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Effects of variable viscosity on MHD boundary layer flow on a continuously moving vertical plate in the presence of radiation and a chemical reaction of order 1

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Abstract

This work presents a study of the flow and heat transfer of an incompressible viscous electrically conducting fluid over a continuously moving vertical infinite plate with uniform suction and heat flux in the presence of radiation and a chemical reaction of order 1 taking into account the effects of variable viscosity. It is found that the velocity increases as the viscosity of fluid or the magnetic parameter decreases and the thermal boundary layer thickness increases as the radiation parameter increases. The skin-friction coefficient is computed and discussed for various values of the parameters. A parametric study was conducted and reported.

Keywords: MHD, variable viscosity, radiation.
Mathematics Subject Classification: 76W05

1.0 Introduction

Flow and heat transfer in the boundary layer induced by a moving surface in a quiescent fluid is important in many engineering applications. For examples, in the extrusion of polymer sheet from a dye, the cooling of an infinite metallic plate in a cooling path, glass blowing continuous casting and spinning of fibres. Sakiadis [15] studied the boundary layer flow over a continuous solid surface moving with constant velocity in an ambient fluid. The flow is quite different from the boundary layer flow over a semi-infinite flat plate due to the entrainment of the ambient fluid. Tsou et al. [18] presented a combined analytical and experimental study of the flow and temperature fields in the boundary layer on a continuous moving surface. Erickson et al. [6] extended Sakiadis problem to include blowing or suction at the moving surface. Crane [5] studied the boundary layer flow caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point on the surface. Gupta and Gupta [8] studied the momentum, heat and mass transfer in the boundary layer over a stretching sheet with suction or blowing. Soundalgekar and Ramana [17] investigated the constant surface case with a power law temperature.

The magnetohydrodynamics of an electrically conducting fluid is encountered in many problems in geophysics, astrophysics, engineering applications and other industrial areas. Hydromagnetic free convection flow have a greet significance for the applications in the fields of steller and planetary magnetospheres, aeronautics. Engineers employ magnetohydrodynamics principles in the design of heat exchangers, pumps, in space vehicle propulsion, thermal protection, control and re-entry and in creating novel power generating systems. However, hydromagnetic flow and heat transfer problems have become more important industrially.

In many metallurgical processes involving the cooling of many continuous strips or filaments by drawing them through an electrically conducting fluid subject to a magnetic field, the rate of cooling can be

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controlled and final product of desired characteristics can be achieved. Another important application of hydromagnetics to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of a magnetic field.

Chakrabartia and Gupta [3] investigated hydromagentic flow, heat and mass transfer over a stretching sheet. Kumar et al. [12] studied hydromagnetic flow and heat transfer on a continuously moving vertical plate. Vajravelu and Hadjinicolaou [19] studied the flow and heat transfer characteristic in an electrically conducting fluid near an isothermal stretching sheet. Sharma and Mathur [16] investigated steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical infinite plate in the presence of heat source or sink.

On the other hand, at high temperature the effects of radiation in space technology, solar power technology, space vehicle re-entry, nuclear engineering applications are very significant. Many processes in industrial areas occur at high temperature and the knowledge of radiation heat transfer in the system can perhaps lead to a desired product with a desired characteristic. Raptis and Massalas [14] studied the radiation effect on the unsteady magnetohydrodynamic flow of an electrically conducting viscous fluid past a plate. Chamkha [4] investigated thermal radiation and buoyancy effects on hydromagnetic flow over an accelerating permeable surface with heat source or sink. Raptis et al.[13] discussed the effect of thermal radiation on MHD asymmetric flow of an electrically conducting fluid past a semi-infinite plate. Okedoye et al. [10, 11], studied MHD flow of a uniformly stretched vertical permeable membrane in the presence of zero order reaction and quadratic heat generation. They show that the flow depends heavily on Grashof numbers, magnetic parameter, reaction parameter and heat generation/absorption parameters.

All the above studies were confined to a fluid with constant viscosity. However, it is known that this physical property may change significantly with temperature. Hossain and Munir [2] analyzed a two-dimensional mixed convection flow of a viscous incompressible fluid of temperature dependent viscosity past a vertical plate. Fang [7] studied the influence of fluid property variation on the boundary layers of a stretching surface. Hossain et al.[1] discussed the effect of radiation on free convection flow of a fluid with variable viscosity from a porous vertical plate.

