Influence of prestress on the response to moving loads of rectangular plates incorporating rotatory inertia correction factor.

S. T. Oni and A. Adedowole
Mathematical Sciences Department, Federal University of Technology, Akure, Nigeria.

Abstract

In this paper, the influence of axial force on the dynamic response to moving concentrated masses of rectangular plate incorporating rotatory inertia correction factor is investigated. The solution technique is based on the versatile two-dimensional generalized integral transform with the normal modes of the plate as the kernel of transformation and a modification of the Struble's asymptotic technique. The closed form solutions are analyzed and numerical analyses in plotted curves are presented. The results show that as the axial force (prestress), N_x and N_y, foundation moduli K and rotatory R_o increase, the response amplitudes of the dynamic system decrease for both illustrative examples. However, higher values of N_x, N_y, K and R_o are required for a more noticeable effect in the case of simple-clamped boundary conditions than those of simply supported boundary conditions. Furthermore, for the same natural frequency, the critical speed for the moving mass problem is smaller than that of the moving force problem. Hence resonance is reached earlier in moving mass problem.

1.0 Introduction

The problem of the response of an elastic system (beam or plate) to a moving load (moving force or moving mass) has been the objective of numerous investigations in Engineering, Mathematical Physics and Applied Mathematics for many years [6]. Stanisic et al [3] made landmark feet when they studied the two-dimensional problems of flexural vibration of plates under the actions of loads, paying more attention to moving mass. Only the inertia term that measures the effect of local acceleration in the direction of the deflection was considered. The method of solution was based on the Fourier sine transform technique suitable only for simply-supported boundary conditions. The solutions so obtained were shown to converge very rapidly. The work of Stanisic et al (1968) was taken up much later by Gbadeyan and Oni [2] who studied the dynamic analyses of an elastic plate continuously supported by an elastic Pasternak foundation traversed by an arbitrary number of concentrated masses. All the components of the inertia terms were considered and the rectangular plate was assumed to be simply supported, the deflection of the plate was calculated for several values of the foundation moduli and shown graphically as a function of time. As in the previous paper, the method of solution is suitable only for simply-supported boundary conditions. More recently, Oni [4] developed a versatile solution technique for solving two-dimensional moving load problems for all variants of classical boundary conditions. The technique involves the use of the modified generalized two-dimensional integral transform to reduce the fourth order differential equation governing the motion of the plate to second order ordinary differential equation which is then treated using the modified asymptotic method of Struble, [7,8]. The elegant method in Oni[4] was extended by Oni[9] to investigate the dynamic behaviour under several masses of rectangular plates resting on a Pasternak elastic foundation and having an arbitrary end supports. The solution method was based on the modified two-dimensional generalized transform and a modification of Sturble's asymptotic method. It was found that the critical speed for the system consisting of a rectangular plate resting on Pasternak's subgrade and traversed by a moving mass is reached prior that traversed by a moving force. Also, a two-dimensional theory, on the correction for rotatory inertia, on flexural motions of plate under moving load was studied by Oni [1]. The generalized two-dimensional integral transform with the normal modes of the plate as the kernel of transformation is used for the solution of the problem. The results show that the moving force solution is not always an upper bound for the accurate solution for the plate problem. However, no attempt was made to extend the theory developed in this study to solve the problem of flexural motions of prestressed rectangular plate under moving loads.

Thus, this paper is concerned with moving concentrated mass problem of a rectangular plate incorporating the effects of rotatory inertia and prestress under a Winkler foundation. The main objective is to classify the effects of prestress and foundation stiffness on the response of the plate. For simplicity in analysis two opposite sides of the plate are simply supported and other (two opposite edges) supported at will. Infact, plate structures of bridges are known usually to have two opposite edges simply supported and the other edges are free [6]. The generalized two-dimentional integral transform with the normal modes of plate as the kernel of transformation is used for the solution of the problem. Analyses of the results are carried for the plate model having simple supports at all edges and that having simple-clamped supports at two opposite edges.

