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Abstract

Mathematical models describing the effects of noise on the level of hearing were formulated. It is shown that sound above 85dB affect the position or orientation of the Hair Cells on the basilar membrane in the inner ear. This change in the orientation of HC affects the ionic exchange at the foot the HC. This allows the release of neuron transmitters, which set up potentials that will transmit impulse to the auditory nerve. The noise so received and transmitted as impulse signal to the auditory nerve is sent to the hypothalamus section of the brain, for interpretations.

1.0 Introduction

We defined noise as that sound made by an object, whose peak is above the normal peak, which should normally be received by the ear. When an object is mechanically or otherwise made to produce this sound, the sound could be classified as low-pitched, normal pitched or high-pitched sound, and it is this type of sound that we shall call noise.

Noise has two major aspects: the frequency or pitch and the intensity or amplitude. The frequency is determined by how rapidly the sound wave vibrates and is measured by the number of its waves passing a given point in one second. The nuisance effects of noise depend on many variables including: duration, pattern, pitch, distance of noise source, time of the day and the individual disposition of the affected person.

In order to understand clearly the mathematical explanation of this mechanism, we shall introduce some vital theoretical concepts.

2.0 The model

The stereocillia (microvilli) in the Basilar membrane are interconnected with cross-links, in such a way that displacement of the bundle by the energy of the incoming wave, changes the stain in it, thus affecting the transduction opening probability. Usually, there exist potential differences across the cell membranes and on displacement of the villi, there will be depolarization [Bell Holmes (1986)], thereby setting up ion-current in the base of the villi.

Right in the endolymph of this inner ear at the cochlea is an elastic structure called the Basilar membrane. At this membrane are distributed the fibres of the auditory nerve. These fibres are called the hair cells (HC). These hair cells' orientations are usually distorted when the wave passes through the endolymph that surrounds them. This distortion allows for ion exchange at the foot of the fibres. It is this ion exchange that actually relays the sound to the auditory nerve, by setting up a potential at the foot of the HC, which will be transmitted to the brain. It has to be further stated that the HC is believed to have a bundle of microvilli, protruding out of it to the surrounding endolymph. These microvilli are here assumed to be Hooke elastic in which case there is an amount of energy which when exerted on it causes it to be partially or permanently distorted or damaged. It is this particular behaviour of the microvilli, and also the Basilar membrane, that actually determines the level of hearing of an individual.

It might be very important to state that it is not actually the wave peak or nature that concerns us, rather the energy that these waves exert or immerse on the objects along their channel of flow should be our major concern.

This compliance structure of the Basilar membrane, which is measured as volume displaced per unit length, per unit pressure difference, increases exponentially with distance. This length variation is most important within the first 7mm distance of its length, since the Basilar membrane is long and narrow. Now, the cochlea is very shallow, about 2mm for each of the half. Von Bekesy (1960 [10]) showed that the wave propagation in the cochlea is not necessarily that of the shallow water wave phenomenon. This is because change in depth of the cochlea model, does not elicit much change in the form of wave in it.

In developing the theories concerning hearing, Helmholts (1954), suggested that each part of the inner ear is tuned to a particular frequency and the discrete system of resonators. However, modern theories of cochlea mechanism are derived from the observation of Von Bekesy (1960 [10]), which are:

i. For a pure tone, there is a traveling wave initiated in the cochlea. The amplitude of this wave has a peak at a position that varies linearly, with the frequency of the tone.

ii. The compliance of the Basilar membrane increase exponentially, with distance from the base of the cochlea;

iii. The Basilar membrane is not under tension and, when it has a load, it resists being bent.

The theory of the Basilar membrane in the endolymph is not necessarily a simple one; thus we shall place our conditions reasonably enough so as to make our study a bit simple and meaningful. We therefore have the following conditions:

a. The cochlea is long and narrow and thus, we shall consider it to be in two-dimensions, so that it will appear only as a line or narrow plate.

b. Each point of this membrane shall respond like a system with mass and density.

c. Elasticity to external forces is such that is displaces the membrane and, thus, the villi from equilibrium.

d. The flow of wave in the cochlea is considered irrotational and inviscid.

e. The motion of the flow is of low amplitude.

