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Transverse vibration under a moving load of a highly prestressed isotropic rectangular plate on a bi-parametric subgrade

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Abstract

This paper investigates the dynamic response of a highly prestressed isotropic rectangular plate resting on a bi-parametric subgrade under the action of a moving load. Using the singular perturbation technique, specifically the Method of Matched Asymptotic Expansion (MMAE), in conjunction with the method of integral transformations and Cauchy Residue theorem, a uniformly valid analytical solution in the entire domain of definition of the rectangular plate is obtained. Analyses of analytical solutions and numerical results show that the leading order solution and the first order correction are affected by the bi-parametric subgrade and anisotropic prestress to the response of 0(□^{1}) of the rectangular plate. It is also found that the critical velocities of the dynamical system increase with prestress for all values of shear modulus and foundation stiffness used. Thus, resonance is reached earlier for lower values of prestress, shear modulus and foundation stiffness than for high values.

1.0 Introduction

The works on dynamic loading of one dimensional solid such as beams have received attention of several researchers. Among several authors that have worked on this subject are Jeffcott [1], Steuding [2] and Odman [3], Milomir et al [4], Leipholz [5], Oni and Gbadeyan [6] to mention but a few. Among the earliest work on moving load plate problem is the work of Holl [7]. He solved the problem of a rectangular plate under the action of uniform moving loads. He indicated that a critical velocity existed for each vibrational mode. Much later Stanisic et al [8] studied the two dimensional problems of flexural vibration of plate under the actions of moving masses. Only the inertia term that measures the effect of local acceleration in the direction of the deflection was considered. The work in Stanisic et al [8] was taken up much later by Gbadeyan and Oni [9] who studied the dynamic analysis of an elastic plate continuously supported by an elastic Pasternak foundation traversed by an arbitrary number of concentrated masses. All the components of the inertia terms were considered and the rectangular plate was assumed to be simply supported. The deflection of the plate was calculated for several values of the foundation moduli and shown graphically as a function of time. More recently, Oni [10] developed a versatile solution technique for solving plate moving load problems for all variants of classical boundary conditions.

In all the aforementioned studies, no consideration has been given to bending effects at the boundaries. In particular, when a plate structure is highly prestressed, a small parameter multiplies the highest derivative in the governing differential equation.

This class of dynamical problem in which a small parameter multiplies the highest derivative in the governing differential equation is not common in literature. However, this class of plate problems has been solved when the plate is executing free vibration or when a static load is acting on such plate, Hutter and Olunloyo [11]. Singular perturbation has to date seen relatively little use in solid mechanics but it is nonetheless being successfully used, Cole [12]. In particular, Hutter and Olunloyo [13] have employed it in investigating rectangular membranes with small bending stiffness. In a more recent article, Gbadeyan and Oyediran [14] compared the two singular perturbation techniques (MCE and MMAE) for initially stressed thin rectangular plate. They found that the results of the MMAE agree with those obtained using generalized MCE and specialized version of MCE when the effect of shearing deformation is $0(\varepsilon)$.

After an earlier work by Oni [15] where he studied the dynamic response to a moving load (using the Method of Matched Asymptotic Expansion MMAE) of a fully clamped prestressed orthotropic rectangular plate, Oni and Tolorunsagba [16] took up the problem of assessing the rotatory inertia influence on the highly prestressed orthotropic rectangular plate when it is under the action of moving load. The method of composite expansion (MCE), an alternate singular perturbation technique is employed in conjunction with the method of integral transformation and Cauchy residue theorem to obtain an approximately uniformly valid solution in the entire domain of definition of the rectangular plate. Analysis showed that the critical velocities of the dynamical system increase with an increase in prestress and rotatory inertia values. However, in the work of Oni [15] and Oni and Tolorunsagba [16], only plates not resting on foundation were considered. Thus, in this work the dynamic response to a moving load of a highly prestressed isotropic rectangular plate resting on a Pasternak-type foundation is considered.

2.0 Problem formulation

The transverse displacement of an isotropic rectangular plate resting on a Pasternak foundation under a moving load is governed by the fourth order partial differential equation

$$D\left[\frac{\partial^4 W(\bar{x},\bar{y};t)}{\partial\bar{x}^4}+\frac{\partial^4 W(\bar{x},\bar{y};t)}{\partial\bar{x}^2\partial\bar{y}^2}+\frac{\partial^4 W(\bar{x},\bar{y};t)}{\partial\bar{y}^4}\right]-N_{\bar{x}}\frac{\partial^2 W(\bar{x},\bar{y};t)}{\partial\bar{x}^2}-N_{\bar{y}}\frac{\partial^2 W(\bar{x},\bar{y};t)}{\partial\bar{y}^2}$$

$$m\frac{\partial^2 W(\bar{x},\bar{y};t)}{\partial\bar{t}^2}+KW(\bar{x},\bar{y};t)+G\left[\frac{\partial^2 W(\bar{x},\bar{y};t)}{\partial\bar{x}^2}+\frac{\partial^2 W(\bar{x},\bar{y};t)}{\partial\bar{y}^2}\right]=P(\bar{x},\bar{y};\bar{t}) \qquad (2.1)$$

Where *D* is the bending stiffness, $N_{\bar{x}}$ is the axial prestress in the $\bar{x}$ direction, $N_{\bar{y}}$ is the axial prestress in the $\bar{y}$ direction, $\bar{x}$, *y* are the position coordinate in the x and y directions respectively , $\bar{t}$ is the time coordinate, W is the deflection of the plate, m is the mass of the plate per unit area, $P(\bar{x},\bar{y};\bar{t})$ is the applied dynamic load, *K* and *G* are the foundation stiffness and shear modulus respectively. The boundaries of the plate are fully clamped, and as such both the deflection and slope varnish identically. Thus,

$$\left.\begin{array}{ll}\bar{x}=0, & 0\le\bar{y}\le B\\ \bar{x}=L, & 0\le\bar{y}\le B\end{array}\right\}W(\bar{x},\bar{y},\bar{t})=0;\qquad \frac{\partial W(\bar{x},\bar{y};t)}{\partial\bar{x}}=0$$

$$\left.\begin{array}{ll}\bar{y}=0, & 0\le x\le L\\ \bar{x}=L, & 0\le\bar{x}\le L\end{array}\right\}W(\bar{x},\bar{y},\bar{t})=0;\qquad \frac{\partial W(\bar{x},\bar{y};t)}{\partial\bar{y}}=0 \qquad (2.2)$$

and for simplicity, the initial conditions are taken to be

$$W(\bar{x},\bar{y};0)=0;\ \frac{\partial W(\bar{x},\bar{y};o)}{\partial\bar{t}}=0 \qquad (2.3)$$

2.1 Non-dimensionalized form

Equation (2.1) is presented in a non-dimensionalized form for the purpose of solution. Substituting $W=\bar{V}L,\ \bar{x}=xL,\ \bar{y}=yL,\ \bar{t}=t/\omega_o$ into equation (2.1), followed by some simplification and arrangement leads to the system of equation

$$\varepsilon^2\left[\frac{\partial^4\bar{V}(x,y;t)}{\partial x^4}+\frac{\partial^4\bar{V}(x,y;t)}{\partial x^2\partial y^2}+\frac{\partial^4\bar{V}(x,y;t)}{\partial y^4}\right]-\frac{\beta_1^2\partial^2\bar{V}(x,y,t;)}{\partial x^2}-\frac{\beta_2^2\partial^2\bar{V}(x,y,t)}{\partial y^2}$$

$$+\frac{\partial^2\bar{V}(x,y,t)}{\partial t^2}+\eta\bar{V}+\sigma_1^2\left[\frac{\partial^2\bar{V}(x,y,t)}{\partial x^2}+\frac{\partial^2\bar{V}(x,y,t)}{\partial y^2}\right]=p_o(x,y;t) \qquad (2.4)$$

where ε is the small parameter multiplying the highest derivative and defined by the relation

$$\varepsilon^2 = \frac{D}{N_o L^2} << 1,\ \eta = \frac{K}{m\omega_0^2},\quad \sigma_1^2 = \frac{G}{N_o},\ \beta_1^2 = \frac{N_x}{N_o},\ \beta_2^2 = \frac{N_y}{N_o} \tag{2.5}$$

$\beta_1^2,\ \beta_2^2$ measures the prestress ratio and the boundary conditions in non–dimensionalized form become:

$$\left.\begin{array}{ll} x = 0, & 0 \le y \le b \\ x = 1, & 0 \le y \le b \end{array}\right\} \overline{V}(x,y,t) = 0; \qquad \frac{\partial \overline{V}(x,y;t)}{\partial x} = 0$$

$$\left.\begin{array}{ll} y = 0, & 0 \le x \le 1 \\ y = b, & 0 \le x \le 1 \end{array}\right\} \overline{V}(x,y,t) = 0; \qquad \frac{\partial \overline{V}(x,y;t)}{\partial y} = 0 \tag{2.6}$$

and the initial conditions are

$$\overline{V}(x,y,0) = 0;\ \frac{\partial \overline{V}(x,y,0)}{\partial t} = 0 \tag{2.7}$$

In this dynamical system, moving load on the rectangular plate moves at a constant velocity u along a straight line parallel to x-axis, say, y_o. Thus $P_a(x,y,t)$ takes the form

$$p_a(x,y;t) = Mg\partial(x-ut)\partial(y-y_o) \tag{2.8}$$

where M is the mass of the moving load, g is the acceleration due to gravity and $\delta(\bullet)$ is the dirac delta function

defined as
$$\partial(x-ut) = \begin{cases} o, & x \ne ut \\ \infty, & x = ut \end{cases} \tag{2.9}$$

When equation (2.8) is substituted into (2.4); one obtains

$$\varepsilon^2\left[\frac{\partial^4\overline{V}(x,y;t)}{\partial x^4} + \frac{\partial^4\overline{V}(x,y;t)}{\partial x^2\partial y^2} + \frac{\partial^4\overline{V}(x,y;t)}{\partial y^4}\right] - \frac{\beta_1^2\partial^2\overline{V}(x,y;t)}{\partial x^2} - \frac{\beta_2^2\partial^2\overline{V}(x,y;t)}{\partial y^2} + \frac{\partial^2\overline{V}(x,y;t)}{\partial t^2}$$

$$+\eta V(x,y;t) + \sigma_1^2\left[\frac{\partial^2\overline{V}(x,y;t)}{\partial x^2} +;t)\frac{\partial^2\overline{V}(x,y;t)}{\partial y^2}\right] = p_a\delta(x-ut)\delta(y-y_o) \tag{2.10}$$

Equations (2.10) together with boundary conditions (2.6) and initial conditions (2.7) define completely the equation of a fully clamped highly prestressed isotropic rectangular plate occupying the domain $0 \le x \le 1$, $0 \le y \le b$ in a non–dimensionalized form.

