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Steady flow of reacting temperature-dependent fluid past an impulsively started porous vertical surface with Newtonian heating

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Abstract

This paper studies the steady flow of a reacting temperature-dependent viscous incompressible fluid past an infinite vertical porous plate, with the flow generated by Newtonian heating and impulsive motion. The resulting momentum equation is non-dimensionalized and the solution is obtained numerically by shooting method. A parametric study of all involved parameters is conducted and illustrated graphically.

Keywords: Reacting fluid, temperature-dependent, heat transfer, Newtonian heating.

1.0 Introduction

The study of fluid flow through porous channel has gained the attention of many researchers in fluid mechanics due to its importance in many real life applications like the bio-circulatory systems, geo-physics and engineering to mention a few. Also the study of flow of fluids whose viscosity depends of temperature is important in many physical problems too such as flow of crude oil in porous medium, flow of engine oil in car engines, flow of air over the aerodynamic surfaces of vehicles and air planes. However, the viscosity of fluids like water, benzene, and crude oil reduces with increasing temperature while it is also important to note that the viscosity of gases like air, helium, or methane increases with temperature (Attia, 2006 [3]). It has also been established that viscosity of some fluids depends on both pressure and temperature (Adesanya and Ayeni, 2008 [1]).

Heat transfer to viscous fluid flow is important in increasing the flow velocity since it reduces the viscosity. The safety of lives and property during flow of many combustible is highly important because large amount of heat is released. This can also affect the quality of the product during highly reactive flow, in such a case the flow through porous channel can allow for suction and injection in order to control the heat of reaction. The influence of plate porosity on the flow profile is evident in the work of (Eldabe et al, 2003), (Makinde and Osalusi, 2006), (Osalusi and Sibanda, 2006).

The Newtonian heating has been used in convective heat transfer, recently (Chaudhary and Jain, 2006 [4]) presented the exact solution to the unsteady free convection boundary-layer flow past an impulsively started vertical surface with Newtonian heating by Laplace-transforms technique. (Merkin, 1994 [9]) considered the steady free-convection boundary-layer over a vertical flat plate immersed in a viscous fluid, also,(Lesnic et al, 1999 [7]), (Ibid, 2000 [5]) and (Lesnic et al, 2003 [7]) also considered the steady free convection of ertical and horizontal surfaces embedded in a porous medium, (Perdikis and Takhar, 1986 [10]) studied the free convection effects on flow past a moving vertical infinite porous plates.

All the studies mention above deals with constant viscosity for free convection. In the present paper, we shall investigate the flow when viscosity depends on temperature and flowing through a porous vertical plate with Newtonian heating, we in addition assumed that the fluid further reacts and the viscous dissipation term is neglected. The paper is organized in three sections; in section 2 the problem is formulated, non-dimensionalized and solved while section 3 of the work comprises of the discussion of result and some concluding remarks.

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Nomenclature
u - Velocity,
T_0 - Ambient temperature,
T - Temperature,
C_p - -Specific heat,
p - Pressure,
Q - Heat per unit mass during reaction,
E - Activation energy,
R - Universal gas constant.
x- Cartesian coordinate along the plate,
y- Cartesian coordinate normal to the plate,
Pr – Prandtl number,
g- Magnitude of acceleration due to gravity,
k- Thermal conductivity,
G - Heat variation parameter,
S - Plate porosity,

Greek symbols
μ - Dynamic viscosity,
λ - Viscosity variation parameter,
ρ - Density of the fluid,
θ - Dimensionless temperature,
β - Coefficient of volumetric expansion,
ϕ - Dimensionless velocity,

2.0 Mathematical analysis

The steady flow of a viscous fluid past an impulsively started porous plate with Newtonian heating is considered, the x-axis is taken along the plate in the vertical upward direction and the Y-axis is chosen normal to the plane. The plate is given an impulsive motion in the vertical upward direction against gravitational field with a characteristic velocity *U*, the viscous becomes fully developed steady state viscous flow after a certain instant of time.

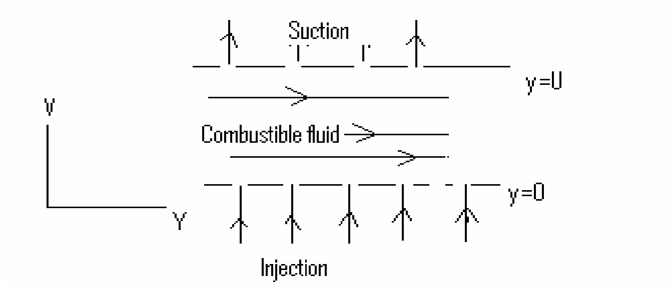

Figure 2.1

The fully developed flow of temperature dependent fluid described above is governed by the Navier-Stokes equation.

$$\rho\frac{\partial u^*}{\partial t^*}+V_0\frac{\partial u^*}{\partial y^*}=\frac{\partial}{\partial y^*}\left(\mu\frac{\partial u^*}{\partial y^*}\right)+\rho g\beta\left(T^*-T_0^*\right) \qquad (2.1)$$

$$\frac{\partial T^*}{\partial t^*}+V_0\frac{\partial T^*}{\partial y^*}=\frac{1}{\rho C_p}\frac{\partial}{\partial y^*}\left(k\frac{\partial T^*}{\partial y^*}\right) \qquad (2.2)$$