The last condition here will enable us neglect the non-linear terms, as well as apply the boundary conditions associated with the Basilar membrane, in the un-distorted position. For further study of the mathematical work of the cochlea and Basilar membrane theories of hearing in man, see Bell and Holmes (1986a, 1986b, 1996c [6]) Brinley (1978 [2]), Carpenter and Reese (1981 [3]).

Having explained all these, let us now look at it mathematically: The equation of sound wave in the air, which transmits the sound to the ear, when the noise is made, is given as:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1\partial^2 y}{c^2 \partial t^2} \tag{2.1}$$

where y is the displacement, and it is a function of distance traveled by the wave and then time, i.e. $y = y(x,t), c$ is the speed of wave. In solving this equation according to Pain (1968 [8]), we have:

$$y(x,t) = f_1(ct - x) + f_2(ct + x) \tag{2.2}$$

If we consider the motion of the Basilar membrane in the fluid as vibrational, then we can consider the wave motion past the villi as oscillatory, so that the displacement of these villi will be that of a simple harmonic oscillator. Consequently, we get the explicit form of equation (2.2) as:

$$y(x,t) = a\sin(\omega t - \phi) = a\sin 2\pi/\lambda(ct - x) \tag{2.3}$$

where $2\pi c/\lambda = \omega$ and $\phi = 2\pi x/\lambda,$ and λ is the wave length, while $'a'$ is the amplitude of the wave.

From the literature, it is this wave that hits the tympanic membrane, thereby transferring certain energy to it. It is this energy that vibrates the tympanic membrane. Let us therefore consider the reflection and transmission of energies at the tympanic membrane boundary.

3.0 Reflection and transmission of energy at the tympanic membrane boundary.

Usually, when an wave meets a boundary, it transfers its energy to the boundary – Pain (1968 [8]). This boundary will ordinarily be between two media of different impedance values. Like our own case study, the impedance values at the outer and middle ear, are not necessarily equal and therefore it satisfies the condition for energy transfer, at the boundaries.

Since air can be considered as a continuum for allowing sound wave to travel, and this wave travels in a straight line. Let us consider the mass of a unit volume of air as ρ, and the maximum amplitude of this air strip simple Harmonic oscillator as A. then the total energy carried by a unit mass of this air is:

$$E = \frac{1}{2}\rho\omega^2 A^2 \tag{3.1}$$

where $\omega 2\pi c/\lambda$ is the wave frequency. Now the wave travels at a velocity c such that each unit volume of airstrip takes up oscillation with the passage of the wave. Thus, the rate at which energy is being carried along a particular airstrip is:

$$Energy\ x\ velocity = \frac{1}{2}\rho\omega^2 A^2 c \tag{3.2}$$

suppose that the masses of the air at the outer and middle ear are ρ_1 and ρ_2, then the energy arriving at the boundary $x = 0$ arrives with the incident wave which is given by:

$$\frac{1}{2}\rho_1\omega^2 c_1 A_1^2 = \frac{1}{2}z_1\omega^2 A_1^2 \tag{3.3}$$

where $z_1 = \rho_1 c_1$ and is called the impedance, since the mass of the air affects the rate of air arriving at the boundary. Now as the wave hits the boundary, not all of it passes through the boundary. Some of it is reflected, while some is transmitted; thus, the rate at which energy leaves the boundary through the reflected and transmitted wave is:

$$\frac{1}{2}\rho_1 c_1\omega^2 A_1^2 = \frac{1}{2}\rho_2 c_2\omega^2 A_1^2 \tag{3.4}$$

as against the total energy of the wave given by equation (2.1), now, this energy is transmitted to the maleus in the middle ear, as the tympanic membrane vibrates. Therefore, ρ_2 will now be denoting the unit mass of the maleus. This is because, though wave is generated in the middle ear due to the pressure of air, we find that the wall enclosing the middle ear and beyond is rigid, such that wave generation due to this vibration by the tympanic membrane cannot set up a wave in air that can effect any appreciable change in the orientation of the cochlea. It is only the energy transmitted to the maleus that causes it to vibrate, via the incus and stapes. The vibration rocks the foot of the stapes and thus the cochlea, thereby setting up a wave in the fluid (endolymph), enclosed in the cavity of the inner ear. Because very relatively loud noise does not destroy our hearing system, we envisage a situation where the impedance is very large, going by the size of ρ_2. However, we know that extremely very loud noise can damage our hearing system.