2.2 Operational Simplification

It is observed that a small parameter multiplies the highest derivatives in (2.10) and as such the problem is amenable to singular perturbation techniques. However, equation (2.10) is considerably simplified by introducing the Laplace transform defined by

$$V = \int \overline{V}e^{-st}dt \tag{2.11}$$

in conjunction with the initial conditions defined in (2.7). Taking t as the principal variable, the Laplace of (2.10) is given as

$$\varepsilon^2\left[\frac{\partial^4 V(x,y;t)}{\partial x^4} + \frac{\partial^4 V(x,y;t)}{\partial x^2\partial y^2} + \frac{\partial^4 V(x,y;t)}{\partial y^4}\right] - \frac{\beta_1^2\partial^2 V(x,y;t)}{\partial x^2} - \frac{\beta_2^2\partial^2 V(x,y;t)}{\partial y^2} + s^2V(x,y;t)$$

$$+\eta V(x,y;t) + \sigma_1^2\left[\frac{\partial^2 V(x,y;t)}{\partial x^2} + \frac{\partial^2 V(x,y;t)}{\partial y^2}\right] = \frac{p_a\delta(y-y_o)}{u}e^{-\frac{s}{u}x} \tag{2.12}$$

Subject to the boundary conditions

$$\left.\begin{array}{ll} x = 0, & 0 \le y \le b \\ x = 1, & 0 \le y \le b \end{array}\right\} V(x,y) = 0; \qquad \frac{\partial V(x,y)}{\partial x} = 0$$

$$\left.\begin{array}{ll} y=0, & 0\le x\le 1 \\ y=b, & 0\le x\le 1 \end{array}\right\} V(x,y)=0; \qquad \frac{\partial V(x,y)}{\partial y}=0 \tag{2.13}$$

2.3 Method of Solution

In equation (2.12), an exact uniformly valid solution in the entire domain is not possible and it is observed that a small parameter, □ say, multiplies the highest derivative in the governing differential equation. This is due to the bending effects at the boundaries. Consequently, solution valid away from the boundaries breaks down near as well as at the boundaries. Thus, only approximate solutions are possible. The two but equivalent approaches that could be used to tackle this type of problem are the method of composite expansion

(MCE) and the Method of Matched Asymptotic Expansion (MMAE). In this paper, MMAE is used. This technique provides an approximate solution to the given problem in terms of two separate expansions which are valid in part of the domain. The two separate solutions, one valid at and near the boundaries and the other valid away from the boundaries are then matched to obtain a uniformly valid solution in the entire domain of definition of the rectangular plate. The Method of Matched Asymptotic Expansion MMAE developed by Bretheton [17] required that the asymptotic solution of equation (2.12) be of the form

$$V=V_O+\varepsilon V_1+\varepsilon^2 V_2+\ldots \tag{2.14}$$

Substituting the expansion (2.14) into equation (2.12) and equating coefficients of like powers of ε, one obtains the recurrence relation

$$\frac{\beta_1^2\partial^2 V_v^o(x,y;t)}{\partial x^2}+\frac{\beta_2^2\partial^2 V_v^o(x,y;t)}{\partial y^2}+s^2 V_v^o(x,y;t)$$

$$-\sigma_1^2\left[\frac{\partial^2 V_v^o(x,y;t)}{\partial x^2}+\frac{\partial^2 V_v^o(x,y;t)}{\partial y^2}\right]-\eta V_v^o(x,y;t)=\begin{cases}\dfrac{p_o\delta(y-y_o)}{u}e^{-\frac{s}{u}x}, & v=0\\ o, & v=1\\ \nabla^4 V_{v-2}^o, & v\ge 2\end{cases} \tag{2.15}$$

where $\nabla^4 V_{v-2}^o=\dfrac{\partial^4 V_{v-2}^o}{\partial x^4}+\dfrac{\partial^4 V_{v-2}^o}{\partial x^2\partial y^2}+\dfrac{\partial^4 V_{v-2}^o}{\partial y^4}$ where the subscripts denote the order in ε. It is remarked here that equation (2.15) are not uniformly valid in the entire domain of the rectangular plate under consideration. In fact, solutions obtained for V_o, V_v, $v\ge 1$ are not valid near the boundaries. The reason for this is simple. The order of the partial differential equation (2.12) has been reduced but the number of boundary condition is not reduced. These solutions are termed outer solutions and the equation (2.15) outer problem.

2.4 Expression near the boundary

In order to obtain an expression that is valid at the boundary, near x = 0, we set the inner variable as $X=x/\varepsilon$ and write the solution valid near $x=0$ as

$$V=\psi^i=\psi_o^i(X,y)+\varepsilon\psi_1^i(X,y)+\varepsilon^2\psi_2^i(X,y)+O(\varepsilon^3) \tag{2.16}$$

where superscript i denote the inner solution. Equation (2.16) is also valid near $x=1$, where we set the inner variable $X=(1-x)/\varepsilon$. Expressions similar to (2.16) can be written down for the solution near $y=0$ and y=b where we set the inner variable as $Y=y/\varepsilon$ and $Y=(b-y)/\varepsilon$ respectively, as

$$V=\psi^i=\psi_o^i(x,Y)+\varepsilon\psi_1^i(x,Y)+\varepsilon^2\psi_2^i(x,Y)+O(\varepsilon^3) \tag{2.17}$$

Using equation (2.16) in (2.12) near either $x=0$ or $x=1$, the differential equation on ψ^i gives

$$\frac{\partial^4\psi_v^i}{\partial X^4}-(\beta_1^2-\sigma_1^2)\frac{\partial^2\psi_v^i}{\partial X^2}=\frac{\beta_2\partial^2\psi_{v-2}^i}{\partial y^2}-\frac{\partial^4\psi_{v-2}^i}{\partial y^2\partial x^2}-S^2\psi_{v-2}^i-\eta\psi_{v-2}^i$$

$$-\sigma_1^2\frac{\partial^2\psi_{v-2}^i}{\partial y^2}-\frac{\partial^4\psi_{v-4}}{\partial y^4}, v=0,1,3,4.... \quad (2.18)$$

$$\frac{\partial^4\psi_v^i}{\partial X^4}-\left(\beta_1^2-\sigma_1^2\right)\frac{\partial^2\psi_v^i}{\partial X^2}=\frac{\beta_2\partial^2\psi_{v-2}^i}{\partial y^2}-\frac{\partial^4\psi_{v-2}^i}{\partial y^2\partial x^2}-S^2\psi_{v-2}^i-\eta\psi_{v-2}^i$$

$$-\sigma_1^2\frac{\partial^2\psi_{v-2}^i}{\partial y^2}+\frac{p_a\delta(y-y_o)}{u}e^{-\frac{s}{u}x},\ v=2 \quad (2.19)$$

Subject to boundary condition $$\psi_v^i=\frac{\partial\psi_v^i}{\partial X}=0,\ v = 0, 1, 2, 3, ... \quad (2.20)$$

The differential equation near $y = 0$, or $y = b$ can similarly be written as

$$\frac{\partial^4\psi_v^i}{\partial Y^4}-\left(\beta_2^2-\sigma_1^2\right)\frac{\partial^2\psi_v^i}{\partial Y^2}=\beta_1^2\frac{\partial^2\psi_{v-2}^i}{\partial x^2}-\frac{\partial^4\psi_{v-2}^i}{\partial x^2\partial y^2}-S\psi_{v-2}^i-\eta\psi_{v-2}^i$$

$$-\sigma_1^2\frac{\partial^2\psi_v^i}{\partial x^2}-\frac{\partial^4\psi_{v-2}^i}{\partial x^4}, v = 0, 1, 3, 4 .... \quad (2.21)$$

$$\frac{\partial^4\psi_v^i}{\partial Y^4}-\left(\beta_2^2-\sigma_1^2\right)\frac{\partial^2\psi_v^i}{\partial Y^2}=\beta_1^2\frac{\partial^2\psi_{v-2}^i}{\partial x^2}-\frac{\partial^4\psi_{v-2}^i}{\partial x^2\partial y^2}-s^2\psi_{v-2}^i-\eta\psi_{v-2}^i$$

$$-\sigma_1^2\frac{\partial^2\psi_v^i}{\partial x^2}+\frac{P_a\partial(y-y_o)}{u}e^{\frac{-s}{u}x}\quad v = 2 \quad (2.22)$$

subject to boundary condition $$\psi_v^i=\frac{\partial\psi_v^i}{\partial Y}=0,\ v = 0, 1, 2, 3, ... \quad (2.23)$$

3.0 Solution procedure

The solutions of equation (2.15) for the function V_v and equations (2.18), (2.19), (2.21) and (2.22) for function ψ_v subject to boundary conditions (2.20) and (2.23) are sought using Fourier transformation techniques.

3.1 Leading order solution

Here the solutions of V_0 and ψ_0 are sought.

