

MHD flow of a uniformly stretched vertical permeable surface under oscillatory suction velocity

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Abstract

The study of unsteady magnetohydrodynamic heat and mass transfer in MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate along with porous medium of time dependent permeability under oscillatory suction velocity normal to the plate has been made. It was considered that the influence of the uniform magnetic field acts normal to the flow and the permeability of the porous medium fluctuate with time. The problem is solved, analytically by asymptotic expansion in order of epsilon for velocity, temperature and concentration fields. The results obtained are discussed for thermal Grashof number ($Gr_T > 0$) corresponding to the cooling of the plate and ($Gr_T < 0$) corresponding to heating of the plate with the help of graphs to observe the effect of various parameters. A parametric study of all parameters involved was considered, and a representative set of results showing the effect of heat generation, reaction parameter, grashof numbers Hartmann number and free stream oscillatory frequency were illustrated.

Keywords: Mass transfer, MHD flow, vertical plate, suction velocity, viscous, oscillatory, Grashof number, permeable surface.

Classification: 76W05

1.0 Introduction

In industries and nature, many transport processes exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect of thermal diffusion and diffusion of chemical species. The phenomenon of heat and mass transfer is observed on buoyancy induced motions in the atmosphere, in bodies of water, quasi-solid bodies, such as earth and so on. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology such flows arise due to either unsteady motion of a boundary or difference in boundary temperature. Besides unsteadiness may also be due to oscillatory free stream velocity and temperature. In the past decades an intensive research effort has been devoted to problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. In addition, the phenomenon of heat and mass transfer is also encountered in chemical process industries such as polymer production and food processing. Many researchers [1-10] have studied the problem on free convection and mass transfer flow of a viscous fluid through porous medium. In these studies, the permeability of the porous medium is assumed to be constant. However, a porous material containing the fluid is a non-homogeneous medium and the porosity of the medium may not necessarily be constant. Recently, Magyari *et al.* [11] have discussed analytical solutions for unsteady free convection in porous media.

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The magnetic current in porous media considered by Raptis *et al.* [12] and Geindreau *et al.* [13]. Muthukumaraswamy *et al.* [14, 15] investigated mass diffusion effects on flow past a vertical surface. Mass diffusion and natural convection flow past a flat plate studied by researchers like Chandrasekhara *et al.* [16] and Panda *et al.* [17]. Magnetic effects on such a flow is investigated by Hossain *et al.* [18] and Israel *et al.* [19]. Sahoo *et al.* [20] and Chamkha *et al.* [21] discussed MHD free convection flow past a vertical plate through porous medium in the presence of foreign mass.

In the present paper, the suction velocity is assumed to be $(1 + BCe^{i\omega t})$ and the permeability is taking to be $\frac{1}{k(1 + \epsilon e^{i\omega t})}$. We carried out the investigation on the flow using asymptotic analysis, which gives the

solution in symbolic form and allow critical analysis of the solution.

Nomenclature

c : non-dimensional concentration

T : fluid

u : fluid axial velocity

C_f : skin-friction coefficient

Gr_c : mass Grashof number

Grt : thermal Grashof number

M : Hartmann number

Nu : Nusselt number

Sh : Sherwood number

Pr : Prandtl number

Sc : Schmidt number

v : fluid transverse velocity

t : time

c_p : specific heat at constant pressure

y : transverse or horizontal coordinate

i : $\sqrt{-1}$: complex identity

K : non-dimensional reaction parameter

C_w : concentration at the wall

T_w : temperature at the wall

Greek symbols

θ : non-dimensional fluid temperature

ϕ : heat generation/absorption coefficient

ω : angular velocity

Dimensionless group

Grt : dimensionless thermal Grashof number

Gr_c : dimensionless mass Grashof number

ϵ : epsilon, $0 \leq \epsilon_0 \ll 1$

Subscripts

w : condition on the wall

∞ : ambient condition

2.0 Mathematical formulation

An unsteady magnetohydrodynamic flow of viscous, incompressible, electrically conducting fluid past an infinite plate in a porous medium of time dependent permeability and suction velocity is considered. In Cartesian co – ordinate system, x-axis is assumed to be along the plate in the direction of the flow and y-axis normal to it. A uniform magnetic field is introduced normal to the direction of the flow. In the analysis, it is assumed that the magnetic Reynold number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. Further, all the fluid properties are assumed to be constant except that of the influence of density variation with temperature. Therefore, the basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the medium. Initially at $t = 0$, the plate as well as fluid is assumed to be at the same temperature and concentration of species is very low so that the Soret and Dofour effect are neglected [16]. When $t = 0$, the temperature of the plate is instantaneously raised (or lowered) to T_w and the concentration of the species raised (or lowered) to c_w . Under the stated assumptions and taking the usual Buossinesqs approximation in to account, the non dimensional governing equations for momentum, energy and concentration are:

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \epsilon e^{i\omega t}) \frac{\partial u}{\partial y} = Gr \tau \theta + Gr \cdot c + \frac{\partial u^2}{\partial y^2} - \left(\frac{1}{k(1 + \epsilon e^{i\omega t})} + M^2 \right) u \quad (2.1)$$

$$\frac{1}{4} \frac{\partial \theta}{\partial \tau} - (1 + \epsilon e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + \phi \theta \quad (2.2)$$

$$\frac{1}{4} \frac{\partial c}{\partial \tau} - (1 + \epsilon e^{i\omega t}) \frac{\partial c}{\partial y} = \frac{1}{Sc} \frac{\partial^2 c}{\partial y^2} - Kc \quad (2.3)$$

Where the parameters ϵ , ω , Grt , Gr_c , k , Pr , Sc , M , ϕ and K and were as defined in nomenclature. The corresponding boundary conditions are

$$\begin{aligned}
 u = 0, \theta = 1 + \varepsilon e^{i\omega t}, c = 1 + \varepsilon e^{i\omega t} \text{ as } y = 0 \\
 u \rightarrow 0, \theta \rightarrow 0, c \rightarrow 0 \text{ as } y \rightarrow \infty
 \end{aligned}
 \tag{2.4}$$

3.0 Method of solution

We seek an asymptotic expansion about ε for our dependent variables of the form:

$$\left. \begin{aligned}
 u(y, t) &= u_o(y) + \varepsilon u_1 e^{i\omega t} + o(\varepsilon^2) + \dots \\
 \theta(y, t) &= \theta_o(y) + \varepsilon \theta_1 e^{i\omega t} + o(\varepsilon^2) + \dots \\
 c(y, t) &= c_o(y) + \varepsilon \theta_1 e^{i\omega t} + o(\varepsilon^2) + \dots
 \end{aligned} \right\}
 \tag{3.1}$$

Substituting (3.1) into equations (2.1) – (2.4) and collect the terms in power of ε , we have the following sets of equations; corresponding to the energy equation we have,

$$\frac{d^2 \theta_o}{dy^2} + \text{Pr} \frac{d\theta_o}{dy} + \text{Pr} \theta_o = 0
 \tag{3.2}$$

$$\theta_o(0) = 1, \theta_o(y) \rightarrow 0 \text{ as } y \rightarrow \infty$$

$$\frac{d^2 \theta_1}{dy^2} + \text{Pr} \frac{d\theta_1}{dy} + \text{Pr} \left(\frac{i\omega}{4} + \phi \right) \theta_1 = -\frac{d^2 \theta_o}{dy^2}
 \tag{3.3}$$

$$\theta_1(y) = e^{i\omega t} \text{ as } y = 0
 \tag{3.3}$$

$$\theta_1(y) \rightarrow 0 \text{ as } y \rightarrow \infty$$

Corresponding to the specie equation we have,

$$\frac{d^2 c_o}{dy^2} + Sc \frac{dc_o}{dy^2} - KSc c_o = 0
 \tag{3.4}$$

$$c_o(0) = 1, c_o(y) \rightarrow 0 \text{ as } y \rightarrow \infty$$

$$\frac{d^2 c_1}{dy^2} + Sc \frac{dc_1}{dy} + Sc \left(\frac{i\omega}{4} - K \right) c_1 = -\frac{dc_o}{dy}
 \tag{3.5}$$

$$c_1(y) = e^{i\omega t} \text{ at } y = 0, c_1(y) \rightarrow 0 \text{ as } y \rightarrow \infty$$