Now, let us see what happens in the inner ear which, actually filters and compile the sound the way we hear it. We should also note that energy reflection at the tympanic membrane boundary somewhat also regulate the level of the noise (sound) that actually gets to our brain for interpretation.

4.0 The inner ear–effects of the wave generated

Ross and Wilson (1981 [9]) gave clear and useful description of the inner ear structure that relates to the mechanism of hearing. When the cochlear is rocked by the vibration of stapes, it sets up a traveling wave in the endolymph where the Basilar membrane is immersed, and this wave travels along the length of this membrane. The nerve fibres or the microvilli on it are effected by this wave by distorting their orientation, since the wave carries some energy with it; and the villi are elastic and, thus, can bend when force is exerted on it. At the foot of the microvilli (stereoallia), there exists the ions k^+, Na^+ and Ca^{++}.

Lewis and Hudspeth (1982 [7]), Evans at el (1985 [4]), Furnes and Hackey (1985 [5]), stated that, a displacement of the microvilli bundle changes the strain in it, thereby affecting the transduction opening probability for the ions $k^+, Na^+\ and\ Ca^{++}$. In this study, we shall measure the displacement in form of a deflection of the villiary membranes, which we shall measure in terms of certain angle θ.

5.0 Effect of cilliary deflection in the level of transmission

Theoretically, it is known that when one is placed in a noisy environment for a long time and is suddenly brought to a noiseless place, one finds it difficult to hear any sound that has pitch lower than the pitch of the sound in the former place.

We have assumed that the Basilar membrane and thus the cilliary membrane are elastic; and can deflect or bend when force or slight energy is exerted on it. Our focus on this first part is the effect of this deflection in the level of excitation that the auditory nerve receives. Now these hair cells (HC) bathe in the fluid contained in the inner ear. The apical surface of the microvilli is k^+ rich, while the surrounding fluid is Na^+ - poor.

As a result of this, there is a potential difference across the cilliary membrane. Thus, when the microvilli deflect, it opens up the channel at its foot, such that there is current flow due to ionic exchange $(\theta > 0)$ and we say that, the HC depolarizes. If however, $\theta > 0$, then the HC hyperpolarizes. In these situations, mechanical stimulation alters the receptor potential of the HC. By study, it was discovered that the fluid at the peribasal space, in contrast to the endolymph, which was Na^+ - poor, is Na^+ - rich, and k^+ - poor. Thus, during depolarization of HC, the basolateral currents contain two outward flowing k^+ current – as 'A' type, and another that is Ca^{++} activated. Lewis and Hudspeth, 1983 [7] showed that there is inward flowing of Ca^{++} current on depolarization of the HC. It is this inward flow of the Ca^{++} current, that increases the intra-cellular calcium concentration in the pre-synaptic membrane and this, in turn triggers off the release of neuron transmitters into the synaptic cleft. These transmitters will then bind with the post – synaptic sites, thereby producing excitatory potentials by changing the membrane and ion permeability. We, for now, have got what we wanted for this aspect, and one can see that it is this bending action of the HC's that enables the establishment of the excitatory potential at the synaptic sites, which gives room for membrane ion permeability to change. Thus, let us now establish an expression for the displacement of the orientation or position of the Basilar membrane, and then the microvilli or the HC from where we may be able to look at angular deflection effect of the HC.

6.0 The energy equation

The cochlea has canals filled with the fluid–endolymph. The vibration in the middle ear establishes a wave motion in the fluid enclosed in these canals. These waves can be viewed as traveling waves and, therefore it exerts energy or forces on the (HC) through the Basilar membrane.

Let us consider the Basilar membrane as a two-dimensionally clamped elastic plate. The undisturbed plate occupied the strip $-\infty < x < \infty,\ -b < z < b$ in the plane $y = 0$, the displacement are given as

$$y = h(x, z, t) \tag{6.1}$$

We regard the displacement as small, and we therefore regard $dxdz$ as the element of area on the plate. Along the boundaries $z = \pm b$. We assume that

$$y = \partial_z h = 0 \tag{6.2}$$

The later condition corresponds to the word "clamped". We assume that h and $\partial_x h \to 0$, sufficiently as rapid as: $|x| = \pm\infty$.