The boundary conditions are

$$\left.\begin{array}{lll} t\le 0: u^*=0, & T^*=T_0^* & \forall \quad y^* \\ t>0: u^*=U_0, & T^*=T_1 & at \quad y^*=0 \\ u^*=0, & T^*=T_0^* & as \quad y^*\to\infty \end{array}\right\} \qquad (2.3)$$

Introducing the following non-dimensional parameters

$$\left.\begin{aligned}&\phi=\frac{u}{U_0},\quad y=\frac{\bar{y}}{h},\quad \theta=\frac{E(T-T_0)}{RT_0^2},\quad S=\frac{V_0}{U_0}\\ &t=\frac{\bar{t}\rho U_0^2}{\mu_0},\ \Pr=\frac{\mu_0 C_p}{k},\ U_0=\frac{\mu_0}{\rho h},\quad G=\frac{\mu_0 g\beta RT_0^2}{\rho E U_0^3}\end{aligned}\right\} \tag{2.4}$$

With the help of (2.4) the governing equations with the boundary conditions (after dropping bars) becomes

$$\frac{\partial\phi}{\partial t}+S\frac{\partial\phi}{\partial y}=\frac{\partial}{\partial y}\left(\exp\left(\frac{\lambda\theta}{1+\in\theta}\right)\frac{\partial\phi}{\partial y}\right)+G\theta \tag{2.5}$$

$$\frac{\partial\theta}{\partial t}+S\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2} \tag{2.6}$$

With the appropriate boundary conditions

$$\begin{aligned}&t\le 0:\phi=0,\quad \theta=0,\quad \forall\quad y\\ &t>0:\phi=1,\quad \theta=1\quad at\quad y=0\\ &\phi\to 0,\ \theta\to 0\ as\quad y\to\infty\end{aligned} \tag{2.7}$$

Taking $\varepsilon=0$ and dropping $\frac{\partial}{\partial t}$ since we are interested in the steady state solution which is independent of time, we obtain a system of second order ordinary differential equations.

$$S\frac{d\phi}{dy}=\frac{d}{dy}\left(\exp\lambda\theta\frac{d\phi}{dy}\right)+G\theta \tag{2.8}$$

$$S\frac{d\theta}{dy}=\frac{1}{\Pr}\frac{d^2\theta}{dy^2} \tag{2.9}$$

$$\begin{aligned}&\phi=1,\quad \theta=1\quad at\quad y=0\\ &\phi\to 0,\ \theta\to 0\ as\quad y\to\infty\end{aligned} \tag{2.10}$$

Then energy equation (2.9) is uncoupled from the momentum equation (2.8) and solved analytically for $\theta(y)$ with the specified boundary condition (2.10) to get

$$\theta=\frac{e^{\Pr Sy}-e^{2\Pr S}}{1-e^{2\Pr S}} \tag{2.11}$$

Then the momentum equation becomes

$$\frac{d^2\phi}{dy^2}=\left(e^{-\lambda\left(\frac{e^{\Pr Sy}-e^{2\Pr S}}{1-e^{2\Pr S}}\right)}-\frac{\lambda\Pr e^{\Pr Sy}}{\left(1-e^{2\Pr S}\right)}\right)S\frac{d\phi}{dy}-G\frac{e^{\Pr Sy}-e^{2\Pr S}}{1-e^{2\Pr S}}e^{-\lambda\left(\frac{e^{\Pr Sy}-e^{2\Pr S}}{1-e^{2\Pr S}}\right)} \tag{2.12}$$

The above equation is a non-linear differential equation which we solve numerically by using shooting method, setting,

$$x_1=y,\quad x_2=\phi,\quad x_3=\phi' \tag{2.13}$$

we obtain,

$$x_1' = 1$$
$$x_2' = x_3 \tag{2.14}$$
$$x_3' = \left(e^{-\lambda\left(\frac{e^{\Pr S x_i} - e^{2\Pr S}}{1-e^{2\Pr S}}\right)} - \frac{\lambda \Pr\left(\Pr + e^{\Pr}\right)}{\left(1-e^{2\Pr S}\right)} \right) S x_3 - G\,\frac{e^{\Pr S x_1} - e^{2\Pr S}}{1-e^{2\Pr S}}\, e^{-\lambda\left(\frac{e^{\Pr S x_1} - e^{2\Pr S}}{1-e^{2\Pr S}}\right)}$$

With $x_1(0) = 0$, $x_2(0) = 1$, $x_2(2) = 0$, $x_3(0) = D$, where D is the guess value set to meet $\phi(\infty) \to 0$. Then equation (2.11) is then iterated using improved Euler Runge-kutta Scheme together with a suitable Pascal code.

3.0 Discussion and conclusion

In order to give a valid discussion of our result, we have taken values of Prandtl number as 0.71, 1.0, and 7.0 which corresponds to air, electrolyte and water respectively. For the numerical validation of our work, we have taken $\lambda = -1$, $G = 1$.