And corresponding to the momentum equation we have,

$$\frac{d^2 u_o}{dy^2} + \frac{du_o}{dy} - \left(\frac{1}{k_0} + M^2 \right) u_o = -Gr \tau \theta_o - Grcc_o
 \tag{3.6}$$

$$u_o(0) = 0, u_o(\infty) = 0$$

$$\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - \left(\frac{1}{k_0} + M^2 \right) u_1 = \frac{i\omega}{4} u_1 - Gr \tau \theta_1 - Gtcc_1 - \frac{1}{k_0} u_0$$

This is simplified as

$$\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - \left(\frac{1}{k_0} + \frac{i\omega}{4} + M^2 \right) u_1 = -Gr \tau \theta_1 - Gtcc_1 - \frac{1}{k_0} u_0 - \frac{du_0}{dy}
 \tag{3.7}$$

$$u_1(0) = 0, u_1(\infty) = 0$$

Now from (3.2), the solution is

$$\theta_o(y) = e^{-my}, \text{ where } m = \frac{1}{2} \left(\text{Pr} + \sqrt{\text{Pr}^2 - 4 \text{Pr} \phi} \right)
 \tag{3.8}$$

From equation (3.3), on substituting (3.8) we have $\frac{d^2\theta_1}{dy^2} + \text{Pr} \frac{d\theta_1}{dy} + \text{Pr}\left(\frac{i\omega}{4} + \phi\right)\theta_1 = me^{-my}$. Letting

$\theta_{1p} = a_1 e^{-my}$, $\Rightarrow \theta'_{1p} = -ma_1 e^{-my}$ and $\theta''_{1p} = m^2 a_1 e^{-my}$. Thus $a_1 \left(m^2 - m \text{Pr} + \text{Pr} \phi + \frac{\text{Pr} i \omega}{4} \right) e^{-my} \equiv m e^{-my}$

$$\Rightarrow a_1 = \frac{m}{\frac{\text{Pr} i \omega}{4}} = \frac{4m}{\text{Pr} i \omega}, \quad a_1 = -\frac{4mi}{\text{Pr} \omega}$$

and for complementary solution $\theta_{1c}(y) = a_2 e^{-m_1 y} + a e^{m_2 y}$, where

$$m_1 = \frac{1}{2} \left(\text{Pr} + \sqrt{\text{Pr}^2 - 4 \text{Pr} \left(\phi + \frac{i\omega}{4} \right)} \right) \text{ and } m_2 = \frac{1}{2} \left(-\text{Pr} + \sqrt{\text{Pr}^2 - 4 \text{Pr} \left(\phi + \frac{i\omega}{4} \right)} \right)$$

Thus, $\theta_1(y) = a_1 e^{-my} + a_2 e^{-m_1 y} + a e^{m_2 y}$ Using the boundary conditions we have

$$\theta_1(0) = a_1 + a_2 + a = e^{i\alpha} \quad \theta_1(\infty) = a e^{m_2 y} \Big|_{y \rightarrow \infty} = 0 \Rightarrow a = 0 \Rightarrow a_2 = e^{i\alpha} - a_1$$

$$\Rightarrow \theta_1(y) = a_1 e^{-my} + a_2 e^{-m_1 y} \tag{3.9}$$

From equations (3.4) and (3.5), we have respectively

$$c_0(y) = e^{-ny}, \quad n = \frac{1}{2} \left(Sc + \sqrt{Sc^2 + 4KSc} \right), \tag{3.10}$$

$$\text{and } c_1(y) = a_3 e^{-ny} + a_4 e^{-n_1 y}, \quad n_1 = \frac{1}{2} \left(Sc + \sqrt{Sc^2 + 4Sc \left(-K + \frac{i\omega}{4} \right)} \right) \tag{3.11}$$

where $a_3 = -\frac{4ni}{Sc\omega}$ $a_4 = e^{i\omega t} - a_3$. Using (3.8) and (3.10) in (3.6), we have

$$\frac{d_2 u_0}{dy^2} + \frac{du_0}{dy} - \left(\frac{1}{k_0} + M^2 \right) u_0 = -Gr \tau e^{-my} - Gr c e^{-ny}$$

Assuming $u_{0p} = a_5 e^{-my} + a_6 e^{-ny}$ and substituting in the above equation we have

$$a_5 = \frac{-Gr\tau}{m^2 - m - \left(\frac{1}{k_0} + M^2 \right)}, \quad a_6 = \frac{-Grc}{n^2 - n - \left(\frac{1}{k_0} + M^2 \right)}$$