Finally, we assume that the elastic energy stored in the deformed plate is given as

$$E = \frac{1}{2} k \int e^{-\lambda x}_{Plate} (\Delta h)^2 d_x d_z \tag{6.3}$$

where $\Delta = \partial_x^2 + \partial_z^2$. The integral sign means, we have summed up the individual energies in each of the (HC). Thus, the energy stored in each of the (HC) will be given as:

$$E_{HC} = \frac{1}{2} K e^{-\lambda x} (\Delta h)^2 \tag{6.4}$$

Now, let us define the force per unit area (i.e. each HC) applied to the plate as $l(x, z)$, where $l > 0$ implies force in the negative $y-$direction. Then, the equation of motion of the plate is given as:

$$K\Delta\left(e^{-\lambda x}\Delta h\right) + l(x, z, t) = 0 \tag{6.5}$$

where we have neglected the mass of the basilar membrane and thus the HC. We shall assume, for simplicity, that the effect of the width of the basilar membrane to the level of hearing is small; then we have:

$$h(x,z,t) = \alpha e^{-\lambda x}\left(1-\sin\theta(x)\right)\cos\omega t \qquad (6.6)$$

Hence, when $\theta = 0$, we have x as being very small, while when $\theta = 90$, then x is large. (Recall that the stiffness of the hair cells (HC) decreases with $x(x\to\infty)$ so that, for a reasonable distance of x, then $\theta \approx 90$ and at such a point, the hair cells (HC) in this region contributes relatively nothing to hearing.

7.0 Analysis of the energy equations

Now let us consider analyzing the energy equation (6.4)

$$E = \frac{1}{2}Ke^{-\lambda x}(\Delta h)^2$$

$$= \frac{1}{2}Ke^{-\lambda x}\left\{\frac{\partial^2}{\partial x^2}\left[\alpha e^{-\lambda x}\left(1-\sin\theta(x)\right)\right]\right\}^2$$

$$= \frac{1}{2}Ke^{-\lambda x}\left\{\frac{\partial}{\partial x}\left[-\lambda\alpha e^{-\lambda x}\left(1-\sin\theta(x)\right)-\alpha\theta e^{-\lambda x}\cos\theta(x)\right]\right\}^2$$

$$= \frac{1}{2}Ke^{-\lambda x}\left\{\lambda^2\alpha e^{-\lambda x}\left(1-\sin\theta(x)\right)-\lambda\alpha\theta e^{-\lambda x}\cos(x)\right.$$

$$\left.-\lambda\alpha\theta' e^{-\lambda x}\cos\theta(x)-\alpha\theta'^2 e^{-\lambda x}\sin\theta(x)-\alpha\theta'' e^{-\lambda x}\cos\theta(x)\right\}^2$$

$$= \frac{1}{2}Ke^{-\lambda x}\left\{\left[\left[-2\lambda\alpha\theta'\cos\theta(x)+\lambda^2\alpha-\left(\theta'^2+\lambda^2\right)\alpha\sin\theta(x)\right]\right.\right.$$

$$\left.\left.-\alpha\theta''\cos\theta(x)\right]e^{-\lambda x}\right\}^2$$

If $0'' = 0$, then

$$E = \frac{1}{2}K\left(e^{-\lambda x}\right)^3\left\{-2\lambda\alpha\theta'\cos\theta(x)+\lambda^2\alpha-\left(\theta'^2+\lambda^2\right)\alpha\sin\theta(x)\right\}^2$$

$$= \frac{1}{2}K\left(e^{-\lambda x}\right)^3\left\{4\lambda^2\alpha^2\theta'^2\cos\theta(x)-2\lambda^3\alpha^2\theta'\cos\theta(x)+\lambda^4\alpha^2\right.$$

$$+2\lambda\alpha^2\theta'\left(\theta'^2+\lambda^2\right)\cos\theta(x)\sin\theta(x)-2\lambda^3\alpha^2\theta'\cos\theta(x)$$

$$-\lambda^2\alpha^2\left(\theta'^2+\lambda^2\right)\sin\theta(x)+2\lambda\alpha^2\theta'\cos\theta(x)\sin\theta(x)$$

$$\left.-\lambda^2\alpha^2\left(\theta'^2+\lambda^2\right)\sin\theta(x)+\left(\theta'^2+\lambda^2\right)^2\alpha^2\sin^2\theta(x)\right\}$$

$$= \frac{1}{2}K\left(e^{-\lambda x}\right)^3\left\{4\lambda^2\alpha^2\theta'^2\cos^2\theta'(x)+\lambda^4\alpha^2+\left(\theta'^2+\lambda^2\right)^2\alpha^2\sin^2\theta(x)\right.$$

$$\left.-2\lambda^2\alpha^2\left(\theta'^2+\lambda^2\right)\sin\theta(x)+4\lambda\alpha^2\theta'\left(\theta'^2+\lambda^2\right)\cos\theta(x)\sin(x)-4\lambda^3\alpha^2\theta'\cos\theta(x)\right\}$$