The purpose of the present work is to study the effects of radiation, reaction parameter and variable viscosity on magnetohydrodynamic boundary layer flow along a continuously moving vertical plate with uniform suction and heat flux.

Nomenclature

u : velocity along x coordinate
v : velocity along y coordinate
g : acceleration due to gravity
T : fluid temperature
T_∞ : ambient temperature
C : species concentration
C_∞ : ambient species concentration
B_0 : transverse magnetic field
C_f : skin – friction coefficient
q : heat flux,
κ : thermal conductivity
c_p : specific heat at constant pressure
q_r : radiative heat flux
v_0 : normal velocity at the plate
k* : mean absorption coefficient
M : Hartmann number
θ : non dimensional fluid temperature
ψ : non dimensional specie concentration
ϕ : non dimensional fluid velocity
γ : reaction parameter
α^*: Stefan-Boltzmann constant
μ : fluid viscosity
B_c : coefficient of mass expansion
B_τ : coefficient of thermal expansion
ρ_∞ : ambient density
σ : electrical conductivity
ρ : density of the fluid
Sc : Schmidt number
Pr : Prandtl number
R : radiation parameter
Grc : mass grashof number
$Gr\tau$: thermal grashof number

2.0 Physical model and governing equations

Consider a steady two- dimensional laminar boundary layer flow of an incompressible electrically conducting viscous fluid on an infinite plate, issuing from a slot and moving vertically with uniform velocity in a fluid and heat is supplied from the plate to the fluid at a uniform rate. The x-axis is taken along the plate in the upwards direction and y- axis is normal to it. A transverse constant magnetic field B_0 is applied, that is, in the direction of y-axis. The physical model of the problem is shown in figure 2.1.

The induced magnetic field is assumed to be negligible. It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible. The viscous dissipation and Joule heating are also neglected.

We further assume that property variations with temperature are limited to viscosity and with the density taken into account only in the buoyancy term in the momentum equation. Since the motion is two – dimensional and the length of the plate is large, therefore, all the physical variables are independent of *x*.

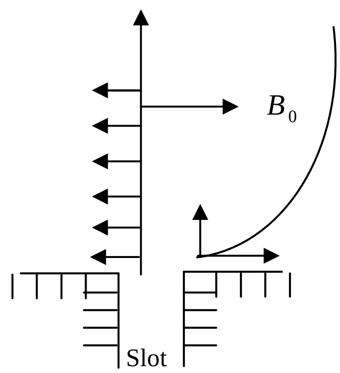

Figure 2.1: Physical model of the problem

Under the above assumptions and Boussinesq approximation the magnetohydrodynamic flow relevant to the problem is governed by the following equations

$$\frac{dv}{dy}=0 \tag{2.1}$$

$$v\frac{du}{dy}=\frac{1}{\rho_\infty}\frac{d}{dy}\left(\mu\frac{du}{dy}\right)+g\beta_\tau(T-T_\infty)+g\beta_c(C-C_\infty)-\frac{\sigma B_0{}^2}{\rho_\infty}u \tag{2.2}$$

$$v\frac{dT}{dy}=\frac{k}{\rho_\infty c_P}\frac{d^2T}{dy^2}-\frac{1}{\rho_\infty c_P}\frac{dq_r}{dy} \tag{2.3}$$

$$v\frac{dC}{dy}=D\frac{d^2C}{dy^2}-A(C-C_\infty) \tag{2.4}$$

The boundary conditions are

$$\left.\begin{aligned} u=u_w,\ v=v_0,\frac{dT}{dy}=-\frac{q}{k},\ C=C_w\ at\ y=0 \\ u\to 0,\ T\to T_\infty,\ C\to C_\infty\quad at\ y\to\infty \end{aligned}\right\} \tag{2.5}$$

From equation (2.1) we take $$v(y)=-v_0 \tag{2.6}$$

where u , v are the velocities along x, y coordinates, respectively. By using Rosseland approximation q_r takes the form [13]

