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Application of thermo-acoustic engines to heat transfer in microcircuits: An analytical approach

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Abstract

As a result of miniaturization, electronic products are shrinking in size and weight but with greater pressure for cost reduction. Heat fluxes have increased considerably and hence thermal management becomes crucial from the reliability point of view. This paper presents an analytical model of heat transfer in microcircuits. The analytical solution is obtained for temperature field of the fin. It is shown that the minimum temperature of the fin decreases as both time and convection coefficient increases and thermal conductivity decreases.

Keywords: heat transfer, engines, microcircuit, variable separation

1.0 Introduction

In recent times electronics engineers have become increasingly interested in the thermal aspects of design. In fact, they have no other choice. Thermal problems can no longer be solved by using a fan to inserting a more powerful fan. In the immediate vicinity of a transistor, heat removal by conduction is the most important. Convection cooling is only important as soon as the heat reaches the hybrid circuit or printed circuit board level [3].

Personal computers are currently air-cooled by either free or forced convection. Fans attached to the central processing unit (CPU) are the most popular cooling devices for low power dissipation systems. They are popular because of reliability, cost, efficiency and ease of implementation. However, for higher frequency chips (above 1000 MHz), the air-cooled heat sinks have some limitations in the form of bulky size, noise and insufficient cooling performance. As a result, current practice of dense packaging of electronics in compact spaces demands novel ways of heat dissipation, which will be able to dissipate as much as 100 W/cm2 at chip levels while maintaining the device at acceptable temperatures, typically below 85°C. One of the proposed solutions for dissipating high heat fluxes is the use of liquid cooling techniques [1].

Cold plates are used to remove heat from the assembly of printed circuit boards. A cold plate consists of an array of rectangular fins, attached between two exterior plates. Fluid passes through the spaces between the fins, to help increase the rate of heat transfer from the fin. A cold plate is called a single stack cold plate if there is no splitter plate between the two exterior plates; double stack if there is one splitter plate and triple stack if there are two splitter plates. A stack is a combination of fins connected together. Thus, the whole stack is made of repeating arrays of fins [4].

If conduction within the fin was infinitely fast, the effective transfer area would be simply the sum of the fin and the effective transfer area. It is also observed that because of the finite conduction rate, a temperature gradient develops within the fin, reducing the local heat transfer driving force associated with the external force [2].

There is still continuous pressure to make circuits ever faster, so that the power per integrated circuit and the total power per package will continue to increase in the future [3].

Lewis et al. [4] carried out an investigation on a generalized formulation of the analysis of cold plates, using dimensionless parameters to replace the dimensional parameters. In addition to that, several unit cells assembled together vertically are analyzed using the finite element method with and without heat losses at the top and bottom of the stack as it happens in reality.

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In this paper we study and model analytically heat transfer in microcircuits. We investigate minimum temperature with respect to time, convection coefficient and thermal conductivity parameters which will be helpful in the design of cold plates used for cooling of electronic systems.

2.0 Mathematical model

Following Lewis et al. [4], the governing equation for a steady state, one-dimensional fin, with conduction in the solid and forced convection in the passage is given by:

$$c\rho A\frac{\partial T}{\partial t} = kA\frac{\partial^2 T}{\partial x^2} - hP\left(T - T_\infty\right) \tag{2.1}$$

with initial and boundary conditions $T(x,0) = T(0,t) = T(1,t) = T_0\,, 0 \le x \le 1\,,$ (2.2)

where

k = thermal conductivity of the fin material
A = cross sectional area perpendicular to the direction of conduction
h = convection coefficient
P = perimeter of the surface where convection takes place
T+ □= ambient temperature
C = specific heat of the fin material
ρ = density of the fin material
T = temperature of the fin at a given location
x = distance measured from the base of a fin
t = time
T_0 = fin wall temperature

With non-dimensionalized equation (2.1) and (2.2) using the non–dimensional variable

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty} \tag{2.3}$$

Then, the dimensionless governing equation with correspond boundary and initial conditions are written as follows:

$$\frac{\partial \theta}{\partial t} = k_1\frac{\partial^2 \theta}{\partial x^2} - h_1\theta \tag{2.4}$$

$$\theta(x,0) = \theta(0,t) = \theta(1,t) = 1,\ 0 \le x \le 1 \tag{2.5}$$

where $k_1 = \dfrac{kA}{\rho cA}$, $h_1 = \dfrac{hP}{\rho cA}$.