For Simplicity, let λ be very small, so that λ of order ≥ 3 will be sufficiently small enough to be negligible. Similarly, we shall let $\left(\theta'^2+\lambda^2\right)$ to be small so that $\left(\theta'^2+\lambda^2\right)\alpha^2\sin^2\theta(x)\approx 0$. Thus

$$E = \frac{1}{2}K\left(e^{-\lambda x}\right)\left\{4\lambda^2\alpha^2\theta'^2\cos\theta^2(x)-2\lambda^2\alpha^2\left(\theta'^2+\lambda^2\right)\sin\theta(x)+4\lambda\alpha^2\theta'\left(\theta'^2+\lambda^2\right)\cos\theta(x)\sin\theta(x)\right\}$$

The force l is a function of θ and thus

$$l = l(\theta,t) = lt\theta(x),t = -\beta e^{-\lambda x}h\{\theta(x)t\} = -2\beta e^{-\lambda x}\left\{\alpha e^{-\lambda x}(1-\sin\theta(x))\cos\omega \mathrm{t}\right\} \quad (7.1)$$

further analysis of this result on the equation of motion (6.5)

$$K\Delta\left(e^{-\lambda x}\Delta h\right) + l(x,zt) = 0 \quad (7.2)$$

Substituting $h(\theta(x),t) = \alpha e^{-\lambda x}(1-\sin\theta(x))\cos\omega \mathrm{t}$, we obtain

$$\Delta h = \left[-2\lambda\alpha\theta'\cos\theta(x) + \lambda^2\alpha - \left(\theta'^2+\lambda^2\right)\alpha\sin\theta\right]e^{-\lambda x}$$

Hence $e^{-\lambda x}\Delta h = \left\{-2\lambda\alpha\theta'\cos\theta(x) + \lambda^2\alpha - \left(\theta'^2+\lambda^2\right)\alpha\sin\theta(x)\right\}\left(e^{-\lambda x}\right)^2$.

Now $K\Delta\left(e^{-\lambda x}\Delta h\right)$ is given as

$$\frac{K\partial^2}{\partial x^2}\left[\left\{-2\alpha\lambda\theta'\cos\theta(x) + \lambda^2\alpha - \left(\theta'^2+\lambda^2\right)\alpha\sin\theta(x)\right\}\left(e^{-\lambda x}\right)^2\right]$$

$$= \frac{K\partial}{\partial x}\Big[\Big\{2\alpha\lambda\theta'^2\sin\theta(x) - 2\lambda\alpha\theta''\cos\theta(x) - \left(\theta'^2+\lambda^2\right)\alpha\cos\theta(x)$$

$$-2\theta'^2\alpha\sin\theta(x) + \left\{-4\alpha\lambda^2\theta'^2\cos\theta(x) - 2\lambda^3\alpha - \lambda\alpha\left(\theta'^2+\lambda^2\right)\sin\theta(x)\right\}\left(e^{-\lambda x}\right)^2\Big]$$

$$= \frac{K\partial}{\partial x}\left[\left\{2(\lambda-1)\alpha\theta'^2\sin\theta(x) - \left(\theta'^2+\lambda^2-4\lambda^2\theta'^2\right)\alpha\cos\theta(x) + 2\lambda\alpha\left(\theta'^2+\lambda^2\right)\sin\theta(x)\right\}\left(e^{-\lambda x}\right)^2\right]$$

since λ is very small and θ' is also small, we find that $\lambda^2\theta'^2$ will be very negligible, so that $4\alpha\lambda^2\theta'^2$ can be neglected. Thus, we have

$$\frac{K\partial}{\partial x}\left[\left\{\sigma\theta'\sin\theta(x) - \delta\cos\theta(x) - 2\lambda\delta\sin\theta(x)\right\}\left(e^{-\lambda x}\right)^2\right]$$

where $\sigma = 2\alpha(\lambda-1)$ and $\delta = \alpha\left(\theta'^2+\lambda^2\right)$