2.0 The basic equation

A rectangular plate of thickness h and lateral dimension L_x and L_y (Respectively in the *x* and *y* direction in the rectangular axis) under the actions of a concentrated load $P(x, y, t)$ of mass M traveling from point $y = y_1$ on the plate along a straight line parallel to the *x*–axis with constant velocity c is considered in this thesis. Neglecting damping and the effects of shear deformation, according to the two-dimensional theory of flexural motions of isotropic elastic rectangular plate, the transverse displacement $W(x, y, t)$, of the mid-surface of such rectangular plate exhibiting anisotropic presstress and under a Winkler foundation is found by solving

$$\left(D\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2-\mu R_o\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 W(x,y,t)-\left(N_x\frac{\partial^2 W(x,y,t)}{\partial x^2}+N_y\frac{\partial^2 W(x,y,t)}{\partial y^2}\right)+$$

$$\mu\frac{\partial^2 W(x,y,t)}{\partial t^2}+KW(x,y,t)=P_f(x,y,t)\left[1-\frac{1}{g}\left(\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial x\partial t}+c^2\frac{\partial^2}{\partial x^2}\right)\right] \quad (2.1)$$

E is the Young's modulus of the plate, v is the Poisson's ratio, t is time, μ is the mass per unit area of the plate and R_o is the measure of rotating inertia effect, where

$$D = \frac{Eh^3}{12(1-\nu)},\ P_f(x, y, t) = Mg\,\delta(x-ct)\delta(y - y_1) \quad (2.2)$$

M is the mass of moving load and δ() is the Dirac delta function.

Furthermore, two opposite sides of the plate are simply supported and the other two opposite edges are taken to be arbitrary.

Thus, at edges $y = 0$ and $y = L_y$ the following conditions pertain

$$W(x,0,t) = 0 = W(x, L_y, t),\ \frac{\partial^2}{\partial y^2}W(x,0,t) = 0 = \frac{\partial^2}{\partial y^2}W(x, L_y, t) \quad (2.3$$

For simplicity, the associated initial conditions are

$$W(x,y,t)_{t=0} = 0 = \frac{\partial W}{\partial t}(x,y,t)_{t=0} = 0 \quad (2.4)$$

3.0 Analytical solution procedure

In order to solve equation (2.1) subject to the end conditions (2.3), equation (2.2) is substituted into equation (2.1) one obtains

$$D_m\left[\frac{\partial^4}{\partial x^4}W(x,y,t)+2\frac{\partial^4}{\partial x^2\partial y^2}W(x,y,t)+\frac{\partial^4}{\partial y^4}W(x,y,t)\right]-N_x^o\frac{\partial^2}{\partial x^2}W(x,y,t)-$$

$$N_y^o\frac{\partial^2}{\partial y^2}W(x,y,t)+K_oW(x,y,t)-R_o\frac{\partial^2}{\partial t^2}\nabla^2W(x,y,t)+\frac{\partial^2}{\partial t^2}W(x,y,t)+\frac{Mg}{\mu}\delta(x-ct)\delta(y-y_1)\bullet \quad (3.1)$$

$$\left(\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial x\partial t}+c^2\frac{\partial^2}{\partial x^2}\right)W(x,y,t)=\frac{Mg}{\mu}\delta(x-ct)\delta(y-y_1)$$

where
$$D_m=\frac{D}{\mu},\ N_x^o=\frac{N_x}{\mu},\ N_y^o=\frac{N_y}{\mu},\ K_0=\frac{K}{\mu} \quad (3.2)$$

The analysis of the dynamic response to the dynamic response to moving concentrated masses of rectangular plates incorporating rotatory inertia correction and prestress factors is carried out in this section employing the solution technique [5]. The transformations technique which is based on two-dimensional Fourier Sine integral transformation is termed generalized two-dimensional integral transformation defined by

$$U(j,k,t)=\int_0^{L_x}\int_0^{L_y}W(x,y,t)Sin\frac{k\pi y}{L_y}W_j(x)dxdy \quad (3.3)$$

with the inverse

$$W(x,y,t)=\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{2}{L_y}\frac{\mu}{w_j}U(j,k,t)\sin\frac{k\pi y}{L_y}W_j(x) \quad (3.4)$$

where
$$W_j = \int_0^{Lx}\mu W_j^2(x)dx \quad (3.5)$$

and $W_j(x)$ is the Jth normal mode in the direction of x-axis vibration of the plate defined as.