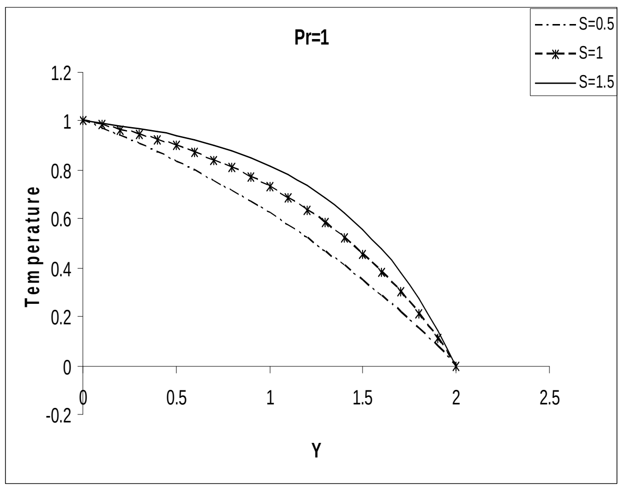

Figure 3.1: Temperature profiles for different values of S

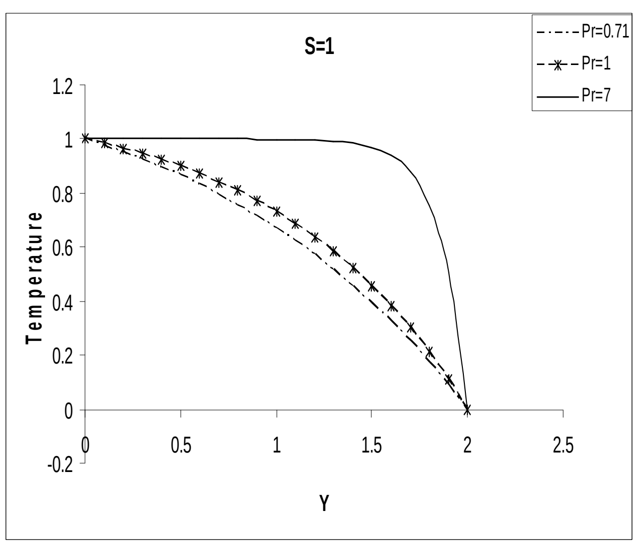

Figure 3.2:- Temperature profiles for various Prandtl number

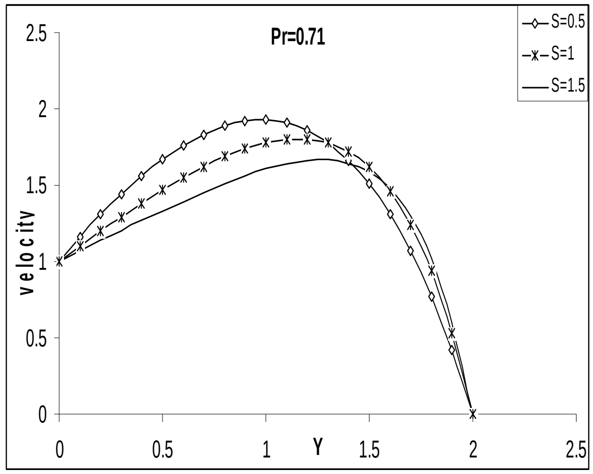

Figure 3.3:- Velocity profiles for different values of the Suction parameter.

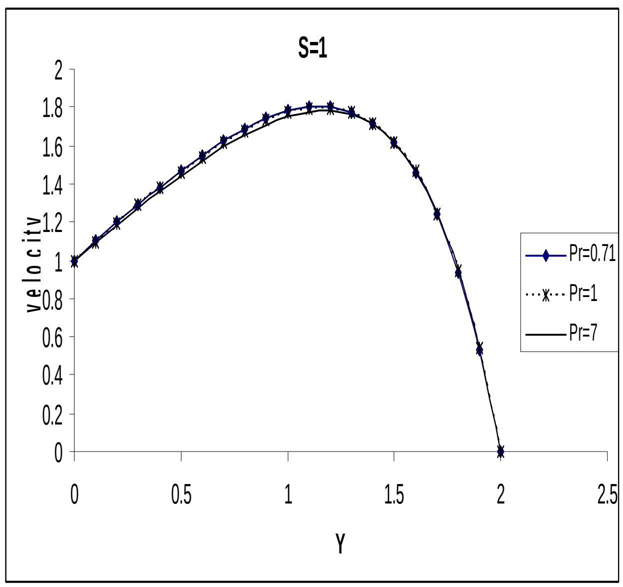

Figure 3.4: -Velocity profiles for different values of Prandtl number

The temperature profiles are presented, in figure 3.1, the effect of plate porosity is shown and figure 3.2 shows the temperature profile for different Prandtl number. Figure 3.3 shows the variation of the velocity profile with the plate porosity and in figure 3.4, the velocity profiles at different Prandtl number is presented.

We have studied the steady flow of temperature-dependent viscous fluid past an impulsively started porous plates with Newtonian heating, Graphical results have presented and discussed.

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