$$= K\Big[\left(\sigma\theta'^3\cos\theta(x) - \theta'\delta\sin\theta(x) - 2\lambda\delta\theta'\sin\theta(x)\right) - 2\lambda\Big(\sigma\theta'^2\sin\theta(x)$$

$$+\delta\cos\theta(x) - 2\lambda\delta\sin(x)\Big)\left(e^{-\lambda x}\right)^2\Big]$$

$$= K\left(e^{-\lambda x}\right)^2\left\{-\theta'\delta\sin\theta(x) - 2\lambda\delta\theta'\cos\theta(x) - 2\lambda\sigma\theta'^2\sin\theta(x) - 2\lambda\delta\cos\theta(x) + 4\lambda^2\delta\sin\theta(x)\right\}$$

$$= K\left(e^{-\lambda x}\right)^2\left\{\left(4\lambda^2\delta - 2\lambda\sigma\theta'^2 - \theta'\delta\right)\sin\theta(x) - (1-\theta')2\lambda\delta\theta'\cos\theta(x)\right\} \quad (7.3)$$

From equation (7.1), we had that $l = \{-2\gamma + 2\gamma\sin\theta(x)\}\left\{e^{-\lambda x}\right\}^2\cos\omega t$ (7.4)

where $\gamma = \alpha\beta$. Substituting (7.3) and (7.4) into (7.2) gives

$$K\left(e^{-\lambda x}\right)^2\left\{\left(4\lambda^2\delta - 2\lambda\sigma\theta'^2 - \theta'\delta\right)\sin\theta(x) - (1+\theta')2\lambda\cos\theta(x)\right\}\left(-2\gamma + 2\gamma\sin\theta^{(x)}\right)\left(e^{-\lambda x}\right)^2 = 0$$

$$= \left\{\left(4\lambda\delta - 2\lambda\sigma\theta'^2 - \theta'\delta\right)\sin\theta(x) - (1+\theta')2\lambda\delta\cos\theta^{(x)} - 2\gamma + 2\gamma\sin\theta(x) = 0\right.$$

Let us consider the boundary condition at $\theta = 0$, then, we obtain

$$-(1+\theta')\,2\lambda\delta k_0 - 2\gamma = 0$$
$$2\gamma = -(1+\theta')\,2\lambda\delta k_0$$
$$\gamma = (1+\theta')\,\lambda\delta k_0$$

Recall that $\gamma = \beta\alpha = -(1+\theta')\lambda\delta k_0$, $\beta = \dfrac{-(1+\theta')\lambda\delta k_0}{\alpha}$. But $l = -2\beta e^{-\lambda x} h\{\theta(x),t\}$.

Therefore, $h(\theta(x)) = -le^{\lambda x}_{\alpha\beta} = \dfrac{1l\alpha e^{\lambda x}}{k_0 2(1+\theta')\lambda\delta}$. Substituting $\delta = \alpha(\theta'^2 + \lambda^2)$, we obtain

$$h(\theta(x)) = \frac{le^{\lambda x}}{k_0 2\lambda(1+\theta')(\theta'^2+\lambda^2)}$$

We shall note that certain assumption were made about $\lambda,\ \theta',\ \theta''$ even $\lambda^2\theta'^2$. These assumptions will definitely affect the final result but actually lead to reasonable simplification of the problem. From the last result therefore, we can get expression for displacement for the basilar membrane which we have considered as a narrow plate and therefore have assumed that $\lambda << 1$ and $l = l\{\theta(x),t\}$ the effective consequence of the distance x, is a function of the hair cells. Therefore, θ is the angular deflection of the hair cells.

Force on the wave, which is also equal to the energy of the wave, is given as $l\{\theta(x),t\} = -2\gamma(1-\sin\theta(x))(e^{-\lambda x})^2 \cos\omega t$.

But when the sound is very loud, it generates a wave with small λ and vice versa. So for a noise, our choice of λ should be small. To see the behaviour of the basilar membrane for a given choice of λ, we then vary the λ and see how the hair cells react to the energy of the wave. For simplicity, let $l\{\theta(x),t\} = 100,\ \gamma = 50,\ \omega = 0$.