Now $u_{0c} = a_7 e^{-ry} + a e^{r_1 y}$, where $r = \frac{1}{2} \left(1 + \sqrt{1 + 4 \left(\frac{1 + k_0 M^2}{k_0} \right)} \right)$ and $r_1 = \frac{1}{2} \left(-1 + \sqrt{1 + 4 \left(\frac{1 + k_0 M^2}{k_0} \right)} \right)$

Combining the complementary and the particular solutions we have $u_0(y) = a_5 e^{-my} + a_6 e^{-ny} + a_7 e^{-ry} + a e^{r_1 y}$

And applying the boundary conditions, we have

$$u_0(0) = a_5 + a_6 + a_7 + a = 0, \quad u_0(\infty) = a = 0 \quad a_7 = -(a_5 + a_6),$$

$$\text{Thus } u_0(y) = a_5 e^{-my} + a_6 e^{-ny} + a_7 e^{-ry} \tag{3.12}$$

From equation (3.7) on substituting equations (3.9), (3.11) and (3.12) we have

$$\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - \left(\frac{1}{k_0} + M^2 + \frac{i\omega}{4} \right) u_1 = -Gr \tau (a_2 e^{-m_1 y} + a_1 e^{-my}) - Gr c (a_3 e^{-ny} + a_4 e^{-n_1 y}) - \frac{1}{k_0} (a_5 e^{-my} + a_6 e^{-ny} + a_7 e^{-ry}) + a_5 m e^{-my} + a_6 n e^{-ny} + a_7 r e^{-ry}$$

That is
$$\frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} - \left(\frac{1}{k_0} + M^2 + \frac{i\omega}{4} \right) u_1 = -(Gr \tau a_1 + a_5 - m a_3) e^{-my} - a_2 Gr c e^{-m_1 y} - \left(Gr c a_3 + \frac{a_6}{k_0} - a_6 n \right) e^{-ny} - Gr c a_4 e^{-n_1 y} - a_7 \left(\frac{1}{k_0} - r \right) e^{-ry}$$

From which $u_{1p} = a_8 e^{-my} + a_9 e^{-m_1 y} + a_{10} e^{-ny} + a_{11} e^{-n_1 y} + a_{12} e^{-ry}$

where
$$a_8 = - \frac{\left(a_1 Gr \tau + a_5 \left(\frac{1}{k_0} - m \right) \right)}{m^2 + m \left(\frac{1}{k_0} + m^2 + \frac{i\omega}{4} \right)}, \quad a_9 = \frac{-a_2 Gr \tau}{m^2_1 - m_1 - \left(\frac{1}{k_0} + M^2 + \frac{i\omega}{4} \right)}$$

$$a_{10} = - \frac{\left(Gr c a_3 + a_6 \left(\frac{1}{k_0} - n \right) \right)}{n^2 - n - \left(\frac{1}{k_0} - M^2 + \frac{i\omega}{4} \right)}, \quad a_{11} = \frac{-a_4 Gr c}{n^2_1 + n_1 - \left(\frac{1}{k_0} + M^2 + \frac{i\omega}{4} \right)}, \quad a_{12} = \frac{-4ia_7(1-rk_0)}{\omega k_0}$$

Now $u_{1c} = a_{13} e^{-r_1 y} + a_{14} e^{r_1 y}$ $u_1(y) = a_8 e^{-my} + a_9 e^{-m_1 y} + a_{10} e^{-ny} + a_{12} e^{-ry} + a_{13} e^{-r_1 y} + a_{14} e^{r_1 y}$

Then using the boundary conditions, $u_1(y) = 0$ at $y = 0$, $u_1(y) = 0$ as $y \rightarrow \infty$,

$$\Rightarrow a_{14} = 0, \quad a_{13} = -(a_8 + a_9 + \dots + a_{12})$$

Hence
$$u_1(y) = a_8 e^{-my} + a_9 e^{-m_1 y} + a_{10} e^{-ny} + a_{11} e^{-n_1 y} + a_{12} e^{-ry} + a_{13} e^{-r_1 y} \tag{3.13}$$