3.0 Method of Solution

We solve equation (2.4) and (2.5) by method of separation of variable. To do this, we seek

$$\theta(x,t) = X(x).T(t) \tag{3.1}$$

Then, equation (2.4) becomes

$$\frac{1}{k_1}\left(XT' + h_1 XT\right) = X''T \tag{3.2}$$

Divide equation (3.2) throughout by XT, we obtain

$$\frac{1}{k_1}\left(\frac{T'}{T} + h_1\right) = \frac{X''}{X} \tag{3.3}$$

Since LHS and RHS of equation (3.3) are function of different variable. Both equal implies they are equal to a constant. So

$$\frac{1}{k_1}\left(\frac{T'}{T} + h_1\right) = \frac{X''}{X} = \lambda^2 \tag{3.4}$$

So that

$$\frac{X''}{X} = \lambda^2 \Rightarrow X'' - \lambda^2 X = 0 \tag{3.5}$$

Seek

$$X(x) = e^{mx} \tag{3.6}$$

we obtain the solution to equation (3.5) as

$$X(x)=\left(\frac{1-e^{-\lambda}}{e^{\lambda}-e^{-\lambda}}\right)e^{\lambda x}+\left(\frac{e^{\lambda}-1}{e^{\lambda}-e^{-\lambda}}\right)e^{-\lambda x} \tag{3.7}$$

Also $$\frac{1}{k_1}\left(\frac{T'}{T}+h_1\right)=\lambda^2 \Rightarrow \frac{dT}{T}=\left(k_1\lambda^2-h_1\right)dt \tag{3.8}$$

Integrating with respect to t, we obtain $$\ln T=\left(k_1\lambda^2-h_1\right)t+\text{constant} \tag{3.9}$$

That is $$T(t)=e^{\left(k_1\lambda^2-h_1\right)t} \tag{3.10}$$

Finally, we have the solution of (2.4) to be

$$\theta(x,t)=\left(\left(\frac{1-e^{-\lambda}}{e^{\lambda}-e^{-\lambda}}\right)e^{\lambda x}+\left(\frac{e^{\lambda}-1}{e^{\lambda}-e^{-\lambda}}\right)e^{-\lambda x}\right)e^{\left(k_1\lambda^2-h_1\right)t} \tag{3.11}$$

The results are presented in figures 3.1 – 3.3.

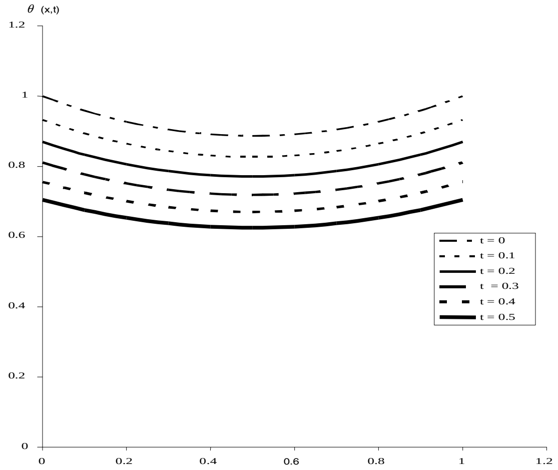

Figure 3.1: Unsteady temperature profile θ(x,t) against position x at various time t,when k_1 = 0.3, h_1 = 1

