+12k)/6p)\sum u_i(0)\sin i\pi x \sin i\pi y)$$
$$((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)) - (\sum u_{itt}(0)\sin i\pi x \sin i\pi y - ((p+12k)/6p)^2 \sum u_i(0)\sin i\pi x \sin i\pi y)/$$
$$[(2(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))[(4p(-p-4(k(p+k))-(p+12k)/6p)]$$
$$-[((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)^2)-((p+12k)/6p)^2 -$$
$$(p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2))^2]\}\exp((p+2k)/6p)t$$

$u(x,y,t)$ =

$$+$$
$$2(\sum u_{itt}(0)\sin i\pi x\sin i\pi y \ -((p+12k)/6p)\sum u_i(0)\sin i\pi x\sin i\pi y)(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))-$$
$$(\sum u_{itt}(0)\sin i\pi x\sin i\pi y-((p+12k)/6p)^2\sum u_i(0)\sin i\pi x\sin i\pi y)/$$
$$2(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)[(4p(-p-4(k(p+k))-(p+12k)/6p)]$$
$$-[((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)^2)-((p+12k)/6p)^2-(p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2))^2]$$
$$\exp(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))t$$
$$\cos(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t$$
$$+$$
$$(\sum u_{itt}(0)\sin i\pi x\sin i\pi y-((p+12k)/6p)^2\sum u_i(0)\sin i\pi x\sin i\pi y))[4p(-p-4k(p+k)/4(p^2-2k)^2+3p^2)$$
$$-((p+2k)/6p)]-(\sum u_{it}(0)\sin i\pi x\sin i\pi y-((p+2k)/6p)\sum u_i(0)\sin i\pi x\sin i\pi y))$$
$$[((-p-4k((p+k))/4(p^2-2k)^2+3p^2)^2)-((p+2k)/6p)^2-((p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2)^2))/$$
$$(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)[(8p(-p-4(k(p+k))-(p+12k)/6p)]$$
$$-[((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)^2)-((p+12k)/6p)^2-(p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2))^2]$$
$$\exp(2p(-p-4k(p+k)))t/4(p^2-2k)^2+3p^2))\sin(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t) \qquad (3.28)$$

Therefore the explicit expression of the state variable, $z(x,y,t)$ is obtained by using the results of equation (3.25), equation (3.26) and equation (3.27) on equation (3.19) in the spirit of equation(2.6). Thus we have:

z(x,y,t) =

$$((p+2k)/6p)\{(\sum u_{it}(0)\sin i\pi x\sin i\pi y - (2(\sum u_{itt}(0)\sin i\pi x\sin i\pi y \ -((p+12k)/6p)\sum u_i(0)\sin i\pi x\sin i\pi y)$$
$$((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))-(\sum u_{itt}(0)\sin i\pi x\sin i\pi y-((p+12k)/6p)^2\sum u_i(0)\sin i\pi x\sin i\pi y)/$$
$$[(2(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))[(4p(-p-4(k(p+k))-(p+12k)/6p)]$$
$$-[((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)^2)-((p+12k)/6p)^2-$$
$$(p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2))^2]\}\exp((p+2k)/6p)t$$
$$+$$
$$2(\sum u_{itt}(0)\sin i\pi x\sin i\pi y \ -((p+12k)/6p)\sum u_i(0)\sin i\pi x\sin i\pi y)(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))-$$
$$(\sum u_{itt}(0)\sin i\pi x\sin i\pi y-((p+12k)/6p)^2\sum u_i(0)\sin i\pi x\sin i\pi y)/$$
$$2(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)[(4p(-p-4(k(p+k))-(p+12k)/6p)]$$
$$-[((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)^2)-((p+12k)/6p)^2-(p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2))^2]$$
$$\exp(2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))t[((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2))$$
$$\cos(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t-(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))$$
$$\sin(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t]$$

$$+$$

$$(\sum u_{itt}(0)\sin i\pi x\sin i\pi y-((p+12k)/6p)^2\sum u_i(0)\sin i\pi x\sin i\pi y))[4p(-p-4k(p+k)/4(p^2-2k)^2+3p^2)$$
$$-((p+2k)/6p)]-(\sum u_{it}(0)\sin i\pi x\sin i\pi y-((p+2k)/6p)\sum u_i(0)\sin i\pi x\sin i\pi y))$$
$$[((-p-4k((p+k))/4(p^2-2k)^2+3p^2)^2)-((p+2k)/6p)^2-((p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2)^2))/$$
$$(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)[(8p(-p-4(k(p+k))-(p+12k)/6p)]$$
$$-[((2p(-p-4k(p+k)))/4(p^2-2k)^2+3p^2)^2)-((p+12k)/6p)^2-(p\sqrt{3}(p+k)/4(p^2-2k)^2+3p^2))^2]$$
$$\exp(2p(-p-4k(p+k)))t/4(p^2-2k)^2+3p^2))[(2p(-p-4k(p+k)))t/4(p^2-2k)^2+3p^2))$$
$$\sin(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t)+(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t)$$
$$\cos(p\sqrt{3}(p+2k)/4(p^2-2k)^2+3p^2))t)] \qquad (3.29)$$

4.0 Conclusion

The functions obtained as u(x,y,t) and z(x,y,t) and given in equation (3.28) and equation (3.29) are the optimal control variable and the optimal state variable for the optimal control model of higher order nondispersive wave, equation(1.1). These functions are indispensable in the simulation of the optimal state and the optimal control variables of the optimal control model of higher order nondispersive wave using the Extended Conjugate Gradient Method due to [2]. The algorithm of the Extended Conjugate Gradient Method is given below:

Step 1:

Guess x_0, u_0

Step 2:

Compute g_0

For $i = 0$

Step 3:

$p_0 = g_0$

Step 4:

$x_{i+1} = x_i + \alpha_i A p_i$

$u_{i+1} = u_i + \alpha_i p_i$;

where $\alpha_i = \dfrac{< g_i, g_i >}{< p_i, Ap_i >}$, for $i = 1, 2, \ldots, n$

Step 5:

$g_{x,\, i+1} = g_{x,i} + \alpha_i A p_{x,i}$

$g_{u,\, i+1} = g_{u,i} + \alpha_i A p_{u,i}$

$p_{x,\, i+1} = p_{x,i} + \beta_i A p_{x,i}$

$p_{u,\, i+1} = p_{u,i} + \beta_i A p_{u,i}$

where $\beta_i = \dfrac{< g_{i+1}, g_{i+1} >}{< g_i, g_i >}$, for $i = 1, 2, \ldots, n$

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