Then, $\theta = \sin^{-1}\left(\dfrac{1-100(e^{-\lambda x})^2}{50}\right)$, which reduces to $\theta = \sin^{-1}\left(1-(e^{-\lambda x})^2\right)$

And we obtain the following graphs as in the Figures 7.1 to 7.2.

7.1 Variation of angular deflection at 4 different points with the same wave length *x*

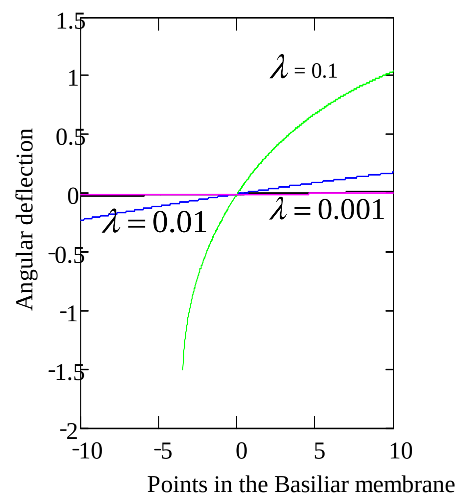

Figure 7.1

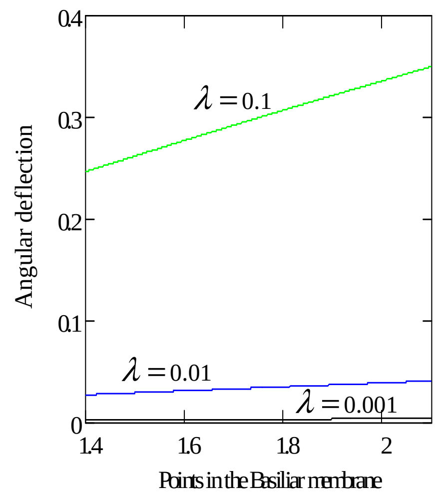

Figure 7.2

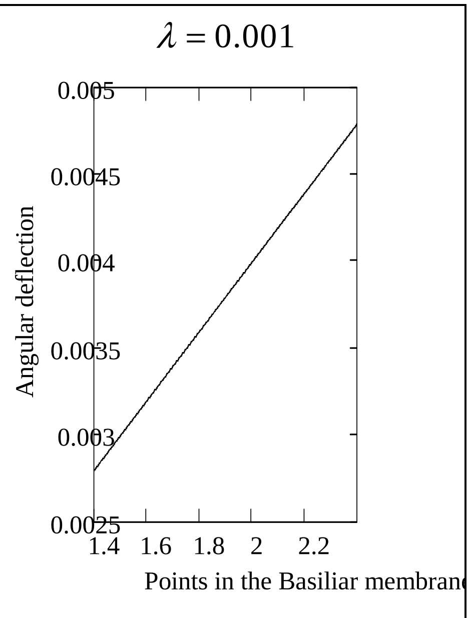

Figure 7.3

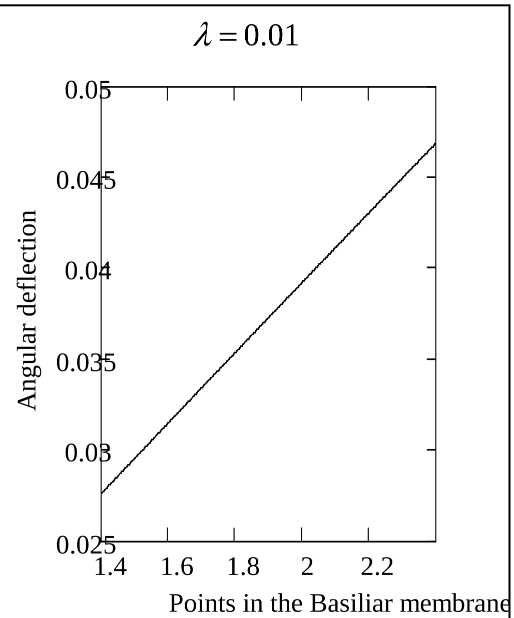

Figure 7.4

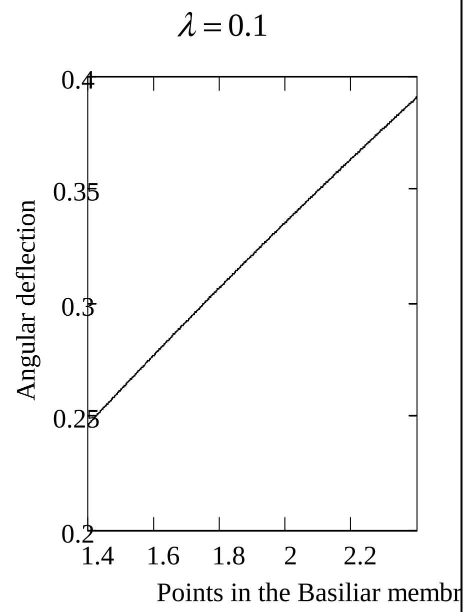

Figure 7.5

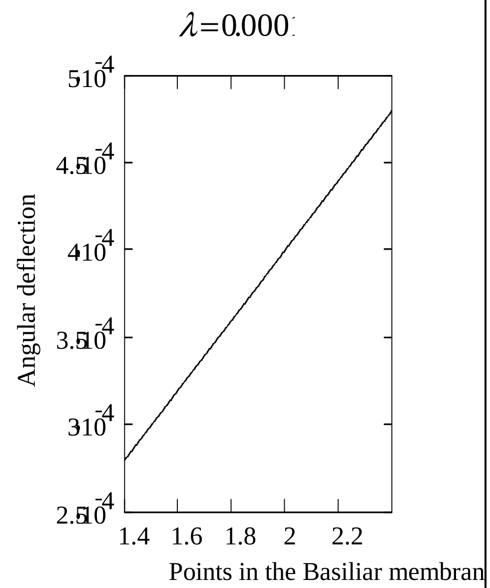

Figure 7.6

8.0 Discussion

Figure 7.1, shows the graph of angular deflections at four different points with various values of λ, while figure 7.2, shows the graph of the 7mm length within the basilar membrane and it increases exponentially with distance. Figures 7.3, 7.4, 7.5 and 7.6 show graphs of different values of λ, which obeys the exponential increase with distance.

Now, when the wave exerts its force on the basilar membrane and thus on the HC, it deflects the HC. Again, we have to note that it is not all the energy generated by the wave that is absorbed by the HC.

It is the excess of the energy generated by the wave and the energy absorbed that gives rise to the deflection. Thus, if we denote the deflection as θ, then θ will be seen to be dependent on these two functions i.e. $\theta\left(E', E\right)$, where E' is the energy generated by the wave, and E is the energy absorbed by the HC. We notice that when $E' >> E$, some form of permanent damage may be done to the HCs, such that they will not return to their former position, even when the wave generation stops or the noise stops. On the other hand, if $E' >> E$, there will be no deflection of the hair cells, meaning that no noise will be heard. From the models (the energy equation), equation (6.5) establishes the equilibrium position of the basilar membrane (and thus, HC) such that, the force on the membrane equals the elastic energy that is stored by the HC. Thus, relaxing this force on the basilar