4.0 Discussion of results

We carried out the analysis using the values $\varepsilon = 0.2$, $Pr = 0.71$, $Sc = 0.6$, $M = 0.5$, $\omega t = \pi/2$, $Gr\tau = 1.2$, $Gr c = 1$, $K_0 = K = 0.2$, $\phi = -0.2$, and $K = -0.5$ for the parameters except where stated otherwise. It should be noted that $K > 0$, $K = 0$ and $K < 0$ represent destructive, no and generative chemical reactions respectively. Also, $\phi > 0$, $\phi = 0$ and $\phi < 0$ indicates heat absorption, no heat generation/absorption and heat generation respectively. The present study limit the choice of Pr and Sc to 0.71 and 0.6 (a case of plasma) respectively. The figures are presented in two forms – (1) 3-dimensional figures and (2) corresponding 2-dimensional figures.

Figure 4.1 show the effect of chemical reaction parameters on concentration field. It is observed that for a generative chemical reaction, there exist oscillations in the field away from the surface. This brings about the presence of minimum and maximum concentration in the field which however less than the surface concentration. For destructive chemical reaction, the boundary layer reduces as the reaction parameters increases. Also there is reduction in concentration field as reaction parameter increases positively. In figure 4.2, we displayed the concentration field as a function of (y, t) , it could be seen that concentration decreases as the flow progresses and decreases faster as we move away from the boundary. While concentration is displayed as a function of (ω, t) in figure 4.3. Oscillation is observed along ω -axis with a steep decrease in the field as y increases.

We displayed in figures 4.4 – 4.7, the temperature profiles for various values of parameters under consideration. It could be seen from figure 4.4 that heat absorption ($\phi < 0$) resulted in decreases in the fluid body temperature, while heat generation ($\phi > 0$) increases the fluid body temperature which lead to presence of extremes temperature in the body of the fluid greater than the surface temperature. In figure 4.5 and 4.6, we shows the temperature profile as a function of (y, t) and (ω, t) respectively. It is observed that the temperature decreases as y increases, and oscillatory field along t and ω -axes which is more pronounced at the initial stage continuous fading as y increases.

Figures 4.7 – 4.15 show the effect of the parameters on the velocity field. It is observed that maximum velocity occurs in the body of the fluid close to the surface. We displayed the effect of reaction parameter on velocity in figure 4.7, it is shown that increase in destructive chemical reaction ($K > 0$) parameter reduces the

velocity field while increase in generative chemical reaction ($K < 0$) parameter increases the velocity. In figure 4.8, it is discovered that increases in ϕ brings about increase in velocity field. While in figure 4.9, effect of Hartmann number M on the velocity is displayed, we discovered that increase in Hartmann number M reduces the velocity field as a result of an opposing force (Lorentz force). Figure 4.10 shows increase in permeability increases the velocity with sudden rise and fall in the velocity field.

We displayed in figures 4.11 and 4.12 the effect of mass and thermal Grashof numbers on the velocity. We discovered that velocity increases as either mass or thermal Grashof number increases. We also noted that heating of the plate ($Gr\tau > 0$) increases the velocity. Figures 4.13 and 4.14 show that increase in ω and t increases the velocity.

In all cases considered here, it could be seen that oscillatory suction velocity affects all the three fields shortly away from the boundary.

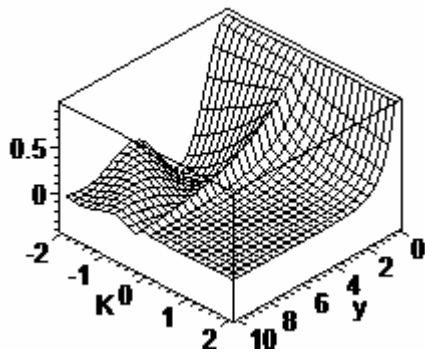


Figure 4.1: Concentration field as function (y, K)

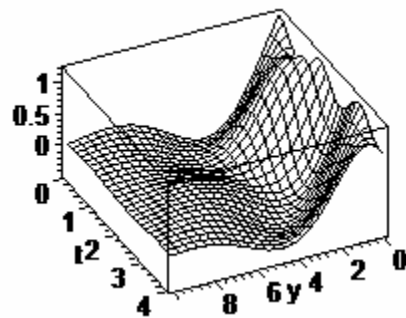


Figure 4.2: Concentration field as function (y, t)