3.2 Solution for V_0^0

Substituting v = 0 in the recurrence equation (2.15), the governing differential equation for V_0 are obtained as

$$\left(\beta_1^2-\sigma_1^2\right)\frac{\partial^2V_o(x,y;t)}{\partial^2x}+\left(\beta_2^2-\sigma_1^2\right)\frac{\partial^2V_o(x,y;t)}{\partial y^2}-S^2V_o(x,y;t)-\eta V_o(x,y;t)=\frac{-P_a\delta(y-y_o)}{u}e^{\frac{-s}{u}x} \quad (3.1)$$

Equation (3.1) is solved for V_0 by introducing the finite Fourier sine transform defined as

$$V(n,y)=\int_0^1V(x,y)\sin n\pi xdx, \text{ with the inverse } V(x,y)=2\sum_{n=1}^{\infty}V\ (n,y)\sin n\pi x \quad (3.2)$$

$$V(m,x)=\int_0^bV(x,y)\sin\frac{m\pi}{b}ydy \text{ with the inverse } V(x,y)=\frac{2}{b}\sum_{m=1}^{\infty}V(m,x)\sin\frac{m\pi}{b}y \quad (3.3)$$

So that the transform of (3.1) with respect to x is

$$V_{o,yy}(n,y)+\alpha^2 V_o(n.y)=T\delta(y-y_o), \tag{3.4}$$

where $\alpha^2=-\left[\frac{(\beta_1^2-\sigma_1^2)n^2\pi^2+s^2+\eta}{\beta_2^2-\sigma_1^2}\right]$, $T=\frac{P_a n\pi u\left[(-1)^n e^{\frac{-s}{u}}-1\right]}{(\beta_2^2-\sigma_1^2)(s^2+n^2\pi^2u^2)}$ while the transform of (3.1) with respect to y is

$$V_{o,xx}(m,x)+\sigma^2 V_o(m,x)=\frac{-P_o}{(\beta_1^2-\sigma_1^2)u}e^{\frac{-s}{u}x}\sin\frac{m\pi}{b}y_o \tag{3.5}$$

where
$$\sigma^2=-\left[\frac{(\beta_2^2-\sigma_1^2)m^2\pi^2+(s^2+\eta)b^2}{(\beta_1^2-\sigma_1^2)b^2}\right] \tag{3.6}$$

The complimentary solution of (3.4) is $V_{oc}(n.y)=C_2\cos\alpha y+D_2\sin\alpha y$ (3.7)

Using the methods of variation parameters, the particular solution of (3.4) can be shown to be

$$V_{op}(n,y)=\frac{-T}{\alpha}\sin\alpha y_o\cos\beta_y+\frac{T}{\alpha}\cos\alpha y_o\sin\beta_y \tag{3.8}$$

Consequently, the general solution of the ordinary differential equation (3.4) can be obtained as $V_o(n,y)=C_2\cos\alpha y+D_2\sin\alpha y+\Gamma_1\sin\alpha(y-y_o)$ (3.9)

where
$$\Gamma_1=\frac{P_a n\pi u\left[(-1)^n e^{-s/u}-1\right]}{\alpha(\beta_2^2-\sigma_1^2)(s^2+n^2\pi^2u^2)} \tag{3.10}$$

Similarly, the complimentary solution of the equation (3.5) is

$$V_{oc}(m,x)=E_3\cos\alpha x+F_3\sin\alpha x \tag{3.11}$$

It is then straight forward to obtain $V_o(m,x)=E_3\cos\alpha x+F_3\sin\alpha x-\Gamma_2 e^{\frac{-s}{u}x}$ (3.12)

where
$$\Gamma_2=\frac{P_a u\sin\frac{m\pi}{b}y}{(\beta_1^2-\sigma_1^2)(s^2+\sigma^2u^2)} \tag{3.13}$$

The inversion of (3.9) and (3.12) gives the general solution of the equation (3.1). Thus,

$$V_o(x,y)=2[C_2\cos\alpha y+D_2\sin\alpha y+\Gamma_1\sin\alpha(y-y_o)]\sin n\pi x$$
$$+\frac{2}{b}\left[E_3\cos\sigma x+F_3\sin\sigma x-\Gamma_2 e^{\frac{-s}{u}x}\right]\sin\frac{m\pi}{b}y \tag{3.14}$$

where C_2, D_2, E_3 and F_3 are arbitrary constants yet to be determined by matching.

3.3 Solution for ψ_0^i

If v = 0 is substituted into equations (2.18) and (2.19), neglecting terms with negative subscripts, we have the leading order inner problem near x = 0 or x = 1 given as

$$\frac{\partial^4\psi_o^i}{\partial X^4}-(\beta_1^2-\sigma_1^2)\frac{\partial^2\psi_o^i}{\partial X^2}=0 \tag{3.15}$$

subjected to $$\psi_o^i = \frac{\partial \psi_o^i}{\partial X} = 0 \qquad (3.16)$$

solving equation (3.15) together with (3.16) gives

$$\psi_o^i = \begin{cases} \overset{\varpi}{q}_o(y)\left\{ X + \dfrac{1}{\sqrt{\beta_1^2 - \sigma_1^2}} e^{-\sqrt{\beta_1^2-\sigma_1^2}\,X} - \dfrac{1}{\sqrt{\beta_1^2 - \sigma_1^2}} \right\} \text{ near } x = 0 \\ \overset{\bar{\varpi}}{q}_o(y)\left\{ X + \dfrac{1}{\sqrt{\beta_1^2 - \sigma_1^2}} e^{-\sqrt{\beta_1^2-\sigma_1^2}\,X} - \dfrac{1}{\sqrt{\beta_1^2 - \sigma_1^2}} \right\} \text{ near } x = 1 \end{cases} \qquad (3.17)$$

Similarly, the leading order inner problem near $y = 0$ and $y = b$ obtained from (2.21) and (2.22) yields

$$\psi_o^i = \begin{cases} \overset{\varpi}{r}_o(x)\left\{ Y + \dfrac{1}{\sqrt{\beta_2^2 - \sigma_1^2}} e^{-\sqrt{\beta_2^2-\sigma_1^2}\,Y} - \dfrac{1}{\sqrt{\beta_2^2 - \sigma_1^2}} \right\} \text{ near } y = 0 \\ \overset{\bar{\varpi}}{r}_o(x)\left\{ Y + \dfrac{1}{\sqrt{\beta_2^2 - \sigma_1^2}} e^{-\sqrt{\beta_2^2-\sigma_1^2}\,Y} - \dfrac{1}{\sqrt{\beta_2^2 - \sigma_1^2}} \right\} \text{ near } y = b \end{cases} \qquad (3.18)$$

In (3.17) and (3.18), exponentially growing terms have been neglected while the functions $\overset{\omega}{q}_o(y)$, $\overset{\bar{\omega}}{q}_o(y)$, $\overset{\omega}{r}_o(x)$, and $\overset{\bar{\omega}}{r}_o(x)$ are to be determined by matching. To this end, Van Dyke's matching principle which requires m-term inner expansion of (the n – term outer expansion) equals the n–term outer expansion of (the m– term inner expansion) is adopted. Thus, matching one term outer expansion written in inner variable (3.17) with one term inner expansion written in outer variable (3.14) (1-1 matching), we immediately have $$\overset{\omega}{q}_0(y) = \overset{\bar{\omega}}{q}_0(y) = \overset{\omega}{r}_0(x) = \overset{\bar{\omega}}{r}_0(x) = 0 \qquad (3.19)$$

$$E_3 = \frac{P_o \sin\frac{m\pi}{b} y}{\left(\beta_1^2 - \sigma_1^2\right)\left(s^2 + \sigma^2 u^2\right)},\; F_3 = \frac{P_o u \sin\frac{m\pi}{b} y_o}{\left(\beta_1^2 - \sigma_1^2\right)\left(s^2 + \sigma^2 u^2\right)}\left[\frac{e^{-\frac{s}{u}}}{\sin\sigma} - \cot\sigma\right] \qquad (3.20)$$

$$C_2 = \frac{P_o n\pi u\left[(-1)^n e^{-\frac{s}{u}} - 1\right]}{\alpha\left(\beta_2^2 - \sigma_1^2\right)\left(s^2 + n^2\pi^2 u^2\right)}\sin\alpha y,\; D_2 = \frac{P_o n\pi u\left[1 - (-1)^n e^{-\frac{s}{u}} - 1\right]}{\alpha\left(\beta_2^2 - \sigma_1^2\right)\left(s^2 + n^2\pi^2 u^2\right)}\cos\alpha y_o \qquad (3.21)$$

It is straight forward to show that $$\psi_o^i = 0 \qquad (3.22)$$

Thus, substituting (3.20) and (3.21) into (3.14) yields the inversion of equation (3.14) and the general solution of equation (3.1) as

$$V_o(x,y) = \frac{2P_o u \sin\frac{m\pi}{b} y_o}{b\left(\beta_1^2 - \sigma_1^2\right)\left(s^2 + \sigma^2 u^2\right)}\left[\cos\sigma x - e^{-\frac{s}{u}x} + e^{-\frac{s}{u}}\frac{\sin\sigma x}{\sin\sigma} - \frac{\cos\sigma \sin\sigma x}{\sin\sigma}\right]\sin\frac{m\pi}{b} y \qquad (3.23)$$

where $\sigma^2 = -\left[\frac{\left(\beta_2^2-\sigma_1^2\right)m^2\pi^2+\eta b^2+s^2b^2}{b^2\left(\beta_1^2-\sigma_1^2\right)}\right]$; $\sigma^c = \left[\frac{\left(\beta_2^2-\sigma_1^2\right)m^2\pi^2+\eta b^2}{b^2\left(\beta_2^2-\sigma_1^2\right)}+s^2\right]^{\frac{1}{2}}$ (3.24)

The Laplace inversion of (3.23) is defined as

$$V_o(x,y)=\frac{2P_o u\sin\frac{m\pi}{b}y_o\sin\frac{m\pi}{b}y}{b\left(\beta_1^2-\sigma_1^2\right)}\left\{F_1(x,y;t)-F_2(x,y;t)+F_3(x,y;t)-F_4(x,y;t)\right\} \quad (3.25)$$

where

$$F_1(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\cosh\sigma^c x}{s^2+\sigma^2u^2}ds, \quad F_2(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{s\left(t-\frac{x}{u}\right)}\cosh\sigma^c x}{s^2+\sigma^2u^2}ds \quad (3.26)$$

$$F_3(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{s\left(t-\frac{1}{u}\right)}\sinh\sigma^c x}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c}ds, \quad F_4(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\cosh\sigma^c\sinh\sigma^c x}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c}ds$$

