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A coupled mathematical model for a temperature dependant blood perfusion in a homogenous biological tissue during microwave heating.

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Abstract

The steady state temperature field of a homogenous biological tissues is discussed when the blood perfusion is temperature dependant. The solution to the steady Maxwell equations provoke a two regional compartments. Our solution is obtained for the temperature field using an adequate matching condition at the interface of the two regions. The results revealed the effects of varying blood perfusion, tissue thickness, Electric field and the matching temperature on the temperature pattern in the tissue. Great care is needed before treatment and modality is administered.

Keywords: Homogenous, Biological tissues, Blood perfusion, Microwave Heating, Temperature dependent

1.0 Introduction

Hyperthermia is the use of heat to treat cancer when the body cells are subjected to higher than normal temperatures. There is no single factor which plays the overall role in achieving success in clinical hyperthermia, hence the countless efforts in the form of research on various thermo physical properties which have been undertaken over the years to find effective methods of therapy in dealing with cancer. Adebile and Ogunmoyela, (1) worked on temperature profile in biological tissues while investigating perfusion effects by approximate series method of solution. Their results showed that the spatial dependence of the blood perfusion did not promote stability in the temperature profile nor give rise to multiple solutions. In another paper Adebile and Ogunmoyela, (2) also worked on the effects of temperature dependent perfusion; and obtained steady State solutions using a series approximation with an adequate matching condition. Their findings revealed the need for greater care in dealing with tissues where perfusion is temperature dependent due to the existence of multiple solutions. Recently, Adebile et al (3) solved the coupled Maxwell's and Penne's Bio heat equations analytically. The effects of thermal conductivity, blood perfusion, the thickness of tissue and the electric field were highlighted. Their results agreed with that obtained by El dabe et al (2003). Gowrishankar et al (6) studied Bio heat transfer requiring evaluation of temporal and spatial distribution of temperature using the Penne's bio heat equation. Transport of heat by Conduction and temperature dependent, spatial heterogeneous blood perfusion was modeled using the transport lattice approach. This method was validated by comparing an analytical Solution for a slab with heterogeneous thermal properties and spatially distributed uniform sink held at constant temperature at the ends. Damage was found to be small even with prolonged skin contact to a surface of up to 45^{O}c. Also revealed was the fact that spatial heterogeneity in skin thermal properties lead to a non uniform temperature distribution during exposure. A realistic two dimensional model of the skin showed that tissue heterogeneity did not lead to a significant local temperature increase when heated by an Iron tip. Liu et al (7) in June 2007 worked on the computer modeling of the effect of perfusion on heating patterns in RF tumor ablation. They performed a computer simulation of RF heating using 2-D and 3-D finite element analysis. This simulation was systematically modeled on clinically relevant application parameters for a range of inner tumor perfusion and outer normal surrounding tissue perfusion for internally cooled single and cluster electrodes over a range of tumor diameters and RF application times. The computer model demonstrated that

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perfusion reduced both RF coagulation and the time to achieve thermal equilibrium. Their results show the importance of considering not only the tumor perfusion but also size and background tissue when attempting to predict the effect of perfusion on RF heating and ablation times.

Nomenclature

Symbol	**Quantity**	**Unit**
ρ	Density	g/cm^3
C_p	Specific heat capacity	*J/kg/k*
t	Time	*S*
C_b	Specific heat Capacity of blood	*J/kg/k*
E	Electric field	*V/m*
X	Space coordinates	*cm*
H	Magnetic field	*T*
T	Tissue temperature	^{O}C
T_a	artery temperature	^{O}C
T_b	Blood temperature	^{O}C
T_c	Core temperature	^{O}C
T_w	Wall temperature	^{O}C
L	Distance from skin surface to core	*cm*
m	Positive Integer	—
ρ_b	Density of Blood	g/cm^{-3}
ω_b	Blood perfusion rate	*M*l/*g*-min
K	Thermal Conductivity of tissue	*Mw/cm* ^{o}c
Q	Body heating coefficient	*J*
μ	Magnetic permeability.	Hm^{-1}
ε	Electric permittivity.	Fm^{-1}
σ	Electric conductivity.	$Js^{-1}M^{-1}k^{-1}/wm^{-1}k^{-1}$
H_o	Magnetic field in free space upon tissue.	*T*
E_o	Electric field in free space upon tissue	Vm^{-1}
P_r	Prandtl's number $Pr = \mu C_p/K$	$Kgm^{-1}s^{-1}$
θ	Kinematics viscosity, $\theta = \mu/\rho$	$Kgsm^{-1}$
θ_t	Viscosity of tissue	Nsm^{-2}
ω	Perfusion	*Ml/g-min*
a, b	Arbitrary constants.	-