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{dT^4}{dy} \tag{2.7}$$

where k^* is the mean absorption coefficient and σ^* is the Stefan-Boltzmann constant. The temperature differences within the fluid assumed sufficiently small such that T^4 may be expressed as a linear function of the temperature. Expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms, we get

$$T^4=4T_\infty{}^3T-3T_\infty{}^4 \tag{2.8}$$

Using equation (2.8), equation (2.7) becomes

$$q_r=-\frac{16\sigma^*T_\infty{}^3}{3k^*}\frac{dT}{dy} \tag{2.9}$$

By using equations (2.6) and (2.9) then equation (2.3) gives

$$-v_0\frac{dT}{dy}=\frac{k}{\rho_\infty c_P}\frac{d^2T}{dy^2}+\frac{1}{\rho_\infty c_P}\frac{16\sigma^*T_\infty{}^3}{3k^*}\frac{dT}{dy} \tag{2.10}$$

Introducing the following non-dimensional quantities

$$\eta = \frac{\rho_\infty v_0}{\mu_\infty} y,\ \phi(\eta) = \frac{u}{u_w},\ \ \theta(\eta) = \frac{T - T_\infty}{\left(\frac{q\mu_\infty}{k\rho_\infty v_0}\right)},\ \ \psi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{2.11}$$

into equations (2.2), (2.4) and (2.11), one gets the following non-dimensional equations governing the flow and the energy distribution:

$$\frac{d^2\psi}{d\eta^2} + Sc\frac{d\psi}{d\eta} - \gamma Sc\,\psi = 0 \tag{2.12}$$

$$\frac{d^2\theta}{d\eta^2} + \frac{\Pr}{1+R}\frac{d\theta}{d\eta} = 0 \tag{2.13}$$

$$\frac{d}{d\eta}\left(\frac{\mu}{\mu_\infty}\frac{d\phi}{d\eta}\right) + \frac{d\phi}{d\eta} + Gr\,\tau\theta + Grc\,\psi - M\phi = 0 \tag{2.14}$$

The appropriate boundary conditions are

$$\left.\begin{aligned} &\phi(\eta) = 1,\ \ \psi(\eta) = 1,\ \ \left.\frac{d\theta}{d\eta}\right|_{\eta=0} = -1\ at\ \eta = 0 \\ &\phi(\eta) \to 0\ \ \theta(\eta) \to 0\ \ \psi(\eta) \to 0\, as\, \eta \to \infty \end{aligned}\right\} \tag{2.15}$$

where $Gr\,\tau = \dfrac{\mu_\infty g\beta_\tau \dfrac{q\mu_\infty}{k\rho_\infty v_0}}{\rho_\infty u_w v_0{}^2}$, $Grc = \dfrac{\mu_\infty g\beta_\tau (C_w - C_\infty)}{\rho_\infty u_w v_0{}^2}$, $M = \dfrac{\sigma B_0{}^2 \mu_\infty}{\rho_\infty{}^2 v_0{}^2}$,

$$\Pr = \frac{\mu_\infty c_p}{k},\ R = \frac{16\sigma^* T_\infty{}^2}{3k^* k},\ Sc = \frac{\mu_\infty}{D\rho_\infty},\ \ \gamma = A\frac{\mu_\infty}{v_0{}^2\rho_\infty}$$

The fluid viscosity μ(θ) was assumed to obey the Reynolds model [9]

$$\frac{\mu}{\mu_\infty} = e^{-\alpha\theta} \tag{2.16}$$

Where α, is parameter depending on the nature of the fluid. Using equation (2.16) in equation (2.14) we obtain

$$\frac{d}{d\eta}\left(e^{-\alpha\theta}\frac{d\phi}{d\eta}\right) + \frac{d\phi}{d\eta} - M\phi = -Gr\,\tau\theta(\eta) - Grc\,\psi(\eta) \tag{2.17}$$

3.0 Method of solution

Solving equations (2.12) and (2.13), we have

$$\theta(\eta) = ne^{-n\eta} \text{ and } \psi(\eta) = e^{-m\eta} \tag{3.1}$$

where $n = \dfrac{\Pr}{1+R}$, $m = \dfrac{1}{2}\left(Sc + \sqrt{Sc^2 + 4\gamma Sc}\right)$

