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# Viscous dissipation effect on the hydro-magnetic flow through a very porous horizontal plate

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***Abstract***

***We analyse heat transfer characteristics of a steady hydro-magnetic flow in a horizontal plate. The effect of the Darcy number, Hartmann number and the Brickman number were effectively determined. Results obtained shows that there is an increase in temperature as the Hartmann number increases and an increase in the Darcy number leads to a decrease in the temperature.***

## 1.0 Introduction

Flow through a porous media has been receiving attention by researchers because of its considerable importance in many practical and technological applications. Many geological flows such as floes in oil reservoir or geothermal power system involve convective heat transfer in porous medium [5]. While the study of heat transfer in hydromagnetic flows have found application in astrophysics, meteorology and many engineering applications. [4].

Several studies have been carried out on the flow through porous horizontal plate. Kafonssias [3] studied flow through a porous medium in the presence of heat transfer but neglected viscous dissipation effect.

The attention of this paper is to investigate the effect of viscous dissipation in a fully developed hydromagnetic flow in a porous horizontal plate.

## 2.0 Governing equations

We consider the two dimensional steady flow of a viscous and electrically conducting fluid through a very porous semi-infinite horizontal surface. Using the boundary layer assumptions the flow and heat transfer are governed by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{2.1}$$

$$U\,\frac{\partial U}{\partial X}+V\,\frac{\partial T}{\partial u}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}\frac{1}{e}\frac{\partial P}{\partial x}+\frac{B_0^2\sigma}{k}\mu u \tag{2.2}$$

$$U\,\frac{\partial T}{\partial x}+V\,\frac{\partial T}{\partial y}=\frac{\alpha}{ec_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{ec_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

So that for a fully developed flow in a free stream we have

$$\alpha\,\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2=0 \tag{2.4}$$

$$\frac{d^2 u}{dy^2}-\frac{B_0^2\sigma u}{k}=-\frac{B_0^2\sigma}{k}u_\infty \tag{2.5}$$

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Subject to the following boundary conditions

$$\left.\begin{array}{r} U(0)=0,\ T(0)=T_0 \\ U(\infty)=U_\infty,\ T(\infty)=T_\infty \end{array}\right\} \qquad (2.6)$$

where $u$ = Fluid velocity in the $x$-direction, $v$ = Fluid velocity in the $y$-direction, $\rho$ = Density of fluid, $\mu$ = Viscosity of fluid, $P$ = Pressure, $k$ = Permeability of the porous medium, $T$ = Temperature, $\alpha$ = Thermal conductivity, $c_p$ = Specific heat, $\sigma$ = Electrical conductivity of the fluid, $B_0$ = Magnetic field strength, $T_0$ = Temperature of the plate, $U_\infty$ = Free stream velocity, $T_\infty$ = Free stream temperature

## 3.0 Problem Solution

Using the following dimensionless variables $M^2=\frac{B_0^2\sigma L^2}{\mu}, U=\frac{u}{u_\infty},\ D_a=\frac{K}{L^2},\ y=\frac{y}{L},\ \theta=$ $\frac{T-T_0}{T_\infty-T_0}$, equations (2.4) and (2.5) reduces to

$$\frac{\partial^2\theta}{\partial y^2}+B_r\left(\frac{\partial u}{\partial y}\right)^2=0. \qquad (3.1)$$

$$\frac{\partial^2 U}{\partial y^2}-\frac{M^2R_e}{D_a}U=-\frac{M^2R_e}{D_a} \qquad (3.2)$$

where

$D_a =$ Darcy Number

$M =$ Hartmann Number

$R_e =$ Reynolds Number

$B_r =$ Brickman Number

We shall now solve (3.1) and (3.2) subject to

$$\left.\begin{array}{r} U(0)=0,\ \theta(0)=\theta_0 \\ U(\infty)=U_1,\ \theta(\infty)=\theta_1 \end{array}\right\} \qquad (3.3)$$

For a bounded solution, we have the solutions to (3.1) and (3.2) to be

$$U=1-e^{-\lambda y} \qquad (3.4)$$

and

$$\theta=\frac{B_r}{4}+\theta_0-\frac{B_r}{4}e^{-2\lambda y} \qquad (3.5)$$

respectively, where

$$\lambda^2=\frac{m^2R_e}{D_a}. \qquad (3.6)$$

The heat transfer rate at the wall is given as

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=0}=\frac{B_rM}{\sqrt{4D_a}}\sqrt{R_e} \qquad (3.7)$$