Subscripts

b : Blood
a : Artery
c : Core
w : Wall
e : Permeability
o : Initial / free space
t : time.

Superscripts

m : Positive integer.
n : positive integers
i : Inner
o : Outer.

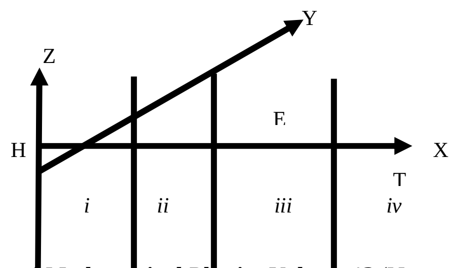
2.0 Mathematical formulation.

This section is concerned with the mathematical formulation of the problem. Physically reasonable assumptions are taken in order to simulate the model based on the governing equations.

2.1 The governing equations

The temperature, electric and magnetic fields are the three dependent variables in the governing equations. $\overline{T} = T(x, t)$; $\overline{E} = E\,(0, E(x, t), 0)$; $\overline{H} = H\,(0, 0, H(x, t))$ (2.1)

Presented below is a one dimensioned tissue model



Key:
i = Epidermis,
ii = Dermis,
iii = Subcutaneous,
iv = inner tissue

Figure 2.1: Multilayered tissue.

In the work done by Hill and Pincombe (1992) the Maxwell's and bio heat equations were given as:

$$\frac{\partial H}{\partial x} + \varepsilon\frac{\partial E}{\partial t} + \sigma E = 0 \quad (2.2)$$

$$\frac{\partial E}{\partial x} + \mu_1\frac{\partial H}{\partial t} = 0 \quad (2.3)$$

$$\rho C_p\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \omega_b\rho_b C_b(T_b - T) + Q(T)|E|^2 \quad (2.4)$$

In (2.4), the first term is the energy gradient; the second term presents the energy stored in the tissue while the third term is the blood transport between the tissue and the blood. Equations (2.2), (2.3) and (2.4) are subject to the following initial and boundary conditions:

$$T(x,0) = \frac{T_c x}{L},\ T(L,t) = \frac{T_c}{T_b},\ T(0,t) = 0$$

$$E(x,0) = \frac{E_0 x}{L},\ E(L,t) = E_o,\ E(0,t) = 0 \quad (2.5)$$

$$H(x,0) = \frac{H_0 x}{L},\ H(L,t) = H_o,\ H(0,t) = 0$$

Merchant and Liu (2001) reported a power law dependence on temperature, so that,

$$Q(T) = T^m \quad (2.6)$$

Introducing the following non dimensional variables,

$$t^* = \frac{t\nu}{L^2},\ x = \frac{x}{L},\ T^* = \frac{Tc}{Tb},\ C_1 = \frac{C_b}{C_p}$$

$$E^* = \frac{E}{E_0},\ H^* = \frac{H}{H_0},\ \rho_1 = \frac{\rho_b}{\rho},\ \omega_1 = \frac{\omega_b L^2}{\upsilon} \quad (2.7)$$

$$\lambda = \frac{L^2 T_b^{m-1}|E_0|^2}{\upsilon\rho C_p},\ \lambda_1 = \frac{\upsilon\varepsilon E_0}{LH_0},\ \lambda_2 = \frac{L\sigma E_0}{H_0},\ \lambda_3 = \frac{\mu\varepsilon H_0\sigma}{L\Sigma_0}$$