(i) ***Case of constant viscosity:***

For $\alpha = 0$, from equation (2.17) we have

$$\frac{d^2\phi}{d\eta^2} + \frac{d\phi}{d\eta} - M\phi = -Gr\,\tau n e^{-n\eta} - Grc\,e^{-m\eta} \tag{3.2}$$

Solving equation (3.2) with the boundary conditions (2.15), we get

$$\phi(\eta) = a_4 e^{-\lambda\eta} + a_6 e^{-n\eta} + a_7 e^{-m\eta} \tag{3.3}$$

$$a_6 = -\frac{nGr\tau}{n^2 - n - M} \quad a_7 = -\frac{Grc}{m^2 - m - M} \quad a_4 = 1 - a_6 - a_7$$

(ii) ***Variable viscosity case:***

On taking into account the solution for temperature, we solved numerically the equation (2.17) under the boundary conditions (2.15) using the fourth order Runge-Kutta method algorithm with systematic guessing $\phi'(\eta)$ by the shooting technique until the boundary condition $\phi(\eta)$ at infinity decay exponentially to zero. If the boundary condition at infinity is not satisfied then the numerical routine uses the Newton-Raphson method to calculate corrections to the estimate value of $\phi'(\eta)$. This process is repeated iteratively until convergence is achieved to a specified accuracy, 10^{-7}. A computer programming language call Pascal and Mathematical software known as Mapple 8.1 are adapted to implement the above process.

The physical quantity of most interest in such problem is the skin- friction coefficient which is defined by

$$c_f = 2\frac{1}{\rho_\infty v_0 u_w}\mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = 2e^{-\alpha\theta(\eta)} \left.\frac{d\phi(\eta)}{d\eta}\right|_{\eta=0} \tag{3.4}$$

4.0 Results and discussion

In order to validate our results, we have compared our numerical results with α = 0 for $\phi'(\eta)$ with those of analytical results. The results are found to be in a good agreement as given in table 4.1.

For the purpose of discussing the effect of various parameters on the flow profiles and the temperature distributions within the boundary layer, numerical calculations have been carried out for various values of M, $Gr\tau$, Grc, α, γ and R with fixed values of Pr and Sc. The value of Pr and Sc were taken to be 0.71 and 0.6 respectively for plasma. These parameters were assigned the values $M = 1$, $R = 0.5$, γ = 0.1, α = 0.1, $Gr\tau = 5$, and $Grc = 1$ except where stated otherwise. It should be noted that increase in α viscosity parameter α leads to decrease in viscosity as given by the relation in equation (2.16). Also γ < 0, γ = 0, and γ > 0 indicate generative, no reaction and destructive chemical reaction respectively. The variation of the skinfriction coefficient $\phi'(\eta)$ for various values of α, M, R and $Gr\tau$ with $Pr = 0.71$ is shown in table 4.2. It can be seen from this table that the skin-friction coefficient increases as the mass Grashof number or the thermal Grashof number increases. Increasing of the magnetic parameter, viscosity parameter, the radiation parameter or reaction parameter leads to a decrease in the skin-friction.

The effect of α on the dimensionless velocity ϕ is illustrated in table 4.3. From this table, one sees that the velocity ϕ increases as the viscosity of the fluid decreases. The fluid velocity increased and reached its maximum value at very short distance from the plate and then decreased to zero. Generative chemical reaction leads to increase in fluid velocity while increase in destructive chemical reaction lowers the velocity as shown in figure 4.1.

Table 4.1: Comparison of analytical and numerical values of $\phi'(0)$ when $\alpha = 0$

M	R	$Gr\tau$	Grc	γ	*Analytical*	*Numerical*
0.0	0.5	5	1	0.1	5.454972247	5.454971336
0.5	0.5	5	1	0.1	2.402963167	2.402963155
1.0	0.5	5	1	0.1	1.316588013	1.316587014
1.0	0.0	5	1	0.1	1.821178808	1.821177917
1.0	0.1	5	1	0.1	1.702310102	1.702312113
1.0	0.5	5	0	0.1	.5504995744	.5504985645
1.0	0.5	5	2	0.1	2.082676450	2.082676541
1.0	0.0	5	4	0.1	3.614853326	3.614853328
1.0	0.1	5	1	-0.12	1.500890979	1.500890980
1.0	0.5	5	1	0.0	1.371494727	1.371494726

