Journal of the Nigerian Association of Mathematical Physics
***Volume* 13 (November, 2008), 189 - 192**

Approximate solution to laminar falling liquid film with variable viscosity along an inclined heated plate

[1]O. K. Koriko and [2]A. J. Omowaye
[1]Department of Mathematical Sciences
Federal University of Technology, Akure, Nigeria.
[2]Department of Mathematical Sciences,
University of Ado-Ekiti, Nigeria.

Abstract

This paper examines the steady-state solution of a laminar falling liquid film with variable viscosity along an inclined heated plate. The existence and uniqueness of solution to this problem was investigated and it was found that the problem has a unique solution. Numerical solution via shooting techniques were employed in tackling the non-linear coupled momentum and energy balance equations. The Brinkman number (λ) and variable viscous parameter (β) play a prominent roll in the construction of the numerical solution .The velocity and temperature profile of the liquid film are displayed in graphs.

***Keywords*:** Numerical solution, laminar, viscosity, film, steady-state.

1.0 Introduction

The study of laminar flow has attracted the interest of many scientist in recent times. Pearson[6] examined unsteady variable viscosity channel flows with high heat generation while Ockendon [5] investigated the steady case. Ayeni [2] investigated the thermal runaway of variable viscosity flows between concentric cylinders ,it was assumed that the inner cylinder rotates with an angular velocity under some tangential stress while the outer cylinder is immobile .

Recently, the second law analysis of heat transfer of laminar falling liquid film of constant viscosity along an inclined heated plate was investigated [7]. Makinde and Osalusi [4] examined the entropy generation rate in a laminar flow through a channel filled with saturated porous media, they assumed that the upper surface of the channel is adiabatic and the lower wall is assumed to have a constant heat flux, Brinkman model was employed, velocity and temperature profiles are obtained from large Darcy number and it was used to obtain the entropy generation number and irreversibility ratio. Adesanya and Ayeni [2] studied the flow of a reacting pressure/temperature dependent viscosity, when air or oxygen is introduced into a channel containing hydrocarbon, oxidation or combustion is induced, the heat transfer in the channel was studied. Bear [3] considered the effect of pressure and temperature on viscosity and concluded that most fluid shows a pronounced variation with temperature but are relatively insensitive to pressure until high pressures have been attained. He also reported that for gases at twice the critical temperature variations of viscosity with temperature are quite small until pressures of the order of the critical pressure have been reached. For gases, at high pressure an increase in temperature causes a decrease in viscosity while a decrease in temperature causes the viscosity of a gas at low density to decrease.

In this paper an approximate solution to laminar falling liquid film with variable viscosity along an inclined heated plate is modeled and the model is tackled using shooting method.

2.0 Mathematical formulation

In this section we introduce the boundary value problem that provides the basis for our investigation of laminar falling liquid with variable viscosity along an inclined heated plate. The mathematical model of the problem is represented by the dimensionless equations

$$\frac{d^2\theta}{dy^2} + \lambda(1-y)e^{\beta\theta} = 0, \quad \frac{du}{dy} = (1-y)e^{\beta\theta} \tag{2.1}$$

with

$$\theta(0) = 0, \ \theta(1) = 1, \ u(0) = 0 \tag{2.2}$$

Here $\theta = \frac{(T - T_0)}{T_f}$ is the dimensionless temperature (T is the absolute, T_0 is the incline plane wall temperature, T_f is the upper free surface temperature), $\lambda = \frac{(\delta^2 \rho g \sin\phi)^2}{\mu_0 \, k(T_F - T_0)}$ is the Brinkman number (δ is the liquid film thickness, ϕ inclination angle, ρ fluid density, g gravitational acceleration, μ_0 fluid viscosity at reference temperature, $\beta = \alpha(T_f - T_0)$ is the variable viscosity parameter (α determines the strength of dependence between μ and T) μ is the dynamic viscosity, u is the velocity

3.0 Existence and uniqueness of solution

In this section we shall show that there exists a unique solution of equation (2.1) subject to equation (2.2), first and for most we shall consider this theorem without proof.

***Theorem* 3.1** (see for example, Williams et al (1978 [8]))

Let D denote the region [in (n + 1) dimensional space, one dimension for t and n dimensions for vector y] $|t - t_0| \leq a, \ \|y - y_0\| \leq b$, if

$$\begin{aligned}
y_1' &= f_1\ (y, y_2 \Lambda\ y_{n_1}, t) \qquad y_1(t_0) = y_{1_0} \\
y_2' &= f_2\ (y, y_2 \Lambda\ y_{n_1}, t) \qquad y_2(t_0) = y_{2_0} \\
\mathrm{M} \qquad & \qquad \mathrm{M} \qquad\qquad \mathrm{M} \\
y_n' &= f_n\ (y,\ y_2 \Lambda\ y_{n_1}, t) \qquad y_n\ (t_0) = y_{n_0}
\end{aligned} \tag{3.1}$$

Then, the system of equations (3.1) *has a unique solution if* $\frac{\partial f_i}{\partial y_j}$, $i, j = 1,2,3\Lambda\ , n$ *are continuous in D.*

(H_1): $\Gamma > 0$, $\lambda, \ \beta > 0$, $0 \leq x_1 \leq 1$, $0 \leq x_2 \leq b$, $d \leq x_3 \leq T$ *and* $e \leq x_4 \leq p$ where *b, d, e, T and p* are positive constants.