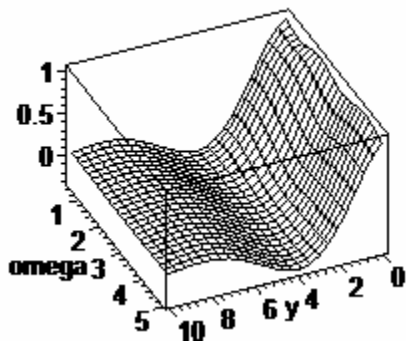


Figure 4.3: Concentration field as function (y, ω)

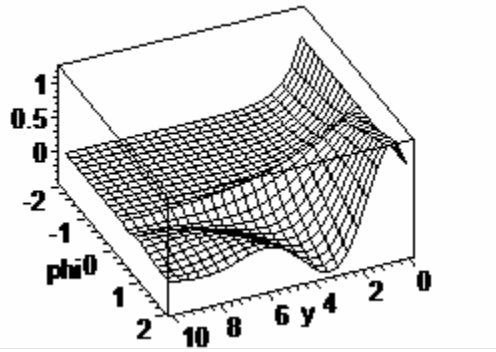


Figure 4.4: Temperature field as function (y, ϕ)

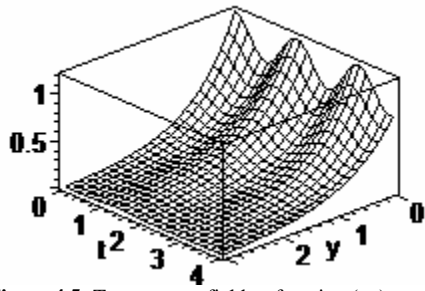


Figure 4.5: Temperature field as function (y, t)

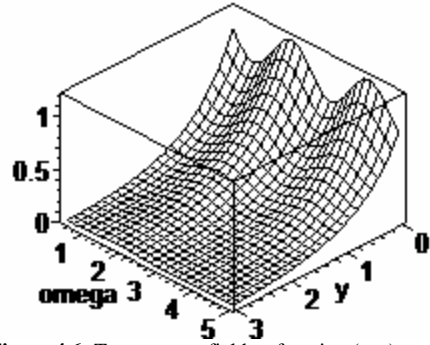


Figure 4.6: Temperature field as function (y, ω)

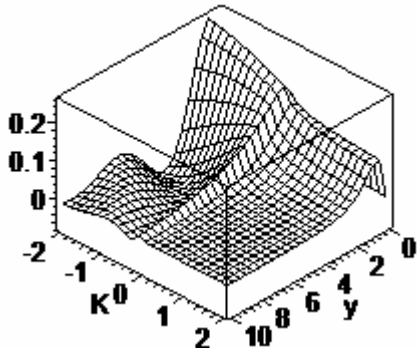


Figure 4.7: Velocity field as function (y, K)

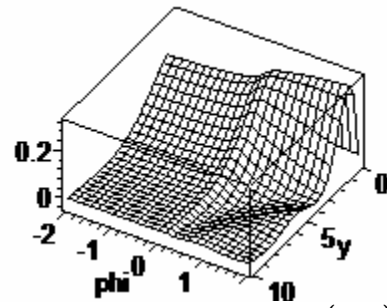


Figure 4.8: Velocity field as function (y, ϕ)

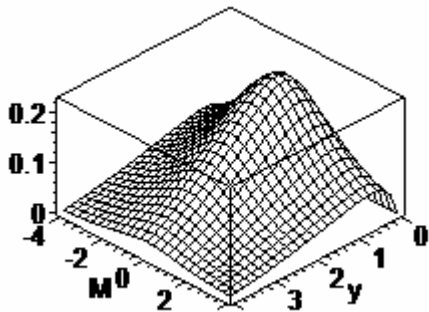


Figure 4.9: Velocity field as function (y, M)

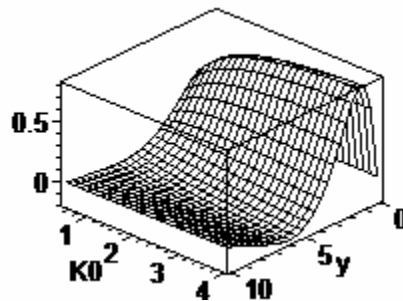


Figure 4.10: Velocity field as function (y, K_0)