$$W_j(x) = Sin\frac{\alpha_j x}{L_x}+A_jCos\frac{\alpha_j x}{L_x}+B_jSinh\frac{\beta_j x}{L_x}+C_jCosh\frac{\beta_j x}{L_x} \quad (3.6)$$

where

$$\alpha_j = L_x\left[\frac{k^2\pi^2}{L_y^2} - \frac{\left(\Omega_{jk}^2\right)^{1/2}}{D_m}\right]^{1/2} \quad , \quad \beta_j = L_x\left[\frac{k^2\pi^2}{L_y^2} + \frac{\left(\Omega_{jk}^2\right)^{1/2}}{D_m}\right]^{1/2} \tag{3.7}$$

are mode frequencies and A_j, B_j and C_j are constants and D_m = D/μ. The parameter $\Omega_{j,k}$ is the natural circular frequency defined by

$$\Omega_{j,k}^2 = D_m\left(\frac{j^4\pi^4}{L_x^4} + 2\frac{j^2k^2\pi^4}{L_x^2L_y^2} + \frac{k^4\pi^4}{L_y^4}\right) \tag{3.8}$$

The function (3.6) satisfies all classical boundary conditions for this class of plate problem in the x direction. Applying the generalized integral transformation (3.3) to (3.1), equation (3.1) can be written as.

$$\begin{aligned} &D_m Z_0(0, L_{x,} L_y) + D_m H_1(j,k,L_{x,}L_y) + U_{tt}(j,k,t) - N_x^o H_2(j,k.L_{x,}L_y) - \\ &N_y^o H_3(j,k,L_x,L_y) - R_o H_4(j,k,L_x,L_y) - R_o H_5(j,k,L_x,L_y) \\ &\frac{M}{\mu}\left[G_1(p,k,t) + G_2(p,k,t) + G_3(p,k,t)\right] = \frac{Mg}{\mu} Sin\frac{k\Pi y_1}{L_y} W_j(ct) \end{aligned} \tag{3.9}$$

where

$$\begin{aligned} Z_o(0, L_x, L_y) = \int_0^{L_y}\Bigg[& W_j(x)\frac{\partial^3 W}{\partial x^3} - W^i{}_j(x)\frac{\partial^2 W}{\partial x^2} + W_j^{iii}(x)\frac{\partial W}{\partial x} - WW_j^{iii}(x) - \\ & \frac{k^2\pi^2}{L_y^2}\frac{\partial W}{\partial x}W_j(x) + \frac{k^2\pi^2}{L_y^2}W_j^i(x)W\Bigg] Sin\frac{k\pi y}{L_y} \end{aligned} \tag{3.10}$$

$$\begin{aligned} H_1(j,k,L_x,L_y) = & \int_0^{L_x}\int_0^{L_y} WW_j^{iv}(x) Sin\frac{k\pi y}{L_y}dxdy - \frac{k^2\pi^2}{L_y}\int_0^{L_x}\int_0^{L_y} WW_j^{ii}(x) Sin\frac{k\pi y}{L_y}dxdy \\ & + \frac{k^4\pi^4}{L_y^4}\int_0^{L_x}\int_0^{L_y} WW_j(x) Sin\frac{k\pi y}{L_y}dxdy \end{aligned} \tag{3.11}$$

$$H_2(j,k,L_x,L_y) = \int_0^{L_x}\int_0^{L_y}\frac{\partial^2 W(x,y,t)}{\partial x^2} Sin\frac{k\pi y}{L_y}W_j(x)dxdy \tag{3.12}$$

$$H_3(j,k,L_x,L_y) = \int_0^{L_x}\int_0^{L_y}\frac{\partial^2 W(x,y,t)}{\partial y^2} Sin\frac{k\pi y}{L_y}W_j(x)dxdy \tag{3.13}$$

$$H_4(j,k,L_x,L_y) = \int_0^{L_x}\int_0^{L_y}\frac{\partial^4 W(x,y,t)}{\partial t^2\partial x^2} Sin\frac{k\pi y}{L_y}W_j(x)dxdy \tag{3.14}$$

$$H_5(j,k,L_x,L_y) = \int_0^{L_x}\int_0^{L_y}\frac{\partial^4 W(x,y,t)}{\partial t^2\partial y^2} Sin\frac{k\pi y}{L_y}W_j(x)dxdy \tag{3.15}$$