In order to evaluate the above integrals, residue theorem is employed. The singularities in the integrals are poles. In particular the denominators of the integrands of $F_1(x,y;t)$ and $F_2(x,y;t)$ have simple poles at $s=\pm\Omega_1$, where

$$, \qquad \Omega_1 = u\sqrt{\frac{\left(\beta_2^2-\sigma_1^2\right)m^2\pi^2+\eta b^2}{b^2\left(\beta_1^2-\sigma_1^2-u^2\right)}} \quad (3.28)$$

it is straight forward to show that

$$F_1(x,y;t)=-\frac{\cosh\Omega_3 x}{2A_1\Omega_1}\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right), \quad F_2(x,y;t)=-\frac{1}{2A_1\Omega_1}\left[e^{\Omega_1\left(t-\frac{x}{u}\right)}-e^{-\Omega_1\left(t-\frac{x}{u}\right)}\right] \quad (3.29)$$

where $\Omega_2=\frac{\left(\beta_2^2-\sigma_1^2\right)m^2\pi^2+\eta b^2}{b^2}$, $\Omega_3=\frac{1}{\left(\beta_1^2-\sigma_1^2\right)^{\frac{1}{2}}}\left(\Omega_1^2+\Omega_2\right)^{\frac{1}{2}}$ (3.30)

Furthermore, to evaluate $F_3(x, y; t)$, its integrand is rewritten to take the form

$$\phi_a=\frac{e^{st}\cosh\sigma^c x}{A_1\left[s^2-u^2\frac{\left[\left(\beta_2^2-\sigma_1^2\right)m^2\pi^2+\eta b^2\right]}{b^2\left(\beta_1^2-\sigma_1^2-u^2\right)}\right]} \quad (3.31)$$

Thus, the simple poles are $s=\Omega_1 \; and \; s=-\Omega_1$. In order to obtain poles emanating from $\sinh\sigma^c$. It is set to zero, i.e. $\sinh\sigma^c=0$;which implies $\sigma^c=\pm iv\pi$, (3.32)

Thus, $s=\pm i\sqrt{\Omega_4}$, where $\Omega_4=v^2\pi^2\left(\beta_1^2-\sigma_1^2\right)+\frac{\left(\beta_2^2-\sigma_1^2\right)m^2\pi^2+\eta b^2}{b^2}$ (3.33)

Thus, the contribution towards $F_3(x, y; t)$ due to simple poles at $s=\pm\Omega_1$ is given by

$$F_{3a}(x,y;t)=\frac{\sinh\Omega_4 x}{2A_1\sinh\Omega_3}\left[e^{\Omega_1\left(t-\frac{1}{u}\right)}-e^{-\Omega_1\left(t-\frac{1}{u}\right)}\right] \quad (3.34)$$

In a similar manner, the contribution due to simple poles at $s=\pm i\sqrt{\Omega_4}$ is given by

$$F_{3b}(x,y;t)=\frac{(-1)^{\nu+1}\left(\beta_1^2-\sigma_1^2\right)\nu\pi\sin\nu\pi x}{A_1\sqrt{\Omega_4}\left(\Omega_4+\Omega_1^2\right)}\left[e^{i\sqrt{\Omega_4}\left(t-\frac{1}{u}\right)}-e^{-i\sqrt{\Omega_4}\left(t-\frac{1}{u}\right)}\right] \tag{3.35}$$

Therefore $$F_3(x,y;t)=F_{3a}(x,y;t)+F_{3b}(x,y;t) \tag{3.36}$$

The contributions toward $F_4(x,y;t)$ are obtained in a similar manner as we have in $F_3(x,y;t)$

$$F_4(x,y;t)=\frac{\sinh\Omega_3 x}{2A_1\Omega_1\sinh\Omega_3}\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right)+\frac{(-1)^{2\nu+1}\left(\beta_1^2-\sigma_1^2\right)\nu\pi\sin\nu\pi x}{A_1\sqrt{\Omega_4}\left(\Omega_4+\Omega_1^2\right)} \tag{3.37}$$

Substituting (3.26), (3.27), (3.36) and (3.37) into (3.25) yields

$$V_o(x,y;t)=\frac{2P_o u\sin\frac{m\pi}{b}y_o\sin\frac{m\pi}{b}y}{bA_1\left(\beta_1^2-\sigma_1^2\right)}\left\{\frac{\cosh\Omega_3 x}{2\Omega_1}\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right)-\frac{e^{\Omega_1\left(t-\frac{x}{u}\right)}}{2\Omega_1}+\frac{e^{-\Omega_1\left(t-\frac{x}{u}\right)}}{2\Omega_1}\right.$$

$$+\frac{\sinh\Omega_3 x}{2\Omega_1\sinh\Omega_3}\left[e^{\Omega_1\left(t-\frac{1}{u}\right)}-e^{-\Omega_1\left(t-\frac{1}{u}\right)}\right]+\frac{(-1)^{\nu+1}\left(\beta_1^2-\sigma_1^2\right)\nu\pi\sin\nu\pi x}{\sqrt{\Omega_4}\left(\Omega_4+\Omega_1^2\right)}\left[e^{i\sqrt{\Omega_4}\left(t-\frac{1}{u}\right)}-e^{-i\sqrt{\Omega_4}\left(t-\frac{1}{u}\right)}\right]$$

$$\left.-\frac{\sinh\Omega_3 x\cosh\Omega_3}{2\Omega_1\sinh\Omega_3}\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right)-\frac{(-1)^{2\nu+1}\left(\beta_1^2-\sigma_1^2\right)\nu\pi\sin\nu\pi x}{\sqrt{\Omega_4}\left(\Omega_4+\Omega_1^2\right)}\left(e^{i\sqrt{\Omega_4}t}-e^{-i\sqrt{\Omega_4}t}\right)\right\} \tag{3.38}$$

The combination of the results (3.22) and (3.38) yield the desired leading order solution of (2.1) which represents the uniformly valid solution of the entire domain of definition of the given plate.

4.0 First order correction

4.1 Solution for V_1^0

The next corrections in outer solution are obtained by setting ν = 1 in equation (2.15). For the outer solution, the governing equation for V_1^0 is given as

$$\left(\beta_1^2-\sigma_1^2\right)\frac{\partial^2 V_1(x,y;t)}{\partial^2 x}+\frac{\left(\beta_2^2-\sigma_1^2\right)\partial^2 V_1(x,y;t)}{\partial y^2}-s^2V_1(x,y;t)-\eta V_1(x,y;t)=0 \tag{4.1}$$

Using the method of finite Fourier sine transform (3.2) on equation (4.1) with respect to *x* yields

$$V_{1,yy}(n,y)+\alpha^2 V_1(n,y)=0 \tag{4.2}$$

The homogeneous solution of (4.2) gives $$V_1(n,y)=C_3\cos\alpha y+D_3\sin\alpha y \tag{4.3}$$

Similarly, if equation (4.1) is subjected to finite Fourier sine transform (3.3) with respect to y, one obtains

$$\left(\beta_1^2-\sigma_1^2\right)V_{1,xx}(m,x)+\frac{m^2\pi^2}{b^2}\left(\beta_2^2-\sigma_1^2\right)V_1(m,x)-s^2V_1(m,x)-\eta V_1(m,x)=0 \tag{4.4}$$

so that $$V_{1,xx}(m,x)+\sigma^2 V_1(m,x)=0 \tag{4.5}$$

The complimentary solution of (4.5) gives $$V_1(m,x)=E_4\cos\sigma x+F_4\sin\sigma x \tag{4.6}$$

The inversion of equation (4.3) together with equation (4.6) gives

$$V_1(n,y)=2\left[C_3\cos\alpha y+D_3\sin\alpha y\right]\sin n\pi x+\frac{2}{b}\left[E_4\cos\sigma x+F_4\sin\sigma x\right]\sin\frac{m\pi}{b}y \tag{4.7}$$

where C_3, D_3, E_4 and F_4 are unknown constants to be determined by matching.

4.2 Solution for ψ_1^i

The first order correction is obtained by setting $v = 1$ in the differential equation (2.18). Doing this and neglecting terms with negative subscripts, we have

$$\frac{\partial^4 \psi_1^i}{\partial X^4} - \left(\beta_1^2 - \sigma_1^2\right)\frac{\partial^2 \psi_1^i}{\partial X^2} = 0 \qquad (4.8)$$

with the boundary condition

$$\psi_1^i = \frac{\partial \psi_1^i}{\partial X} = 0 \qquad (4.9)$$

Following usual argument in equation (3.17) and (3.18), the first order correction of the inner problem can be written as:

$$\psi_1^i = \begin{cases} \hat{q}_1(y)\left\{ X + \dfrac{1}{\sqrt{\beta_1^2 - \sigma_1^2}} e^{-\sqrt{\beta_1^2 - \sigma_1^2}\,X} - \dfrac{1}{\sqrt{\beta_1^2 - \sigma_1^2}} \right\} \text{near } x = 0 \\ \bar{q}_1(y)\left\{ X + \dfrac{1}{\sqrt{\beta_1^2 - \sigma_1^2}} e^{-\sqrt{\beta_1^2 - \sigma_1^2}\,X} - \dfrac{1}{\sqrt{\beta_1^2 - \sigma_1^2}} \right\} \text{near } x = 1 \\ \hat{r}_1(x)\left\{ Y + \dfrac{1}{\sqrt{\beta_2^2 - \sigma_1^2}} e^{-\sqrt{\beta_2^2 - \sigma_1^2}\,Y} - \dfrac{1}{\sqrt{\beta_2^2 - \sigma_1^2}} \right\} \text{near } y = 0 \\ \bar{r}_1(x)\left\{ Y + \dfrac{1}{\sqrt{\beta_2^2 - \sigma_1^2}} e^{-\sqrt{\beta_2^2 - \sigma_1^2}\,Y} - \dfrac{1}{\sqrt{\beta_2^2 - \sigma_1^2}} \right\} \text{near } y = b \end{cases} \qquad (4.10)$$

where exponentially growing terms have been neglected as unmatchable. The functions $\hat{q}_1(y)$, $\bar{q}_1(y)$, $\hat{r}_1(x)$ and $\bar{r}_1(x)$ will be determined by matching. By matching one term outer solution with two terms inner solution expansion written in outer variable, we obtained as follows:

$$\hat{q}_1(y) = \frac{2P_o u \sin\frac{m\pi}{b} y_o}{b\left(\beta_1^2 - \sigma_1^2\right)\left(s^2 + \sigma^2 u^2\right)}\left[\frac{e^{-\frac{s}{u}}}{\sin\sigma} - \cot\sigma + \frac{s}{u\sigma} e^{-\frac{s}{u}}\right]\sigma \sin\frac{m\pi}{b} y \qquad (4.11)$$

$$\bar{q}_1(y) = \frac{2P_o u \sin\frac{m\pi}{b} y_o}{b\left(\beta_1^2 - \sigma_1^2\right)\left(s^2 + \sigma^2 u^2\right)}\left[\sigma \sin\sigma - \sigma e^{-\frac{s}{u}} \frac{\cos\sigma}{\sin\sigma} + \sigma\cos\sigma\cot\sigma - \frac{s}{u} e^{-\frac{s}{u}}\right]\sin\frac{m\pi}{b} y \qquad (4.12$$

$$\hat{r}_1(x) = \frac{2m\pi P_o u \sin\frac{m\pi}{b} y_o}{b^2\left(\beta_1^2 - \sigma_1^2\right)\left(s^2 + \sigma^2 u^2\right)}\left[\cos\sigma x + e^{-\frac{s}{u}} \frac{\sin\sigma x}{\sin\sigma} - \cot\sigma \sin\sigma x - e^{-\frac{s}{u}x}\right] \qquad (4.13)$$

$$\bar{r}_1(x) = \frac{2(-1)^m m\pi P_o u \sin\frac{m\pi}{b} y_o}{b^2\left(\beta_1^2 - \sigma_1^2\right)\left(s^2 + \sigma^2 u^2\right)}\left[e^{-\frac{s}{u}x} - \cos\sigma x - e^{-\frac{s}{u}} \frac{\sin\sigma x}{\sin\sigma} + \cot\sigma \sin\sigma x\right] \qquad (4.14)$$

We seek an asymptotic outer solution of the form $V^o = V_0^o + \varepsilon V_1^o$ which implies that (4.15)

$$V^o = A_4\left[\cos\sigma x - e^{-\frac{s}{u}x} + e^{-\frac{s}{u}} \frac{\sin\sigma x}{\sin\sigma} - \frac{\cos\sigma \sin\sigma x}{\sin\sigma}\right]\sin\frac{m\pi}{b} y$$

$$+2\varepsilon[C_3\cos\alpha y + D_3\sin\alpha y]\sin n\pi x + \frac{2}{b}\varepsilon[E_4\cos\alpha x + F_4\sin\alpha x]\sin\frac{m\pi}{b}y \qquad (4.16)$$

where
$$A_4 = \frac{2P_o u \sin\frac{m\pi}{b}y_o}{b\left(\beta_1^2-\sigma_1^2\right)\left(s^2+\sigma^2u^2\right)} \qquad (4.17)$$

By matching two terms outer solution with two terms inner solution (2 – 2 matching) of equation (4.10) as $\varepsilon \to 0,$ one obtains

$$E_4 = \frac{-P_o u\sin\frac{m\pi}{b}y_o}{\left(\beta_1^2-\sigma_1^2\right)^{\frac{3}{2}}\left(s^2+\sigma^2u^2\right)}\left[\frac{\sigma e^{-\frac{s}{u}}}{\sin\sigma} - \frac{\sigma\cos\sigma}{\sin\sigma} + \frac{s}{u}\right] \qquad (4.18)$$

$$F_4 = \frac{-P_o u\sin\frac{m\pi}{b}y_o}{\left(\beta_1^2-\sigma_1^2\right)^{\frac{3}{2}}\left(s^2+\sigma^2u^2\right)}\left[\sigma - 2e^{-\frac{s}{u}}\frac{\cot\sigma}{\sin\sigma} + 2\sigma\cot^2\sigma - \frac{se^{-\frac{s}{u}}}{u\sin\sigma} - \frac{s}{u}\cot\sigma\right] \qquad (4.19)$$

$$C_3 = \frac{P_0 um\pi\sin\frac{m\pi}{b}y_0}{b^2(\beta_1^2-\sigma_1^2)(\beta_2^2-\sigma_1^2)^{\frac{1}{2}}(s^2+\sigma^2u^2)\sin n\pi x}\Big\{\cos\sigma x - e^{\frac{-s}{u}}$$
$$+e^{-\frac{s}{u}}\frac{\sin\sigma x}{\sin\sigma} - \frac{\cos\sigma\sin\sigma x}{\sin\sigma}\Big\} \qquad (4.20)$$

$$D_3 = \frac{2P_o um\pi\sin\frac{m\pi}{b}y_o}{b^2\left(\beta_1^2-\sigma_1^2\right)\left(\beta_2^2-\sigma_1^2\right)^{\frac{1}{2}}\left(s^2+\sigma^2u^2\right)\sin n\pi x}\Big\{(-1)^m\frac{\cos\sigma x}{\sin\alpha b} - \frac{(-1)^m e^{-\frac{s}{u}x}}{\sin\alpha b}$$
$$+\frac{(-1)^m e^{-\frac{s}{u}}\sin\sigma x}{\sin\sigma\sin\alpha b} - \frac{(-1)^m\cos\sigma\sin\sigma x}{\sin\sigma\sin\alpha b} + \cos\sigma x\cot\alpha b$$
$$-e^{-\frac{s}{u}}\cos\alpha b + \frac{e^{-\frac{s}{u}}\sin\sigma x\cot\alpha b}{\sin\sigma b} - \cot\sigma\sin\sigma x\cot\alpha b\Big\} \qquad (4.21)$$

substituting equations (4.18), (4.19), (4.20) and (4.21) into (4.7), after some substitution and arrangement yields

$$V_1(x,y) = \frac{2P_o um\pi\sin\frac{m\pi}{b}y_o}{b^2\left(\beta_1^2-\sigma_1^2\right)\left(\beta_2^2-\sigma_1^2\right)^{\frac{1}{2}}\left(s^2+\sigma^2u^2\right)}\Big\{-\cos\sigma x\cos\alpha y + e^{-\frac{s}{u}x}\cos\alpha y$$
$$-e^{-\frac{s}{u}x}\frac{\sin\sigma x\cos\alpha y}{\sin\sigma} + \cot\sigma\sin\sigma x\cos\alpha y + \frac{(-1)^m\cos\sigma x\sin\alpha y}{\sin\alpha b} - (-1)^m e^{-\frac{s}{u}x}\frac{\sin\alpha y}{\sin\alpha b}$$
$$+(-1)^m e^{-\frac{s}{u}}\frac{\sin\sigma x\sin\alpha y}{\sin\sigma\sin\alpha b} - (-1)^m\frac{\cos\sigma\sin\sigma x\sin\alpha y}{\sin\sigma\sin\alpha b} + \cos\sigma x\cot\alpha b\sin\alpha y$$

$$-e^{-\frac{s}{u}x}\cot\alpha b\sin\alpha y+e^{-\frac{s}{u}}\frac{\sin\sigma x\cot\alpha b\sin\alpha y}{\sin\sigma}-\cot\sigma\sin\sigma x\cot\alpha b\sin\alpha y\}$$

$$+\frac{2P_o u\sin\frac{m\pi}{b}y_o\sin\frac{m\pi}{b}y}{\left(\beta_1^2-\sigma_1^2\right)^{\frac{3}{2}}\left(s^2+\sigma^2u^2\right)}\left\{\frac{\sigma\cos\sigma\cos\sigma x}{\sin\sigma}-\sigma e^{-\frac{s}{u}}\frac{\cos\sigma x}{\sin\sigma}-\frac{s}{u}\cos\sigma x\right.$$

$$\left.-\sigma\sin\sigma x+2e^{-\frac{s}{u}}\frac{\cot\sigma\sin\sigma x}{\sin\sigma}-2\sigma\cot^2\sigma\sin\sigma x+\frac{s}{u}e^{-\frac{s}{u}}\frac{\sin\sigma x}{\sin\sigma}+\frac{s}{u}\cot\sigma\sin\sigma x\right\} \tag{4.22}$$

where $$P_{a_1}=\frac{2P_o um\pi\sin\frac{m\pi}{b}y_o}{b^2\left(\beta_1^2-\sigma_1^2\right)\left(\beta_2^2-\sigma_1^2\right)^{\frac{1}{2}}},\quad P_{a_2}=\frac{2P_o u\sin\frac{m\pi}{b}y_o\sin\frac{m\pi}{b}y}{\left(\beta_1^2-\sigma_1^2\right)^{\frac{3}{2}}} \tag{4.23}$$

The Laplace inversion of (4.22) is given by

$$V_1(x,y;t)=P_{a1}\{-G_1(x,y;t)+G_2(x,y;t)-G_3(x,y;t)+G_4(x,y;t)+G_5(x,y;t)-G_6(x,y;t)$$
$$+G_7(x,y;t)-G_8(x,y;t)+G_9(x,y;t)-G_{10}(x,y;t)+G_{11}(x,y;t)-G_{12}(x,y;t)\}$$
$$+P_{a_2}\{-G_{13}(x,y;t)-G_{14}(x,y;t)-G_{15}(x,y;t)+G_{16}(x,y;t)+G_{17}(x,y;t)\} \tag{4.24}$$

where $$G_1(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\cosh\sigma^c x\cosh\alpha^c y}{s^2+\sigma^2u^2}ds \tag{4.25}$$

$$G_2(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{s\left(t-\frac{x}{u}\right)}\cosh\alpha^c y}{s^2+\sigma^2u^2}ds \tag{4.26}$$

$$G_3(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{s\left(t-\frac{1}{u}\right)}\sinh\sigma^c x\cosh\alpha^c y}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c}ds \tag{4.27}$$

$$G_4(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\coth\sigma^c\sinh\sigma^c x\cosh\alpha^c y}{s^2+\sigma^2u^2}ds \tag{4.28}$$

$$G_5(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{(-1)^m e^{st}\cosh\sigma^c x\sinh\alpha^c y}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c b}ds \tag{4.29}$$