Hence the dimensionless Maxwell's and Penne's bio heat equation after ignoring the star mark is written as,

$$\frac{\partial H}{\partial x} + \lambda_1\frac{\partial E}{\partial t} + \lambda_2 E = 0 \quad (2.8)$$

$$\frac{\partial E}{\partial x} + \lambda_3\frac{\partial H}{\partial t} = 0 \quad (2.9)$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\left(\frac{\partial^2 T}{\partial x^2}\right) + \omega_1\rho_1 C_1(1 - T) + \lambda|E|^2 T^m \quad (2.10)$$

Subject to the non dimensional initial and boundary conditions:

$$T(x,0) = \frac{T_c}{T_b}x,\ T(1,t) = \frac{T_c}{T_b},\ T(0,t) = 0$$

$$E(x,0) = x,\ E(1,t) = 1,\ E(0,t) = 0 \quad (2.11)$$

$$H(x,0) = x,\ H(1,t) = 1,\ H(0,t) = 0$$

3.0 Method of solution

Some of the assumptions taken to simplify the model are:

1. The rate of blood perfusion is temperature dependent $\omega = (a_0 + b_0 T)^n$, $n \in R$, $b_0 <<< 1$
2. For the body heating coefficient, $Q(T) = T^m$, $m = 1$

Therefore the steady states Maxwell's and Penne's bio-heat equations to be solved are;

$$\frac{dH}{dx} + \lambda_2 \text{ E} = 0 \tag{3.1}$$

$$\frac{dE}{dx} = 0 \tag{3.2}$$

$$\beta \frac{d^2T}{dx^2} + (a_0 + b_0 T)^n (1 - T) + \lambda |E|^2 T = 0 \tag{3.3}$$

subject to: $T(0) = 0,\ E(0) = 0,\ H(0) = 0$ (3.4)

$$T(1) = \frac{T_c}{T_b},\ E(1) = 1,\ H(1) = 1 \tag{3.5}$$

The solution to equation (3.2) is **E** = Constant. This provokes seeking solution in two regions. Region 1([0,*a*)) is region of no electric effect ($\mathbf{E_1}$) and region 2([*a*,1]) is the region where electric field is active ($\mathbf{E_2}$).

For region of no electric effect, $\mathbf{E}_1$ = constant = 0, hence, the steady state Penne's bio-heat equation to be solved is:

$$\beta \frac{d^2T_1}{dx^2} + (a_0 + b_0 T_1)^n (1 - T_1) = 0 \tag{3.6}$$

$$T_1(0) = 0,\ T_1(a) = h \tag{3.7}$$

Consider a solution for: $$T_1^i = \sum_{i=1}^{\infty} p_i x^i,\ 0 \le x \le a \tag{3.8}$$

substituting (3.8) into (3.6) comparing and collecting the coefficients of,

order x^0, $p_2 = -\frac{f}{2\beta} \equiv \vartheta$

order x^1; $p_3 = -\frac{gp_1}{6\beta} \equiv \vartheta_0 P_1$;

order x^2, $p_4 = -\frac{gp_2 - hP_1^2}{12\beta} \equiv \vartheta_2 P_1^2 + \vartheta_3$ (3.9)

where $f = \alpha a_0^n;\ g = \alpha a_0^{n-1} b_0 - \alpha a_0^n;\ h_1 = \alpha \frac{n(n-1)}{2} a_0^{n-2} b_0^2 - \alpha n a_0^{n-1} b_0;\ J = \frac{n(n-1)}{2} a_0^{n-2} b_0^2$

$$\vartheta = -\frac{f}{2\beta};\ \vartheta_1 = -\frac{g}{6\beta};\ \vartheta_2 = \frac{-h}{12\beta};\ \vartheta_3 = \frac{-gp_2}{12\beta} \tag{3.10}$$

Substituting the compressed forms of p_2, p_3 and p_4 in (3.9) into (3.8) subject to (3.7), the quadratic equation obtained is,

$$\vartheta_2 a^4 p_1^2 + (a + \vartheta_1 a^3) p_1 + \vartheta a^2 + \vartheta_3 a^4 - h = 0 \tag{3.11}$$

which is solved to get; $$p_1 = -((a + \vartheta_1 a^3)) \pm \frac{\sqrt{(a + \vartheta_1 a^3)^2 - 4(\vartheta_2 a^4)(\vartheta a^2 + \vartheta_3 a^4 - h) \equiv D_{13}^s}}{2(\vartheta_2 a^4)} \tag{3.12}$$