$$G_1(p,k,t) = \int_0^{L_x}\int_0^{L_y}\delta(x-ct)\delta(y-y_1)\frac{\partial^2 W(x,y,t)}{\partial t^2} Sin\frac{k\pi y}{L_y}W_j(x)dxdy \tag{3.16}$$

$$G_2(p,k,t) = 2c\int_0^{L_x}\int_0^{L_y}\delta(x-ct)\delta(y-y_1)\frac{\partial^2 W(x,y,t)}{\partial x\partial t} Sin\frac{k\pi y}{L_y}W_j(x)dxdy \tag{3.17}$$

$$G_3(p,k,t) = c^2\int_0^{L_x}\int_0^{L_y}\delta(x-ct)\delta(y-y_1)\frac{\partial^2 W(x,y,t)}{\partial x^2} Sin\frac{k\pi y}{L_y}W_j(x)dxdy \tag{3.18}$$

it is remarked at this juncture that when

$$W(x,y,t) = Sin\frac{k\pi y}{L_y}W_j(x)Sin\Omega_{j,k}t \tag{3.19}$$

is the natural circular frequency of a rectangular plate is substituted into the equation of free vibration of plate namely

$$D\left[\frac{\partial^4 W(x,y,t)}{\partial x^4} + 2\frac{\partial^4 W(x,y,t)}{\partial x^2\partial y^2} + \frac{\partial^4 W(x,y,t)}{\partial y^4}\right] + \mu\frac{\partial^2 W(x,y,t)}{\partial t^2} = 0 \tag{3.20}$$

one obtains $D\left[W_j^{iv}(x)-2\frac{k^2\pi^2}{L_y^2}W_j^{ii}(x)+\frac{k^4\pi^2}{L_y^2}W_j^{ii}(x)+\frac{k^4\pi^4}{L_y^4}W_j(x)\right]=\Omega_{j,k}^2W_j(x)$ (3.21)

Equation (3.22) implies

$$D_m\left[\int_0^{L_x}\int_0^{L_y}WW_j^{iv}(x)Sin\frac{k\pi y}{L_y}dxdy-2\frac{k^2\pi^2}{L_y^2}\int_0^{L_x}\int_0^{L_y}WW_j^{ii}(x)Sin\frac{k\pi y}{L_y}dxdy\right.$$

$$\left.+\frac{k^4\pi^4}{L_y^4}\int_0^{L_x}\int_0^{L_y}WW_j(x)Sin\frac{k\pi y}{L_y}dxdy\right]=\Omega_{j,k}^2U(j,k,t)(3.22) \quad (3.22)$$

Consequently, $$H_1(j,k,L_xL_y)=\frac{\mu}{D}\Omega_{j,k}^2U(j,k,t) \quad (3.23)$$

In order to evaluate $H_2(j,k,L_x,L_y)$, it is noted that for any arbitrary subscripts $j = p$, $k = q$, equation (3.4) can be written as

$$\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{2}{L_x}\frac{\mu}{W_p}U(p,q,t)Sin\frac{q\pi y}{L_y}W_p(x) \quad (3.24)$$

It follows that

$$W_{xx}(x,y,t)=\frac{2}{L_y}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{\mu}{W_p}U(p,q,t)Sin\frac{q\pi y}{L_y}\frac{d^2}{dx^2}W_p(x) \quad (3.25)$$

therefore, $$H_2(j,k,L_xL_y)=\sum_{p=1}^{\infty}\frac{\mu}{W_p}W(p,k,t)\Lambda^2(p,j) \quad (3.26)$$

similarly $$H_3(j,k,L_xL_y)=\frac{-k^2\pi^2}{L_y^2}\sum_{p=1}^{\infty}\frac{\mu}{W_p}W(p,k,t)\Lambda(p,j) \quad (3.27)$$

where $$\Lambda^2(p,j)=\int_0^{L_x}\frac{d^2}{dx^2}W_p(x)W_j(x)dx\,,\Lambda(p,j)=\int W_p(x)W_j(x)dx \quad (3.28)$$