***Theorem* 3.2**

If (H_1) holds then problem (2.1) *satisfying* (2.2) *has a unique solution*.

Proof

Equation (2.1) can be written as system of first order equations as $x_1 = y, \ x_2 = \theta, x_3 = \theta', \ x_4 = u$

Then

$$\begin{pmatrix} x_1' \\ x_2' \\ x_2' \\ x_4' \end{pmatrix} = \begin{pmatrix} 1 \\ \theta' \\ \theta'' \\ u' \end{pmatrix} \tag{3.2}$$

We define

$$
\begin{aligned}
f_1(x_1, x_2, x_3, x_4) &= 1 \\
f_2(x_1, x_2, x_3, x_4) &= x_3 \\
f_3(x_1, x_2, x_3, x_4) &= -\lambda(1-x_1^2)\,e^{\beta x_2} \\
f_4(x_1, x_2, x_3, x_4) &= (1-x_1)\,e^{\beta x_2}
\end{aligned}
\tag{3.3}
$$

The relation in (3.2) and (3.3) can be represented as

$$
\begin{pmatrix} x_1' \\ x_2' \\ x_2' \\ x_4' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ -\lambda(1-x_1^2)e^{\beta\theta} \\ (1-x_1)e^{\beta\theta} \end{pmatrix} \tag{3.4}
$$

Satisfying the initial condition

$$
\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \Gamma \\ 0 \end{pmatrix} \tag{3.5}
$$

Where Γ is guessed such that θ(1) = 1 is satisfied. Now,

$$
\left|\frac{\partial f_1}{\partial x_j}\right| \le 0\ ,\ j = 1()4\ ,\ \left|\frac{\partial f_2}{\partial x_j}\right| \le 0\ ,\ j = 1,2,\ 4\ ,\ \left|\frac{\partial f_2}{\partial x_3}\right| \le 1\ ,\left|\frac{\partial f_3}{\partial x_j}\right| \le 0\ ,\ j = 3,4
$$

$$
\left|\frac{\partial f_3}{\partial x_1}\right| \le \left|-\lambda(1-2x_1)e^{\beta x_2}\right| \le \left|\lambda e^{\beta x_2}\right|\ ,\ \left|\frac{\partial f_3}{\partial x_2}\right| \le \left|-\lambda\beta(1-2x_1^2)e^{\beta x_2}\right| \le \left|(1-2x_1^2)e^{\beta x_2}\right|,
$$

$$
\left|\frac{\partial f_3}{\partial x_j}\right| \le 0\ \ j = 3,4,\quad \left|\frac{\partial f_4}{\partial x_1}\right| \le \left|-e^{\beta x_2}\right|,\ \left|\frac{\partial f_4}{\partial x_2}\right| = \left|\beta(1-2x_1)e^{\beta x_2}\right| \le \left|\beta e^{\beta x_2}\right|,\ \left\|\frac{\partial f_4}{\partial x_j}\right\| \le 0\ \ j = 3,4
$$

Therefore, $\frac{\partial f_i}{\partial x_j}, i, j$ is bounded and there exist k (where k = max ($k_{i,j}$) such that $\left|\frac{\partial f_i}{\partial x_j}\right| \le k$, where $0 < k < \infty$.

Therefore, $f_i(x, x_{21}, x_3, x_4),\ i = 1,2,3,4$ are Lipschitz continuous for different conditions on β,λ and Γ. Hence there exist a unique solution of the system (3.4) subject to (3.5).

4.0 Method of solution

Equation (2.1) subject to equation (2.2) does not have close form solution. We therefore employed the numerical method called shooting techniques. The basic idea of shooting method for solving boundary value ordinary differential equation is to try to find approximate initial condition for which the computed solution "hits the target" so that the boundary conditions at other points are satisfied. In other words, the boundary value problem (3.1) subject to (3.2) is transform to system of initial value problem (3.4) subject to (3.5) where Γ in (3.5) is guessed such that θ(1) = 1 is satisfied. The numerical result is presented in figures 4.1 – 4.4 below.

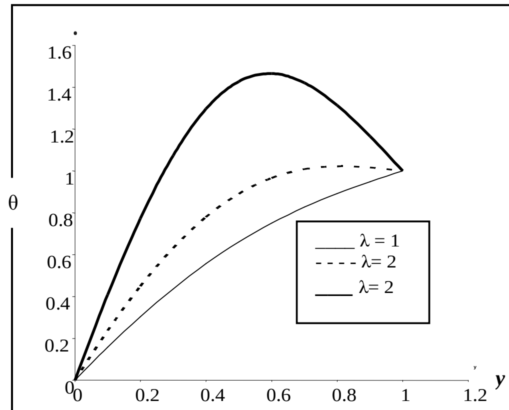

Figure 4.1: Graph of temperature profile θ against position *y* for various λ when β = 1.