membrane implies relaxing the HC form stress, as it returns to its former position. Also, applying larger forces imply that the force exerted on the HC, will be greater than the elastic energy stored, thus, the HC will respond to this extra force by bending or deflecting. Therefore, for one to hear a sound, E' is usually greater to a large extent than the elastic energy stored by the HC.

Concluding this point therefore, it might be necessary to state that the elasticity of the basilar membrane and then the HC has a limit. If force is exerted on it to a very large extent and for a long time, there is the tendency that it will lose this elasticity. When this is done, and it the noise or sound eventually stops, a non-reclaimable damage has already been done to the structure and orientation of the HC, so that it cannot return to its normal position and orientation. This is why when such a person eventually finds himself in a noiseless zone, and sound is made in a low pitch, such sounds/noise can not exert enough force on the present orientation of the HC, so as to cause any further change in the form of deflection.

When this is the situation, then there will be no ion current established at the foot of the HC, so as to allow for the release of the neurotransmitters which are supposed to set up a potential that will send impulse to the auditory nerve, informing the brain that a sound has been made.

In another situation, the sound made, may not be too loud as to exert so much force on the HC, thereby causing no permanent damage. Rather, it may cause temporary damage which may be reclaimable, meaning that the elastic limit of the HC has not been broken or reached. For this kind of situation, we discover that when one, with such condition, finds oneself out of this noisy zone, one will be temporarily deaf with the same situation as discussed above, until the HC reclaims its usual orientation. When the usual orientation is reclaimed, the sound/noise can now be heard by the normal process, as has been discussed in this study.

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