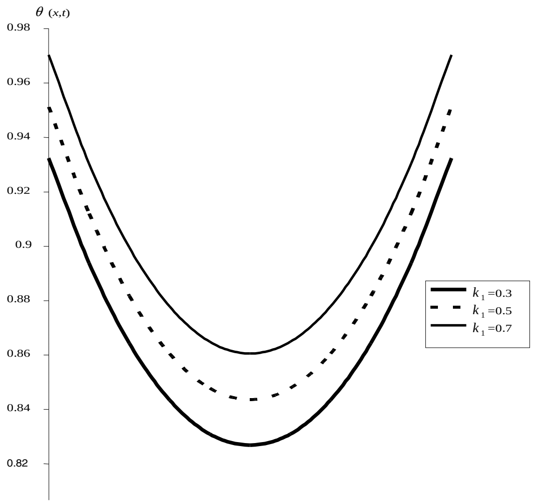

Figure 3.2: Unsteady temperature profile θ(*x*,*t*) against position *x* for various values of k_1.vwhen $t = 0.1$. $h_1 = 1$.

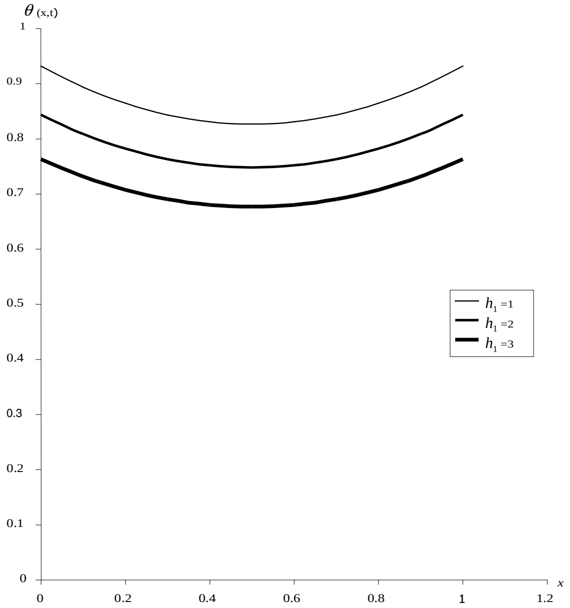

Figure 3.3: Unsteady temperature profile θ(*x*,*t*) against position *x* for various values of h_1, when $t = 0.1$, $k_1 = 0.3$.

4.0 Discussion of results

The unsteady of temperature profiles of the fin at a given location are shown in figure s 3.1 – 3.3. Figure 1 shows that there is a decrease in minimum temperature of the fin as the time t increase. Figure 2 shows that there is a decrease in minimum temperature of the fin as values of thermal conductivity of the fin material decrease.

Figure 3 shows that there is a decrease in minimum temperature of the fin as values of convection coefficient increase.

5.0 Concluding remarks

An analytical approach to application of thermo-acoustic engines to heat transfer in microcircuits is studied. We examined the effect of parameters on the temperature of the fin. It is observed that the minimum temperature of the fin decreases as both time and convection coefficient increases and thermal conductivity decreases.

References

[1] Aniruddha P., Yogendra J., Monem H.B., Chandrakant D.P and Todd W. (2002) "Design and Performance Evaluation of a Compact Thermosyphon", http://www.hpl.hp.com/research/papers/2002/thermosyphon.pdf

[2] Basmadjian D.(1999) " The art of modeling in Science and Engineering", Chapman and HALL/CRC Boca Raton Florida U.S.A.

[3] Kos A. and De Mey G. (2005) "Thermal Modelling and Optimization of Power Microcircuits", Electrochemical Publications, IOM

[4] Lewis J.L., Van Maanen J.J., Benco D., Moore K.L. and Roe A.L. (2005) "Application of thermo-acoustic engines to heat transfer in microcircuits", http://eurodyn2005.univ.mlv.fr/complete-example-of-a-full-length-paper.pdf