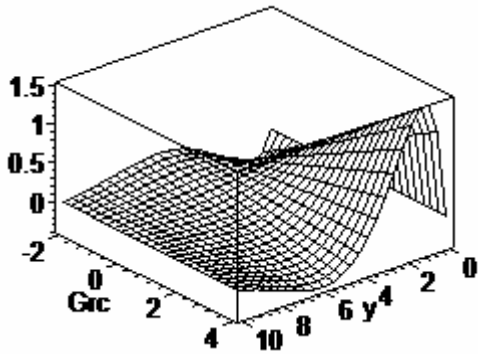


Figure 4.11: Velocity field as function (y, Gr_c)

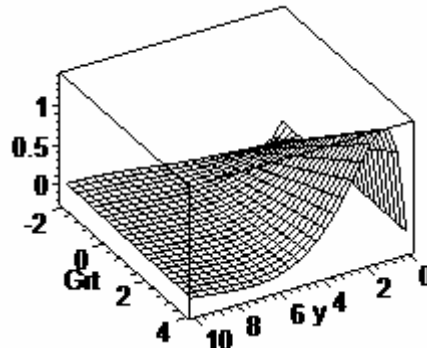


Figure 4.12: Velocity field as function (y, Gr_t)

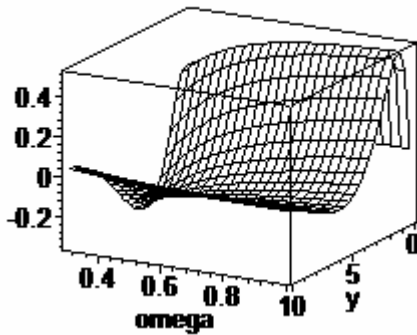


Figure 4.13: Velocity field as function (y, ω)

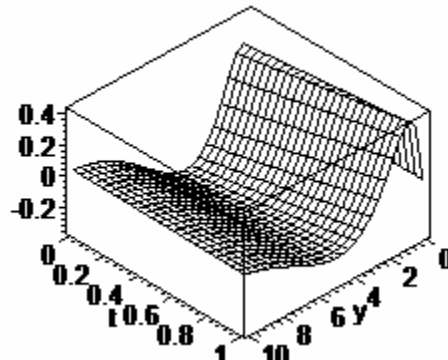


Figure 4.14: Velocity field as function (y, t)

4.1 Analysis of dimensionless number on MHD flows

We now move to examining some important fluid parameters that are of importance to this work. Such parameters include Sherwood number, Nusselt number and Skin-friction coefficient. We therefore denote and define respectively, Sherwood number, Nusselt number and Skin-friction coefficient as

$$Sh = \frac{J_\omega v}{(c_\omega - c_\infty) D v_\omega} = \frac{-d}{dy} c(0), \quad J_\omega = -D \frac{dc}{dy} \Big|_{y=0}, \quad Nu = \frac{q_\omega v}{(T_\omega - T_\infty) K v_\omega} = \frac{-d}{dy} \theta(0), \quad q_\omega = -K \frac{dT}{dy} \Big|_{y=0},$$

$$c_f = \frac{T_f}{\rho u_\omega v_\omega} = \frac{d}{dy} u(0), \quad \tau_f = \mu \frac{du}{dy} \Big|_{y=0}$$

Now, we revisit the analytical solutions reported from which we obtain Sh , Nu and c_f as follows;

$$Sh = -n + \epsilon e^{i\alpha} (-a_3 m - a_4 n_1)$$

$$Nu = -m + \epsilon e^{i\alpha} (-a_1 m - a_2 m_1)$$

$$c_f = -a_5 m - a_6 n - a_7 r + \epsilon e^{i\alpha} (-a_8 m - a_9 m_1 - a_{10} n - a_{11} n_1 - a_{12} r - a_{13} r_1)$$