Using similar arrangements, noting that equation (3.14) can be rewritten as

$$H_4(j,k,L_x,L_y)=\frac{\partial^2}{\partial t^2}\int_0^{L_x}\int_0^{L_y}\frac{\partial^2W(x,y,t)}{\partial x^2}Sin\frac{k\pi y}{L_y}W_j(x)dxdy \quad (3.29)$$

It is straight forward to show that $$H_2(j,k,L_xL_y)=\sum_{p=1}^{\infty}\frac{\mu}{W_p}W(p,k,t)\Lambda^2(p,j) \quad (3.30)$$

In a similar manner $$H_5(j,k,L_xL_y)=\frac{-k^2\pi^2}{L_y^2}\sum_{p=1}^{\infty}\frac{\mu}{W_p}U_{tt}(p,k,t)\Lambda(p,j) \quad (3.31)$$

To evaluate integral (3.9), use is made of the property of the Dirac Delta function as a function to express it as a Fourier Cosine Series, namely

$$\delta(x-ct)=\frac{1}{L_x}+\frac{2}{L_x}\sum Cos\frac{n\pi ct}{L_x}Cos\frac{n\pi x}{L_x} \quad (3.32)$$

Similarly, $$\delta(y-y_1)=\frac{1}{L_y}+\frac{2}{L_y}\sum_{n=i}^{\infty}Cos\frac{n\pi y_1}{L_y}Cos\frac{n\pi y}{L_y} \quad (3.33)$$

It follows from (3.4), that $$W_{tt}(x,y,t)=\frac{2}{L_y}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{\mu}{W_p}U_{tt}(p,q,t)Sin\frac{q\pi y}{L_y}W_p(x) \quad (3.34)$$

Thus, using (3.32), (3.33) and (3.34) into the integrals (3.12) to (3.18) which when simplified and rearranged becomes

$$G_1(p,k,t)=\frac{4}{L_xL_y}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mu}{W_p}U_{tt}(p,q,t)\cos\frac{n\pi ct}{L_x}\sin\frac{k\pi y_1}{L_y}$$

$$\sin\frac{q\pi y_1}{L_y}\Lambda(n,j,p)+\frac{2}{L_xL_y}\sum_{p=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mu}{W_p}U_{tt}(p,q,t)\cos\frac{n\pi ct}{L_x}\Lambda(n,j,p)$$

$$+\frac{2}{L_xL_y}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{\mu}{W_p}U_{tt}(p,q,t)\sin\frac{q\pi y_o}{L_y}\Lambda(j,p)+\frac{1}{L_xL_y}\sum_{p=1}^{\infty}\frac{\mu}{W_p}U_{tt}(p,q,t)\Lambda(j,p) \tag{3.35}$$

where
$$\Lambda(n,j,p)=\int_o^{L_x}\cos\frac{n\pi x}{L_x}W_j(x)W_p(x)dx \tag{3.36}$$

$$G_2(p,k,t)=\frac{4}{L_xL_y}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mu}{W_p}U_t(p,q,t)\cos\frac{n\pi ct}{L_x}\sin\frac{k\pi y_1}{L_y}\sin\frac{q\pi y_1}{L_y}$$

$$\Lambda^1(n,j,p)+\frac{2}{L_xL_y}\sum_{p=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mu}{W_p}U_t(p,k,t)\cos\frac{n\pi ct}{L_x}\Lambda^1(n,j,p)$$

$$+\frac{2}{L_xL_y}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{\mu}{W_p}U_t(p,q,t)\sin\frac{k\pi y_1}{L_y}\sin\frac{q\pi y_1}{L_y}\Lambda^1(j,p)+\frac{1}{L_xL_y}\sum_{p=1}^{\infty}\frac{\mu}{W_p}U_t(p,k,t)\Lambda^1(j,p) \tag{3.37}$$

where
$$\Lambda^1(j,p)=\int_o^{L_x}W_j(x)\frac{d}{dx}W_p(x)dx\ ,\ \Lambda^1(n,j,p)=\int_o^{L_x}\cos\frac{n\pi x}{L_x}W_j(x)\frac{d}{dx}W_p(x)dx \tag{3.38}$$

and $G_3(p,k,t)=\frac{4}{L_xL_y}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mu}{W_p}U(p,q,t)\cos\frac{n\pi ct}{L_x}\sin\frac{k\pi y_1}{L_y}\sin\frac{q\pi y_1}{L_y}\times$