We consider a case for which the discriminant, $D_{13}^s = 0$, so that a unique solution is obtained otherwise there will be multiple solutions. Again for the second region, E_2 = constant = 1, hence the steady state Penne's bio-heat equation to be solved is:

$$\beta \frac{d^2T_2}{dx^2} + (a_0 + b_0 T_2)^n (1 - T_2) + \lambda T_2 = 0 \tag{3.13}$$

$$T_2(a) = h,\ T_2(1) = \frac{T_c}{T_b} \tag{3.14}$$

We seek a solution for; $$T_2^o = \sum_{i=1}^{\infty} q_i (1 - x)^i,\ a \le x \le 1 \tag{3.15}$$

Introducing (3.14) into (3.12), comparing and comparing the coefficients of, order Z^0, $q_2 = \dfrac{Jq_0^3 - hq_0^2 - (g+\lambda)q_0 - f}{2\beta} \equiv \partial$

$$\text{order } Z,\ q_3 = \frac{3Jq_0^2 - 2hq_0 - (g+\lambda)q_1}{6\beta} \equiv \partial_1 q_1 \tag{3.16}$$

order Z^2, $q_4 = \dfrac{(3Jq_0^2 - 2hq_0) - (g+\lambda)q_2 + (3Jq_0 - h)q_1^2}{12\beta} \equiv \partial_2 q_1^2 + \partial_3$

where: $f = \alpha\ a_0^n$; g = $\alpha\ na_0^{n-1} - \alpha a_0^n$; $h_1 = \alpha\ \dfrac{n(n-1)}{2} a_0^{n-2} b_0^2 - \alpha n a_0^{n-1} b_0$; J= $\dfrac{n(n-1)}{2} a_0^{n-2} b_0^2$

$$\partial = \frac{Jq_0^3 - hq_0^2 - (g+\lambda)q_0 - f}{2\beta};\ \partial_1 = \frac{3Jq_0^2 - 2hq_0 - (g+\lambda)}{6\beta}$$

$$\partial_2 = \frac{3Jq_0 - h}{12\beta};\ \partial_3 = \frac{3Jq_0^2 - 2hq_0 - (g+\lambda)q_2}{12\beta},\ Z = 1 - x \tag{3.17}$$

Substituting (3.16) into (3.15), the quadratic equation obtained is;

$$\partial_2 a^4 q_1^2 + (a + \partial_1 a^3) q_1 + \partial a^2 + \partial_3 a^4 - h = 0 \tag{3.18}$$

$$q_1 = -((a + \partial_1 a^3)) \pm \frac{\sqrt{(a + \partial_1 a^3)^2 - 4(\partial_2 a^4)(\partial a^2 + \partial_3 a^4 - h) \equiv D_{14}^s}}{2(\partial_2 a^4)} \tag{3.19}$$

We consider a case for which the discriminant, $D_{14}^s = 0$, so that a unique solution is obtained otherwise there will be multiple solutions

4.0 Result and discussions.

4.1 Graphs of temperature dependent blood perfusion for *m* = 1

Figure 4.1: Temperature distribution plotted against the space coordinate for a temperature dependent blood perfusion, for different tissue thickness, *L*, where $a_0 = 1$, $b_0 = 1$, $n = 2$, $c_b = 3770$, $c_p = 3590$, $w_b = 0.00125$, $k = 0.24$, $g = 1050$, $E_0 = 2$ and $g_b = 1060$

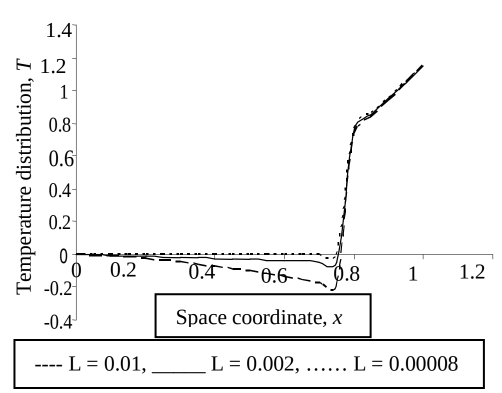


Figure 4.2: Temperature distribution plotted against the space coordinate for a temperature dependent blood perfusion, for different electric field in the free space upon issue, where $a_0 = 1$, $b_0 = 1$, $n = 2$, $c_b = 3770$,