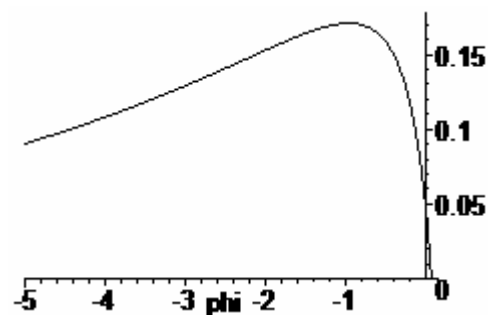
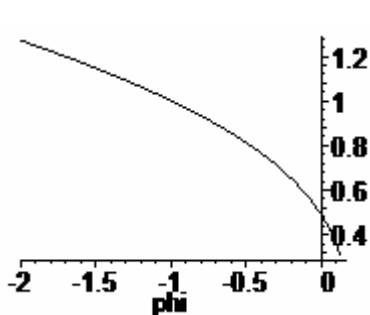


Figure 4.15: Nusselt number profile against heat generation/absorption coefficient

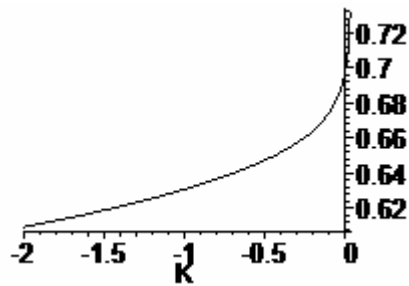


Figure 4.17: Skin friction profile against reaction parameter

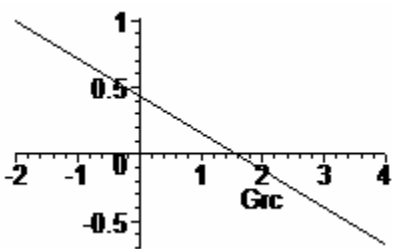


Figure 4.19: Skin friction profile against mass Grashof number

Figure 4.16: Skin friction profile against heat generation/absorption coefficient

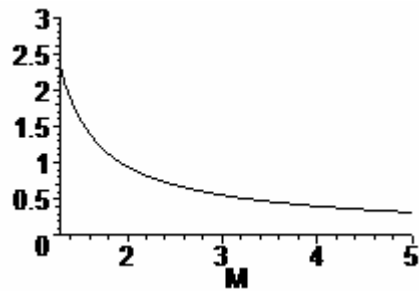


Figure 4.18: Skin friction profile against Hartmann number

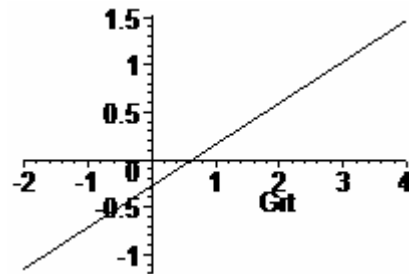


Figure 4.20: Skin friction profile against thermal Grashof number

Figure 4.15 shows the Nusselt number as a function heat generation/absorption coefficient ϕ . We discovered that as heat generation coefficient increases, the Nusselt number also increases. We also show in figure 16 the effect of generation/absorption coefficient on the skin friction coefficient. It could be seen that maximum values of skin friction coefficient occurs for $-1 < \phi < 0$, which is a case of heat generation. While in figure 17, it is observed that reduction in generative chemical reaction parameter increases the skin friction coefficient. Figure 18 shows that as Hartmann number M increases, the skin friction coefficient reduces asymptotically with maximum value only when $M=0$. We show in figures 19 and 20 that increase in mass and thermal Grashof number reduces and increases respectively the skin friction coefficient.

4.2 Concluding remarks

Analytical solution for unsteady hydromagnetic heat and mass transfer in MHD flow of an incompressible, electrically conducting, viscous fluid past an infinite vertical porous plate along with porous medium of time dependent permeability under oscillatory suction velocity normal to the plate has been made. The various combinations of parameters give much insight into the behaviour of magnetohydrodynamic flow. Based on the obtained graphical results, the following conclusions were deduced that:

- (1) the fluid temperature increases during heat generation and decreased during heat absorption.
- (2) the concentration of chemical species increases with increase in generative chemical reaction ($K < 0$), and decrease with increase in destructive chemical reaction ($K > 0$).
- (3) the boundary layer of the velocity, temperature and concentration reduces as either time or position of the flow elements increases.
- (4) the velocity reduces as Hartmann number increases.
- (5) the maximum velocity occurs in the body of fluid.
- (6) the velocity increases as the plate is heated ($Gr_t > 0$).
- (7) increases in heat generation coefficient increases the Nusselt number
- (8) reduction in generative chemical reaction parameter increases the skin friction coefficient.

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