$$G_6(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{(-1)^m e^{s\left(t-\frac{x}{u}\right)}\sinh\alpha^c y}{\left(s^2+\sigma^2u^2\right)\sinh\alpha^c b}ds \tag{4.30}$$

$$G_7(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{(-1)^m e^{s\left(t-\frac{1}{u}\right)}\sinh\sigma^c x\sinh\alpha^c y}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c\sinh\alpha^c b}ds \tag{4.31}$$

$$G_8(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{(-1)e^{st}\cosh\sigma^c\sinh\sigma^c x\sinh\alpha^c y}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c\sinh\alpha^c b}ds \tag{4.32}$$

$$G_9(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\cosh\sigma^c x\coth\alpha^c b\sinh\alpha^c y}{s^2+\sigma^2u^2}ds \tag{4.33}$$

$$G_{10}(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{s\left(t-\frac{x}{u}\right)}\coth\alpha^c b\sinh\alpha^c y}{\left(s^2+\sigma^2u^2\right)}ds \tag{4.34}$$

$$G_{11}(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{s\left(t-\frac{1}{u}\right)}\sinh\sigma^c x\coth\alpha^c b\sinh\alpha^c y}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c}ds \tag{4.35}$$

$$G_{12}(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\coth\sigma^c\sinh\sigma^c x\coth\alpha^c b\sinh\alpha^c y}{s^2+\sigma^2u^2}ds \tag{4.36}$$

$$G_{13}(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{st}\sigma\cosh\sigma^c\cosh\sigma^c x}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c}ds \tag{4.37}$$

$$G_{14}(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{e^{s\left(t-\frac{1}{u}\right)}\sigma\cosh\sigma^c x}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c}ds \tag{4.38}$$

$$G_{15}(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}e^{st}H(x;s)ds \tag{4.39}$$

where

$$H(x;s)=\frac{\frac{s}{u}\cosh\sigma^c x+\sigma\sinh\sigma^c x-\frac{s}{u}\coth\sigma^c\sinh\sigma^c x+2\sigma\coth^2\sigma^c\sinh\sigma^c x}{s^2+\sigma^2u^2} \tag{4.40}$$

$$G_{16}(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{2e^{s\left(t-\frac{1}{u}\right)}\coth\sigma^c\sinh\sigma^c x}{\left(s^2+\sigma^2u^2\right)\sinh\sigma^c}ds \tag{4.41}$$

$$G_{17}(x,y;t)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{se^{s\left(t-\frac{1}{u}\right)}\sinh\sigma^c x}{\left(s^2+\sigma^2u^2\right)u\sinh\sigma^c}ds \tag{4.42}$$

where
$$\alpha^c=\frac{1}{\left(\beta_2^2-\sigma_1^2\right)^{\frac{1}{2}}}\left[\left(\beta_1^2-\sigma_1^2\right)n^2\pi^2+\eta+s^2\right]^{\frac{1}{2}} \tag{4.43}$$

$$\sigma^c=\frac{1}{\left(\beta_1^2-\sigma_1^2\right)^{\frac{1}{2}}}\left[\left(\beta_2^2-\sigma_1^2\right)\frac{m^2\pi^2}{b}+\eta+s^2\right]^{\frac{1}{2}} \tag{4.44}$$

In order to evaluate the integrals $G_1(x,y;t)-G_{17}(x,y;t)$ the procedure outlined for $F_1(x,y;t)$ – $F_4(x,y;t)$ shall be followed. On simplification and rearrangement, one obtains

$$G_1(x,y;t)=\frac{1}{2A_1\Omega_1}\cosh\Omega_3 x\cosh\Omega_6 y\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right) \tag{4.45}$$

$$G_2(x,y;t)=\frac{1}{2A_1\Omega_1}\cosh\Omega_6 y\left[e^{\Omega_1\left(t-\frac{x}{u}\right)}-e^{-\Omega_1\left(t-\frac{x}{u}\right)}\right] \tag{4.46}$$

$$G_3(x,y;t)=\frac{\sinh\Omega_3 x\cosh\Omega_6 y}{2A_1\Omega_1\sinh\Omega_3}\left[e^{\Omega_1\left(t-\frac{x}{u}\right)}-e^{-\Omega_1\left(t-\frac{x}{u}\right)}\right]$$

$$+\frac{(-1)^{v+1}\left(\beta_1^2-\sigma_1^2\right)v\pi\sin v\pi x\cosh\alpha^{c^*}y}{A_1\Omega_7\Omega_8}\left[e^{i\Omega_{71}\left(t-\frac{1}{u}\right)}-e^{-i\Omega_7\left(t-\frac{1}{u}\right)}\right] \tag{4.47}$$

$$G_4(x,y;t)=\frac{1}{2A_1\Omega_1}\coth\Omega_3\sinh\Omega_3 x\cosh\Omega_6 y\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right) \tag{4.48}$$

$$G_5(x,y;t)=\frac{(-1)^m\cosh\Omega_3 x\sin\Omega_6 y}{2A_1\Omega_1\sinh\Omega_6 b}\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right)$$

$$+\frac{(-1)^{m+k}\left(\beta_2^2-\sigma_1^2\right)k\pi\cosh\sigma^{c^*}x\sin\frac{k\pi}{b}y}{bA_1\Omega_9\Omega_{10}}\left(e^{i\Omega_9 t}-e^{-i\Omega_9 t}\right) \tag{4.49}$$

$$G_6(x,y;t)=\frac{(-1)^m\sinh\Omega_6 y}{2A_1\Omega_1\sinh\Omega_6 y}\left[e^{\Omega_1\left(t-\frac{x}{u}\right)}-e^{-\Omega_1\left(t-\frac{x}{u}\right)}\right]$$

$$+\frac{(-1)^{m+k}\left(\beta_2^2-\sigma_1^2\right)k\pi\sin\frac{k\pi}{b}y}{bA_1\Omega_9\Omega_{10}}\left[e^{i\Omega_9\left(t-\frac{x}{u}\right)}-e^{-i\Omega_9\left(t-\frac{x}{u}\right)}\right] \tag{4.50}$$

$$G_7(x,y;t)=\frac{(-1)^m\sinh\Omega_3 x\sinh\Omega_6 y}{2A_1\Omega_1\sinh\Omega_3\sinh\Omega_6 b}\left[e^{\Omega_1\left(t-\frac{1}{u}\right)}-e^{-\Omega_1\left(t-\frac{1}{u}\right)}\right]$$

$$G_7(x,y;t)=\frac{(-1)^m\sinh\Omega_3 x\sinh\Omega_6 y}{2A_1\Omega_1\sinh\Omega_3\sinh\Omega_6 b}\left[e^{\Omega_1\left(t-\frac{1}{u}\right)}-e^{-\Omega_1\left(t-\frac{1}{u}\right)}\right]$$

$$+\frac{(-1)^{v+m}\left(\beta_1^2-\sigma_1^2\right)v\pi\sin v\pi x\sinh\alpha^{c^*}y}{A_1\Omega_7\Omega_8\sinh\alpha^{c^*}b}\left[e^{i\Omega_7\left(t-\frac{1}{u}\right)}-e^{-i\Omega_7\left(t-\frac{1}{u}\right)}\right]$$

$$+\frac{(-1)^{k+m}\left(\beta_2^2-\sigma_1^2\right)k\pi\sin\frac{k\pi}{b}y\sinh\sigma^{c^*}x}{bA_1\Omega_9\Omega_{10}\sinh\sigma^{c^*}}\left[e^{i\Omega_9\left(t-\frac{1}{u}\right)}-e^{-i\Omega_9\left(t-\frac{1}{u}\right)}\right] \tag{4.51}$$

$$G_8(x,y;t)=\frac{(-1)^m\cosh\Omega_3\sinh\Omega_3 x\sinh\Omega_6 y}{2A_1\Omega_1\sinh\Omega_3\sinh\Omega_6 b}\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right)$$

$$+\frac{(-1)^{2v+m}\left(\beta_1^2-\sigma_1^2\right)v\pi\sin v\pi x\sinh\alpha^{c^*}y}{bA_1\Omega_7\Omega_8\sinh\alpha^{c^*}b}\left(e^{i\Omega_7 t}-e^{-i\Omega_7 t}\right)$$

$$+\frac{(-1)^{k+m}\left(\beta_2^2-\sigma_1^2\right)k\pi\cosh\sigma^{c^*}\sinh\sigma^{c^*}x\sin\frac{k\pi}{b}y}{bA_1\Omega_9\Omega_{10}\sinh\sigma^{c^*}}\left(e^{i\Omega_9 t}-e^{-i\Omega_9 t}\right) \tag{4.52}$$

$$G_9(x,y;t)=\frac{1}{2A_1\Omega_1}\cosh\Omega_3 x\sinh\Omega_6 b\sinh\Omega_6 y\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right) \tag{4.53}$$

$$G_{10}(x,y;t)=\frac{1}{2A_1\Omega_1}\coth\Omega_6 b\sinh\Omega_6 y\left[e^{\Omega_1\left(t-\frac{x}{u}\right)}-e^{-\Omega_1\left(t-\frac{x}{u}\right)}\right] \tag{4.54}$$

$$G_{11}(x,y;t)=\frac{\sinh\Omega_3 x\coth\Omega_6 b\cosh\Omega_6 y}{2A_1\Omega_1\sinh\Omega_6}\left[e^{\Omega_1\left(t-\frac{1}{u}\right)}-e^{-\Omega_1\left(t-\frac{1}{u}\right)}\right]$$
$$+\frac{(-1)^{v+1}\left(\beta_1^2-\sigma_1^2\right)v\pi\sin v\pi x\coth\alpha^{c*}\sinh\alpha^{c*}y}{A_1\Omega_7\Omega_8}\left[e^{i\Omega_{71}\left(t-\frac{1}{u}\right)}-e^{-i\Omega_7\left(t-\frac{1}{u}\right)}\right] \tag{4.55}$$

$$G_{12}(x,y;t)=\frac{1}{2A_1\Omega_1}\coth\Omega_3\sinh\Omega_3 x\coth\Omega_6\sinh\Omega_6 y\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right) \tag{4.56}$$