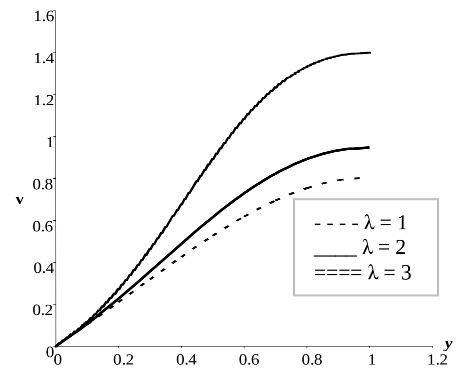

Figure 4.2: The graph of velocity profile against position y for various values of λ when β = 1

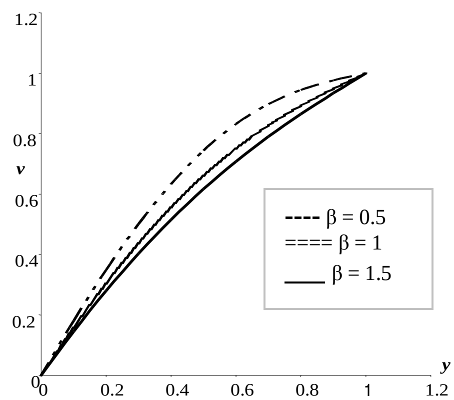

Figure 4.3: The graph of temperature θ profile against position *y* for various values of β when λ = 1

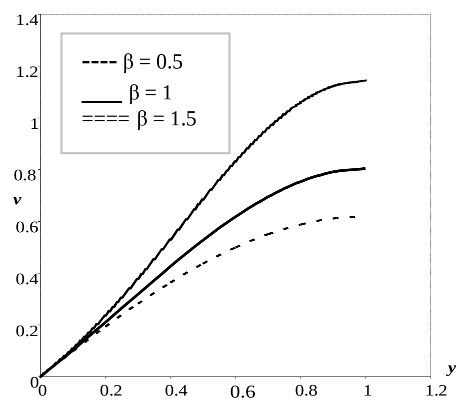

Figure 4.4: The graph of velocity profile *v* against position *y* for various values of β when λ = 1

5.0 Discussion of result

A model that described the laminar falling liquid film with variable viscosity along an inclined heated plate has been developed. The existence and uniqueness theorem for the model was developed and proved. The theorem showed there exist a unique solution for the model. In the numerical analysis that follows, some of the parameters were varied to simulate physically realistic situations and for space consideration only the graphs will be displayed.

Figure 4.1 shows the graph of temperature profile θ against position y for various λ when β = 1 and the shooting guessed values are 1.6297, 2.480, and 4.2612 respectively. We observed that an increase in Brinkman number λ causes an increase in temperature. Figure 4.2 shows the graph of velocity profile against position y for various values of λ Brinkman number, when the variable viscosity parameter β is one this also causes an increase in the velocity while figure 4.3 shows the graph of temperature profile θ against position y for various value of β variable viscosity parameter and fixed value of λ Brinkman number and the shooting guessed value are 1.6297, 1.8634 and 1.5014 respectively there is an increase in the temperature as β increases also, figure 4.4 shows the graph of velocity against position *y* for various value of β,the velocity also increases with increase in β. We must remark here that for β greater than 1.5 both the velocity and the temperature in figures 4.3 and 4.4 start to oscillate.

6.0 Conclusion

The existence of solution of laminar falling liquid film with variable viscosity along an inclined heated plate implies that the problem represent a physical problem under specified condition.

References

[1] Adesanya, S.O and Ayeni, R. O (2008) Heat Transfer to poiseuille flow of a reacting pressure and temperature dependent viscosity in a channel .J. Modern Maths and Statistics no2 vol. 2 pp 40-44.

[2] Ayeni, R. O (1982) On the thermal runaway of variable viscosity flows between concentric cylinders. J. Appl. Maths. Phy. Vol. 33 pp 408-413.

[3] Bear, J. (1972) Dynamics of fluid in porous media.. Elsevier,New Yolk.

[4] Makinde,O.D and Osalusi,E(2005)Second law Analysis of Laminar flow in a channel filled with saturated porous media.Entropy no 7 vol.12 pp 148-168.

[5] Ockendron, H (1979) J. Mech. Vol 83 pp737.

[6] Pearson, J. R. A (1977) J. Fluid Mech. Vol 83 191.

[7] Saouli, S, a iboud- Saouli,S (2004) Second law analysis of Laminar falling liquid film along an inclined heated plate. Int.Comm.Heat transfer vol31 pp687-699.

[8] Williams, R. D and Stanley, A. G (1978) Elementary Differential Equations with applications. Addison-Wesley Publishing company