0.5	0.5	5	1	0.2	1.277089806	1.277089803
1.0	0.5	8	1	0.1	2.617708152	2.617708142
1.0	0.0	10	1	0.1	3.485121575	3.485121564

In figure 4.2, we show the distribution of velocity for various value of radiation parameter. It could be seen that increase in radiation parameter reduces the velocity. We displayed in figures 4.3 and 4.4 the effect of thermal and mass grashof numbers respectively. It is observed that increase in the values of both parameters leads to increase the velocity and vice versa. The maximum value increased with the increasing *Grτ* and *Grc*. The velocity ϕ at any vertical plane near the plate decreases as the magnetic parameter *M* increases as shown in figure 4.5. It is observed that the velocity increased to its maximum value near the plate and then decreased to zero. Figure 4.7 shows the temperature θ profile for various value of radiation parameter. It could be seen that increase radiation parameter *R* reduces temperature of the fluid. Also it is noticed that a decreases in the fluid temperature with maximum value at the plate and minimum at a distance away from the plate. The effect of reaction parameter on the concentration of chemical species is shown in figure 4.8. We noticed that increase in reaction parameter reduces the concentration of the chemical species.

Table 4.2: The values of $\phi'(0)$ with *Pr* = 0.71 and *Sc* = 0.6, when $\alpha \neq 0$

α	*M*	*R*	*Grτ*	*Grc*	γ	$\phi'(0)$
-0.2	1.0	0.5	5	1	0.1	1.98561
-0.1	1.0	0.5	5	1	0.1	1.52380
0.0	1.0	0.5	5	1	0.1	1.31658
0.1	1.0	0.5	5	1	0.1	0.57833
0.2	1.0	0.5	5	1	0.1	0.60750
0.1	0.0	0.5	5	1	0.1	4.90000
0.1	0.5	0.5	5	1	0.1	2.12050
0.1	1.0	0.5	5	1	0.1	1.10200
0.1	1.0	0.0	5	1	0.1	1.66554
0.1	1.0	0.1	5	1	0.1	1.52930
0.1	1.0	0.5	5	0	0.1	0.57833
0.1	1.0	0.5	5	2	0.1	1.62567
0.1	1.0	0.0	5	4	0.1	2.67301
0.1	1.0	0.1	5	1	-0.12	1.27950
0.1	1.0	0.5	5	1	0.0	1.15208
0.1	0.5	0.5	5	1	0.2	1.06731
0.1	1.0	0.5	8	1	0.1	2.45597
0.1	1.0	0.0	10	1	0.1	3.35860

Table 4.3: Velocity $(\phi(\eta))$ distribution for various values of α

η	ϕ at α=-0.2	ϕ at α=-0.1	ϕ at α =0.0	ϕ at α =0.1	ϕ at α = 0.2
0	1	1	1	1	1
1	1.1101945	1.11281359	1.11527855	1.11789245	1.12034
2	0.74127546	0.75244925	0.75053917	0.74903271	0.747296
3	0.471132	0.46965492	0.46779761	0.46675	0.465323
4	0.2905447	0.28979349	0.28832033	0.28833389	0.287626

5	0.17946252	0.17913306	0.17744776	0.1785048	0.178206
6	0.11114181	0.11099644	0.10832835	0.11074104	0.110632
7	0.06885575	6.88E-02	6.40E-02	6.87E-02	6.87E-02
8	0.04247193	4.24E-02	3.36E-02	4.24E-02	4.24E-02
9	0.02576829	2.57E-02	9.43E-03	2.56E-02	2.57E-02
10	0.01480355	1.46E-02	0.00E+00	1.46E-02	1.48E-02
11	0.00691527	0.00656344	0.00E+00	6.53E-03	6.87E-03
12	3.2091E-05	0.00E+00	0.00E+00	0.00E+00	0.00E+00