$$G_{13}(x,y;t)=\frac{i\Omega_3\cosh\Omega_3\cos\Omega_3 x}{2A_1\Omega_1\sinh\Omega_3}\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right)$$
$$+\frac{(-1)^{2v+1}\left(\beta_1^2-\sigma_1^2\right)v^2\pi^2\cos v\pi x}{2A_1\Omega_7\Omega_8}\left(e^{i\Omega_7 t}-e^{-i\Omega_7 t}\right) \tag{4.57}$$

$$G_{14}(x,y;t)=\frac{i\Omega_3\cosh\Omega_3 x}{2A_1\Omega_1\sinh\Omega_3}\left[e^{\Omega_1\left(t-\frac{1}{u}\right)}-e^{-\Omega_1\left(t-\frac{1}{u}\right)}\right]$$
$$+\frac{(-1)^{v}\left(\beta_1^2-\sigma_1^2\right)v^2\pi^2\cos v\pi x}{A_1\Omega_7\Omega_8}\left[e^{i\Omega_7\left(t-\frac{1}{u}\right)}-e^{-i\Omega_7\left(t-\frac{1}{u}\right)}\right] \tag{4.58}$$

$$G_{15}(x,y;t)=\frac{1}{2A_1\Omega_1}\left(G_{15a}(x,y;t)+G_{15b}(x,y;t)+G_{15c}(x,y;t)\right)\left(e^{\Omega_1 t}-e^{-\Omega_1 t}\right)$$

$$G_{15a}(x,y;t)=\frac{\Omega_1}{u}\cosh\Omega_3 x+i\Omega_3\sinh\Omega_3 x,\quad G_{15b}(x,y;t)=-\frac{\Omega_1}{u}\coth\Omega_3\sinh\Omega_3 x$$
$$G_{15c}(x,y;t)=2i\Omega_3\coth^2\Omega_3\sinh\Omega_3 x \tag{4.59}$$

$$G_{16}(x,y;t)=\frac{\coth\Omega_3\sinh\Omega_3 x}{A_1\Omega_1\sinh\Omega_3}\left[e^{\Omega_1\left(t-\frac{1}{u}\right)}-e^{-\Omega_1\left(t-\frac{1}{u}\right)}\right]$$
$$+\frac{(-1)^{2v}\left(\beta_1^2-\sigma_1^2\right)\sin v\pi x}{A_1\Omega_7\Omega_8}\left[e^{i\Omega_7\left(t-\frac{1}{u}\right)}-e^{-i\Omega_7\left(t-\frac{1}{u}\right)}\right] \tag{4.60}$$

$$G_{17}(x,y;t)=\frac{\frac{\Omega_1}{u}\sinh\Omega_3 x}{2A_1\Omega_1\sinh\Omega_3}\left[e^{\Omega_1\left(t-\frac{1}{u}\right)}-e^{-\Omega_1\left(t-\frac{1}{u}\right)}\right]$$

$$+\frac{(-1)^v\left(\beta_1^2-\sigma_1^2\right)\frac{v\pi}{u}\Omega_7\sin v\pi x}{A_1\Omega_7\Omega_8}\left[e^{i\Omega_7\left(t-\frac{1}{u}\right)}-e^{-i\Omega_7\left(t-\frac{1}{u}\right)}\right] \tag{4.61}$$

where
$$\Omega_5=\left(\beta_1^2-\sigma_1^2\right)n^2\pi^2+\eta\,;\;\Omega_6=\frac{1}{\left(\beta_1^2-\sigma_1^2\right)^{\frac{1}{2}}}\left(\Omega_1^2+\Omega_5\right)^{\frac{1}{2}} \tag{4.62}$$

$$\Omega_7=\sqrt{\Omega_4}=\left(v^2\pi^2\left(\beta_1^2-\sigma_1^2\right)+\Omega_2\right)^{\frac{1}{2}};\;\Omega_8=\left(v^2\pi^2\left(\beta_1^2-\sigma_1^2\right)+\Omega_2+\Omega_1^2\right)^{\frac{1}{2}} \tag{4.63}$$

$$\Omega_9=\left[\left(\beta_1^2-\sigma_1^2\right)n^2\pi^2+\eta+\frac{k^2\pi^2}{b^2}\left(\beta_2^2-\sigma_1^2\right)\right]^{\frac{1}{2}};\;\Omega_{10}=\left[\Omega_1^2+\Omega_9^2\right] \tag{4.64}$$

$$\alpha^{c*}=\frac{1}{\left(\beta_1^2-\sigma_1^2\right)^{1/2}}\left[-v^2\pi^2\left(\beta_1^2-\sigma_1^2\right)-\left(\beta_2^2-\sigma_1^2\right)\frac{m^2\pi^2}{b^2}+\left(\beta_1^2-\sigma_1^2\right)n^2\pi^2\right]^{\frac{1}{2}} \tag{4.65}$$

$$\sigma^{c*}=\frac{1}{\left(\beta_1^2-\sigma_1^2\right)^{\frac{1}{2}}}\left[-\left(\beta_1^2-\sigma_1^2\right)n^2\pi^2-\frac{k^2\pi^2}{b^2}\left(\beta_2^2-\sigma_1^2\right)+\left(\beta_2^2-\sigma_1^2\right)\frac{m^2\pi^2}{b^2}\right]^{\frac{1}{2}} \tag{4.66}$$

Substitution of integrals $G_1(x,y;t)$ - $G_{17}(x,y;t)$ into equation (4.24) gives the complete inversion of $V_1(x,y;t)$.

From equation (2.14), the perturbation scheme of a uniformly valid solution in the entire domain of definition of the plate problem is given by

$$V(x,y;t)=V_o(x,y;t)+\varepsilon V_1(x,y;t) \tag{4.67}$$

where $V_o(x, y, t)$ is the leading order solution and $V_1(x, y, t)$ is the first order correction. These are given respectively as (3.38) and (4.24). Thus substituting (3.38) and (4.24) into equation (4.67) gives the required solution.

5.0 Remark on theory

Equations (3.38) and (4.24) are the leading order and first order (transformed) solutions of the problem. The leading order and the first order solutions are combined in equation (4.67) to form the composite solution which is uniformly valid in the entire domain of the highly prestressed plate.

From equation (3.38), it is found that the anisotropic prestress, shear modulus and the foundation stiffness affect the response to $o(\varepsilon)$ of the rectangular plate. In an undamped system such as this, it is pertinent to examine the phenomenon of resonance. It is observed from the leading order and the first order correction results that fully clamped prestressed isotropic plate resting on a Pasternak-type foundation and transversed by a moving force reaches the state of resonance whenever $\beta_1^2=\sigma_1^2$ (5.1)

Other conditions when the system reaches a state of resonance are

$$\beta_2^2=\sigma_1^2 \text{ and } \left(\beta_1^2-\sigma_1^2\right)^{\frac{1}{2}}=u \tag{5.2}$$

$$u^2\left[\frac{\left(\beta_2^2-\sigma_1^2\right)m^2\pi^2+\eta b^2}{b^2\left(\beta_1^2-\sigma_1^2-u^2\right)}\right]^{\frac{1}{2}}=-\left[v^2\pi^2\left(\beta_1^2-\sigma_1^2\right)+\left(\beta_2^2-\sigma_1^2\right)\frac{m^2\pi^2}{b^2}+\eta\right] \tag{5.3}$$

$$u^2\left[\frac{\left(\beta_2^2-\sigma_1^2\right)m^2\pi^2+\eta b^2}{b^2\left(\beta_1^2-\sigma_1^2-u^2\right)}\right]^{\frac{1}{2}}=-\left[n^2\pi^2\left(\beta_1^2-\sigma_1^2\right)+\left(\beta_2^2-\sigma_1^2\right)\frac{k^2\pi^2}{b^2}+\eta\right] \tag{5.4}$$

From (5.1) to (5.4), it is observed that the resonance conditions of the plate are dependent on the anisotropic prestress and the elastic foundation. It is also evident that to any order of calculation, resonance conditions are affected by both the shear modulus G and foundation stiffness *K*.

At this juncture, the critical velocities for the system of a highly prestressed isotropic rectangular late on an elastic foundation traversed by a moving load are sought. The three distinct critical velocities that exist in the dynamical system are given as

$$U_1(\nu, m, \pi) = -\frac{1}{\nu \pi b}\left[\nu^2 \pi^2 b^2 \left(\beta_1^2 - \sigma_1^2\right) + \left(\beta_2^2 - \sigma_1^2\right) m^2 \pi^2 + \eta b^2\right]^{\frac{1}{2}} \tag{5.5}$$

$$U_2(k, n, \pi) = -\frac{1}{\pi}\left[\frac{\left(\beta_1^2 - \sigma_1^2\right)\left[n^2 \pi^2 b^2 \left(\beta_1^2 - \sigma_1^2\right) + \left(\beta_2^2 - \sigma_1^2\right) k^2 \pi^2 + \eta b^2\right]}{\left(k^2 - m^2\right)\left(\beta_2^2 - \sigma_1^2\right) + n^2 b^2 \left(\beta_1^2 - \sigma_1^2\right)}\right]^{\frac{1}{2}} \tag{5.6}$$

$$U_3 = \left(\beta_1^2 - \sigma_1^2\right)^{\frac{1}{2}} \tag{5.7}$$

6.0 Numerical calculations

In order to illustrate the analytical results, for example, the isotropic rectangular plate is taken to be of length L_X = 1.0m and width 0.5*m*. Other values used for the analysis in this section are b = 0.5 m, v = 1, $\pi = \frac{22}{7}$. The values of the prestress ratio in x - direction β_1^2 range between 0 and 100000. The critical velocities are plotted against prestress and foundation stiffness for various values of shear modulus G and subgrade *K*. Values of shear modulus G between 0 and 100000 were used while the values of foundation stiffness *K* were varied between 0N/m^3 and 2000000 N/m^3.