## 3.0 Numerical results and discussion

Results for various values of the flow parameter are as shown. Figure 3.1 shows the influence of the Hartmann number on the dimensionless temperature for $D_\alpha = 31$, $B_r = 5$, $R_e = 10$, $\theta_0 = 1$. It can be seen that as the Hartmann number increases the temperature also increases. Figure 3.2 shows a plot of temperature variation with the Darcy number for different values of the Hartman number for $M = 0.5$, $B_r = 5$, $\theta_0 = 1$, $R_e = 10$. It is observed that the temperature decreases for higher values of the Hartman number

**Figure 3.1**: Graph of dimensionless temperature against distance for values Hartmann

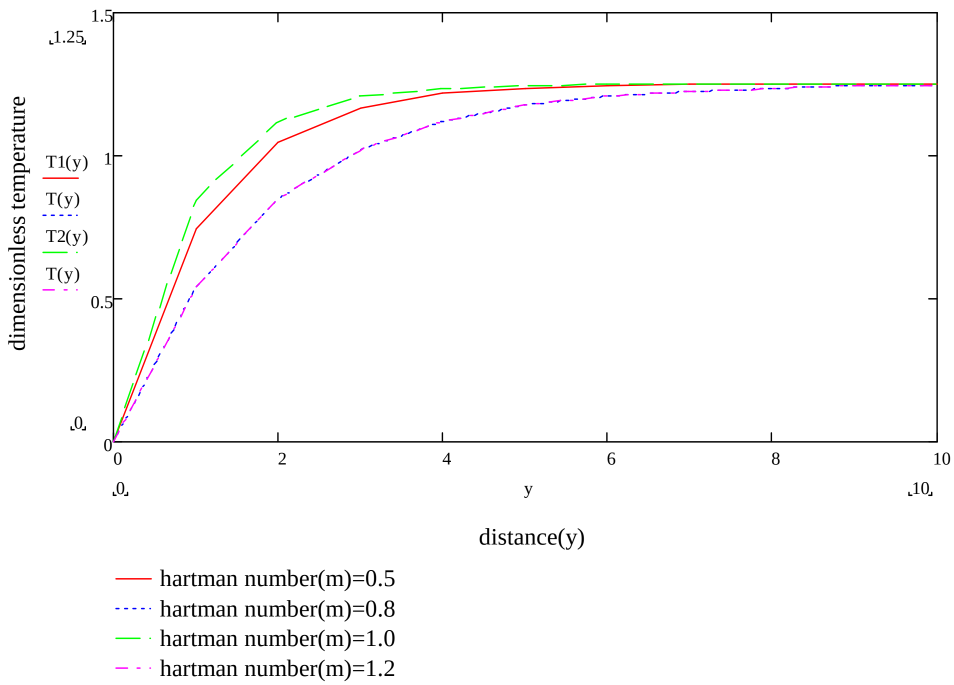
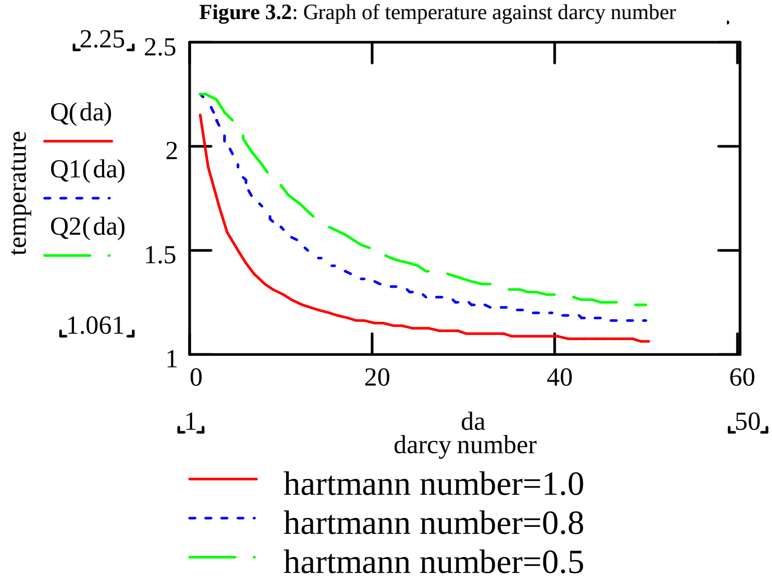



**Figure 3.2**: Graph of temperature against darcy number




## 4.0 Conclusion

A solution for the effect of viscous dissipation on the hydromagnetic flow in a porous horizontal plate has been presented. The effect of the Hartmann and Darcy numbers has been effectively studied. Result obtained shows that there is an increase in the temperature as the Hartmann number increases. This increase is due partly to the viscous effect and the external electric field for higher Hartmann number the temperature rise may be high enough to cause ionization and the gas may become an ionized plasma [2]. We also observed that an increase in the Darcy number causes a decrease in the temperature.

## ***References***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