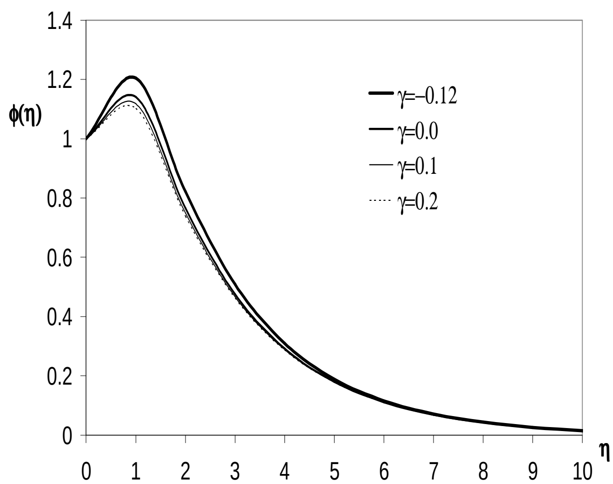

Figure 4.1: Velocity $(\phi(\eta))$ distribution for various values of γ

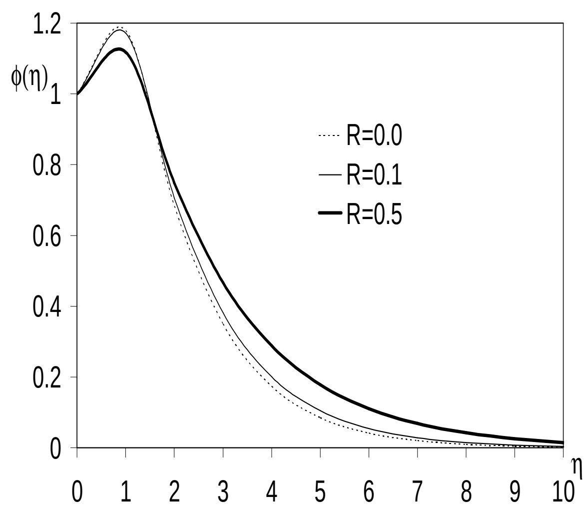

Figure 4.2: Velocity $(\phi(\eta))$ distribution for various values of R

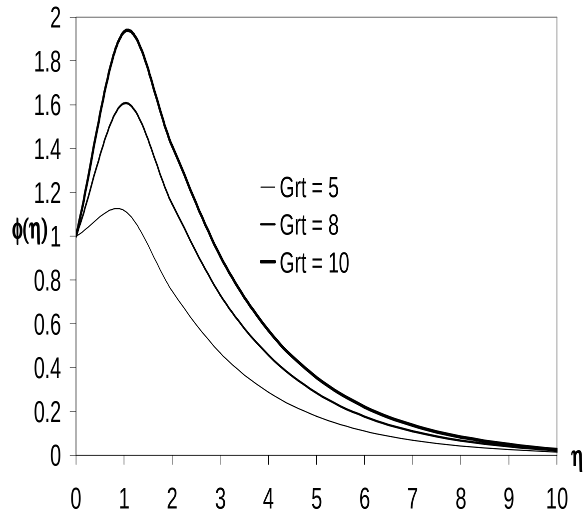

Figure 4.3: Velocity $(\phi(\eta))$ distribution for various values of $Gr\tau$

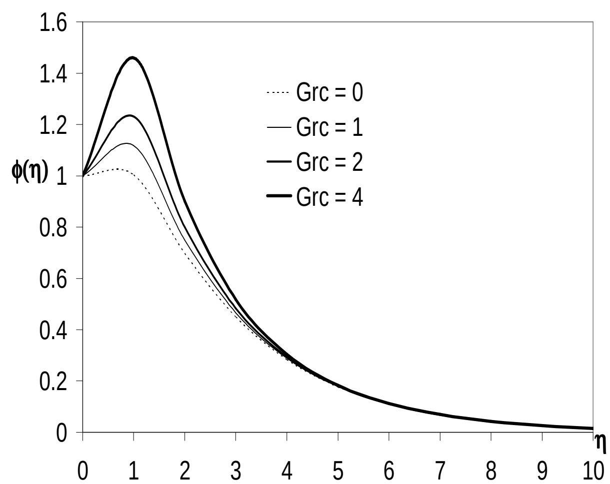

Figure 4.4: Velocity $(\phi(\eta))$ distribution for various values of *Grc*

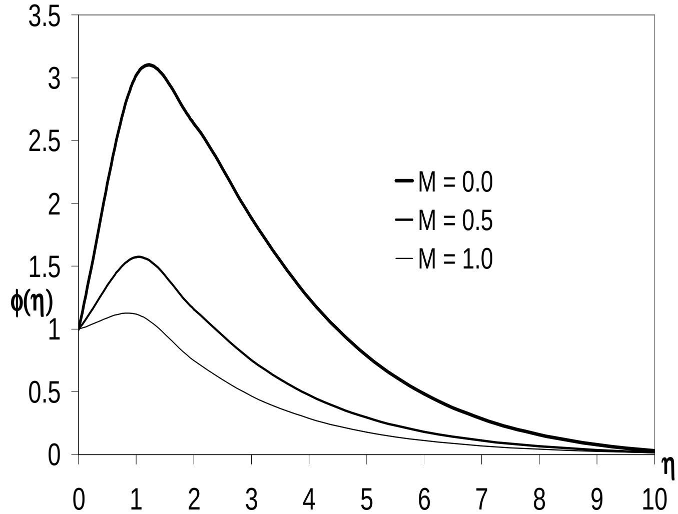

Figure 4.5: Velocity $(\phi(\eta))$ distribution for various values of M

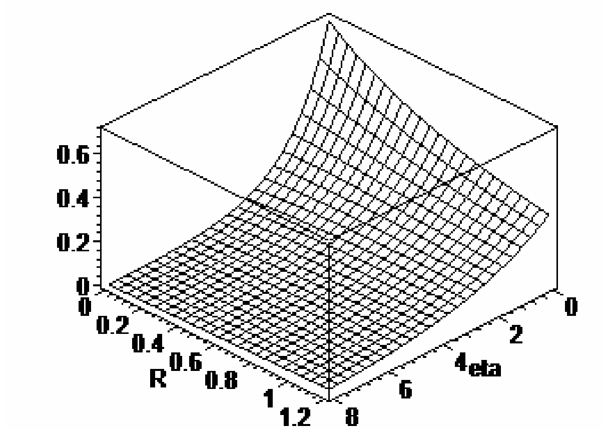

Figure 4.6: Temperature $(\theta(\eta))$ distribution for various values of R with Pr = 0.71

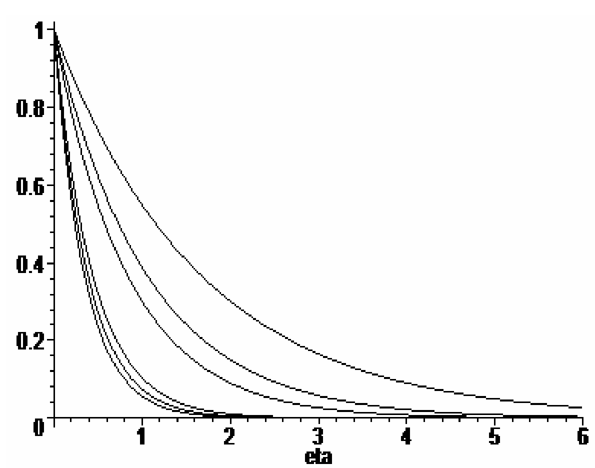

Figure 4.7: Concentration $(\psi(\eta))$ distribution for various values of γ

5.0 Conclusion

In this works, the problem of boundary layer flow of a steady viscous, incompressible electrically conducting fluid with variable viscosity over a continuously moving vertical porous plate in the presence of magnetic field and radiation has been investigated. The major results from this study can be summarized thus:

1. the velocity increases as the viscosity parameter reduces, while it decreases as the magnetic parameter increases.
2. the maximum value of the velocity increases as the Grashof numbers increases.
3. the thermal boundary layer thickness decreases as the radiation parameter increases.
4. the skin-friction coefficient increases as the Grashof number increases, while it decreases as the magnetic parameter increases.
5. the temperature reduces radiation parameter increases.
6. the fluid temperature decreases with maximum value at the plate and minimum at a distance away from the plate.
7. increase in reaction parameter reduces the concentration of the chemical species.
8. the velocity increased to its maximum value near the plate and then decreased to zero.
9. generative chemical reaction leads to increase in fluid velocity while increase in destructive chemical reaction lowers the velocity.

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