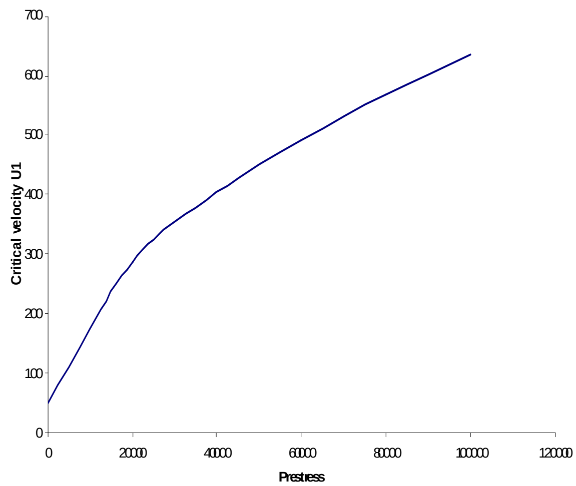

Figure 6.1: The graph of Critical Velocity U_1 against Prestress for G=100000 and K = (2000000)

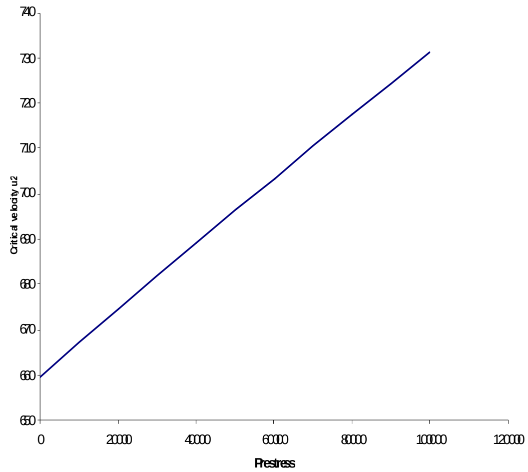

Figure 6.2: The graph of critical velocity U_2 against prestress for K=2000000 and G=100000

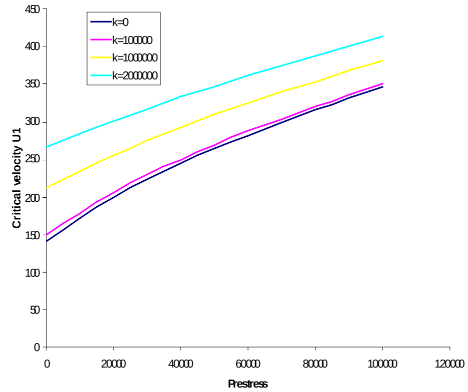

Figure 6.3: The graph of critical velocity U_2 against prestress for G=100000 and various values of K

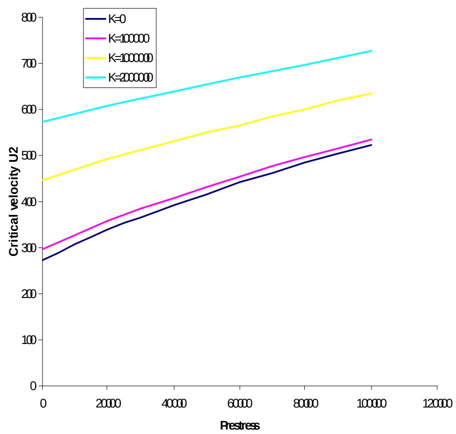

Figure 6.4: The graph of critical velocity U_1 against prestress for *G* =100000 and various values of *K*

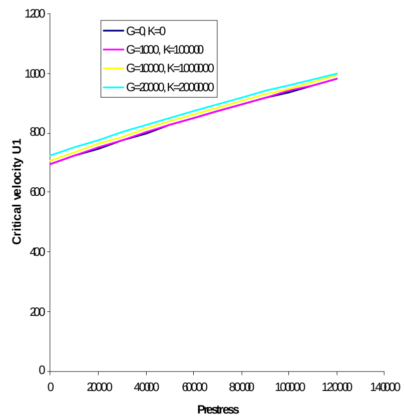

Figure6.5: The graph of critical velocity U_1 against prestress for various values of G and *K*.

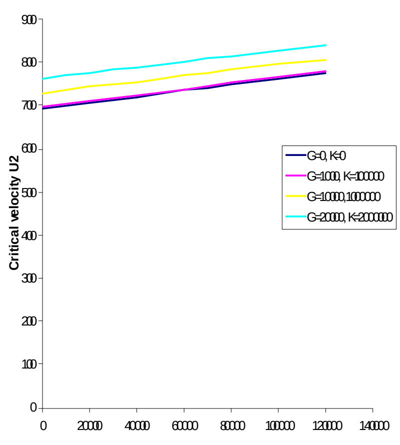

Figure 6.6: The graph of critical velocity U_2 against prestress for various values of *G* and *K*

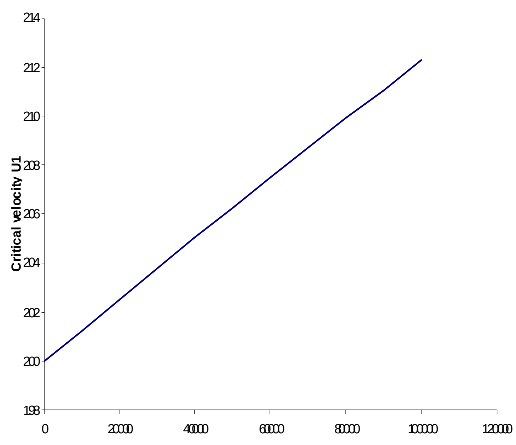

Figure 6.7: The graph of critical velocity U_1 against foundation stiffness *K* for *G* = 100000

Figure 6.1 displays the graph of critical velocity U_1 against prestress. From the graph, it is observed that the critical velocity U_1 increases with prestress for fixed values of shear modulus G and foundation stiffness K.

Thus, for high value of prestress, our design is more stable and reliable. In a similar manner, in figure 3.2, the critical velocity U_2 behaves exactly the same way as U_1. Results and analysis similar to those of figure 6.1 are obtained. The graph of U_1 against the restress for various values of foundation stiffness is shown in figure 6.3.

Evidently, the critical velocity increases with prestress for all values of foundation stiffness used. Thus resonance is reached earlier for lower values of prestress than for high values of prestress. Thus the design is more stable and the risk of resonance is remote for high values of prestress. Also, the graph of the critical velocity U_2 against the foundation stiffness behaves the same way as U_1; as evident in figure 6.4.

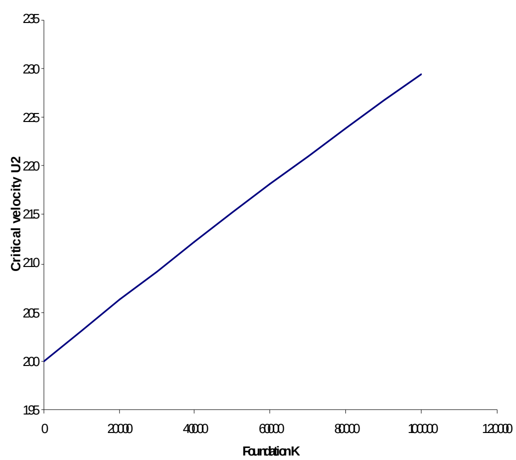

Figure 6.8: T he graph of critical velocity U_2 against foundation stiffness *K* for *G* = 100000

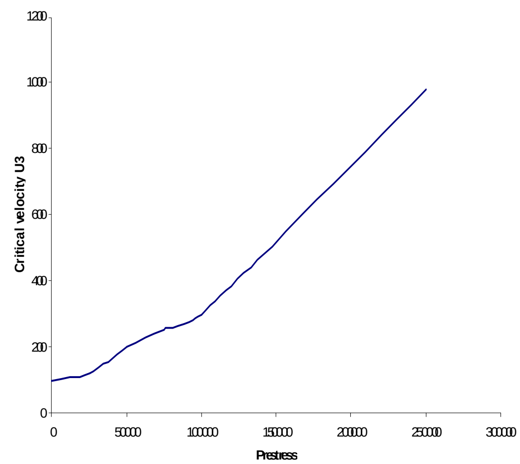

Figure 6.9: The graph of critical velocity U_3 against prestress for fixed value of *G* = 1000000

Figure 6.5 shows the plotted curves of U_1 against prestress for various values of shear modulus and foundation stiffness. The graph shows that as prestress increases, the critical velocity increases as well. Thus, with these, the likelihood of collapsed structure is very remote. The critical velocity U_2 in figure 6.6 behaves exactly the same way as U_1. Results and analysis similar to those of figure 6.5 are obtained.

The graph of U_1 against foundation is shown in figure 6.7. Evidently, the critical velocity increases as the foundation stiffness increases. Thus resonance is reached earlier for lower values of foundation stiffness. The critical velocity U_2 in figure 6.8 behaves exactly the same way as U_1. Results and analysis similar to those of figure 6.7 are obtained. Figure 6.9 displays the critical velocity U_3 against prestress. From the graph, the critical velocity increases with prestress for fixed value of shear modulus. Thus resonance is reached earlier for lower values of prestress.

7.0 Conclusion

This study concerns the problem of the dynamic response of a highly prestressed isotropic rectangular plate under a travelling load. The problem is governed by a fourth order non–homogenous differential equation. For the purpose of solution, the equation is presented in a non–dimensionalized form. It is observed that a small parameter multiplies the highest derivatives in the governing differential equation. Thus, this type of dynamical problem is usually amenable to singular perturbation technique. In particular the Method of Matched Asymptotic Expansion (MMAE) is used. This technique constructs outer (core) and inner (boundary layer) solutions that are valid in partly disjoint domains. These solutions are then matched in an intermediate domain where both asymptotic expansions are valid. Consequently, an approximate uniformly valid solution in the entire domain of definition of the rectangular plate is obtained with the rigorous use of Laplace transformation and the Cauchy residue theorem. This solution is analysed and five distinct resonance conditions are obtained in the dynamical system.

Numerical analysis is carried out and the study exhibits the following results:

(i) The leading order solutions and the first order correction are affected by the bi- parametric subgrade moduli anisotropic prestress.

(ii) As the foundation stiffness increases, the critical velocities of the isotropic rectangular plate transversed by moving load increases.

(iii) The critical velocities of the dynamical system increases with increase in prestress for all values of shear modulus and foundation stiffness used.

(iv) There may be more than one resonance condition in a dynamical system such as this which involves plate flexure under moving loads.

Finally, this work has showcased the use of a valuable method for the solution of this class of dynamical problems.

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