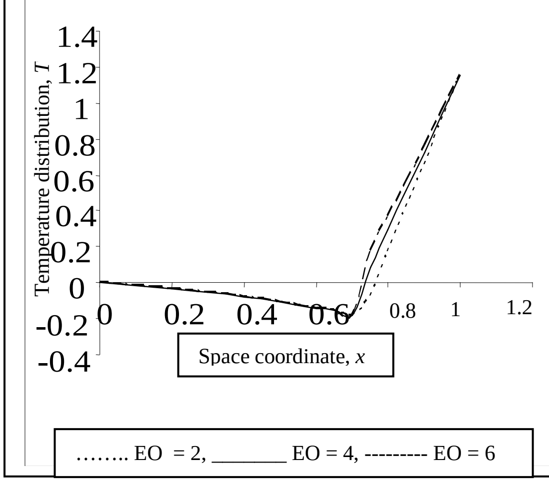
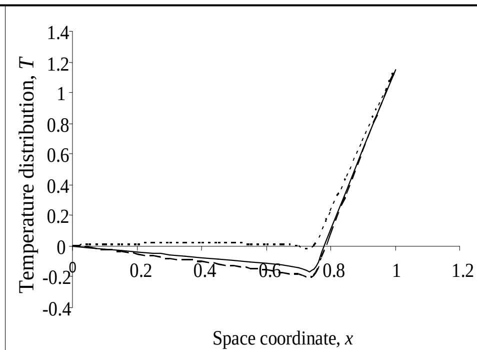


Figure 4.3: Temperature distribution plotted against the space coordinate, x for different values of the blood perfusion in w = (a_0 +b_0T)'n where a = 75, L = 0.00008, a_0 = 1, b_0 = 1, n = 2, k = 0.24, E_0 = 2, g_0 = 1060, g = 3770, and c_p = 3590.c_p = 3590, w_b = 0.00125, k = 0.24, g = 1050, E_0 = 2 and g_b = 1060



----- *wb* = 0.0005, ___ *wb* = 5, …..*wb* = 10

In figure 4.1, thicker skin types tend to attain lower temperatures while less thick skin types record higher temperatures. In region I, where the electric field in free space upon tissue is zero, there is no significant effect for the different values of E_0. While in region II a direct proportion is observed so that higher values of E_0 attain higher temperatures in figure 4.2. Higher blood perfusion rates imply higher temperatures as clearly displayed in figure 4.3

References

[1] Adebile E.A. and Ogunmoyela J.K (2005): Thermoregulation in biological tissue during microwave hyperthermia. Part I: Spatial dependent blood perfusion. Journal of the Mathematical Association of Nigeria, Vol. 32, No 2A. Mathematical series pp 9-23.

[2] Adebile E.A and Ogunmoyela J.K (2005): thermoregulation in biologic tissues during microwave hyperthermia blood perfusion effects. Journal of the mathematical Association of Nigerian, vol. 32 No 2A, mathematical series pp. 53-67

[3] Adebile E.A. and Akintewe B.N. (2006): On the steady state temperature profiles of biological tissues during microwave heating. Journal of the Nigerian Association of Mathematical Physics vol. 10 pp 223-228

[4] Gowrishankar T.R, Donald A.S.Gregory T.M and James C.W (2004): Transport lattice models of heat transport in skin with spatially heterogeneous temperature dependent perfusion Biomedical engineering Journal online vol. 3, issue 44.

[5] Nabil T.M, El Dabe. Mona A.A Mohammed and Asma F Elsayed (2003): effects of microwave on the thermal states of biological tissue; African Journal of Biotechnology, vol. 2.

[6] S.C. Hwang and D Lemmonier (1995): Coupling numerical solution of bio heat transfer equation and Maxwell's equations in biological tissues during hyperthermia. Transactions of Wessex institute. Biomedicine and health Vol. 2

[7] Z Liu, M Ahmed, A Sabir, S Humpries, S.N. Goldberg (2007): Radiology 2007 June; 243 (3):712-9 17517930 Online