$$\Lambda^2(n,j,p)+\frac{2}{L_xL_y}\sum_{p=1}^{\infty}\sum_{n=1}^{\infty}\frac{\mu}{W_p}U(p,k,t)\cos\frac{n\pi ct}{L_x}\Lambda^2(n,j,p)$$

$$+\frac{2}{L_xL_y}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{\mu}{W_p}U(p,q,t)\sin\frac{k\pi y_1}{L_y}\sin\frac{q\pi y_1}{L_y}\Lambda^2(j,p)+\frac{1}{L_xL_y}\sum_{p=1}^{\infty}\frac{\mu}{W_p}U(p,k,t)\Lambda^2(j,p) \tag{3.39}$$

where $\Lambda^2(j,p)=\int_o^{x}W_j(x)\frac{d^2}{dx^2}W_p(x)dx,\ \Lambda^2(n,j,p)=\int_o^{x}\cos\frac{n\pi x}{L_x}W_j(x)\frac{d^2}{dx^2}W_p(x)dx$ (3.40)

It is remarked at this juncture that for all classical boundary conditions.

$$Z_o(0,L_x,L_y)=0 \tag{3.41}$$

4.0 Solution of the transformed equation

In order to solve equation (3.10) after same simplification and rearrangements yields

$$U_{tt}(j,k,t)+(\Omega_{j,k}^2+k)U(j,k,t)+D_mZ_o(0,L_x,L_y)-\sum_{p=1}^{\infty}\frac{\mu}{W_p}U(p,k,t)\Big[N_x^o\Lambda^2(p,j)$$

$$\left.-N_y^o\frac{k^2\pi^2}{L_y^2}\Lambda(p,j)\right]-R_o\sum_{p=1}^{\infty}\frac{\mu}{W_p}U_{tt}(p,k,t)\left[\Lambda^2(p,j)-\frac{k^2\pi^2}{L_y^2}\Lambda(p,j)\right]$$

$$+\Gamma_0L_xL_y\{G_1(p,k,t)+2cG_2(p,k,t)+c^2G_3(p,k,t)\ \}=\frac{Mg}{\mu}\sin\frac{k\pi y_1}{L_y}W_j(ct) \tag{4.1}$$

$$\Gamma_0=\frac{M}{\mu L_xL_y} \tag{4.2}$$

In what follows, two special cases of equation (4.1) are discussed. They are termed “moving force” and “moving mass” problems.

(a) Case I

If inertia effect of moving load is neglected, i.e setting Γ_0 in equation (4.1) to zero, obviously, an exact analytical solution of this equation is not possible. Consequently, an approximate analytical solution technique which is a modification of the asymptotic method of Struble discussed in [4] and [9] shall be used. To this end, we rearrange (4.1) to take the form

$$\left[1+\Gamma\frac{\mu}{W_p}\left(\frac{k^2\pi^2}{L_y^2}\Lambda(j,j)-\Lambda^2(j,j)\right)\right]U_{tt}(j,k,t)+\left[\alpha_{j,k}^2-\frac{\mu}{W_p}\left(N_x^0\Lambda^2(j,j)-N_y^0\frac{k^2\pi^2}{L_y^2}\Lambda(j,j)\right)\right]\times$$

$$U\ (j,k,t)+\Gamma\sum_{\substack{p=1\\p\neq 1,}}^{\infty}\frac{\mu}{W_p}\left\{\left(\frac{k^2\pi^2}{L_y^2}\Lambda(p,j)-\Lambda^2(p,j)\right)U_{tt}(p,k,t)-\right.$$

$$\left.\frac{1}{Dh}U\ (p,k,t)\left(N_x^0\Lambda^2(p,j)-N_y^o\frac{k^2\pi^2}{L_y^2}\Lambda(p,j)\right)\right\}=P_L Sin\ \frac{K\pi y_1}{L_y}W_j(ct) \tag{4.3}$$

Set $$P_L=\frac{Mg}{\mu},\ \Gamma=Dh,\ \ p=j \tag{4.4}$$

By means of this technique, we seek the modified frequency corresponding to the frequency of the free system due to the presence of the effect of rotatory inertia. An equivalent free system operator defined by the modified frequency then replaces equation (4.3). To this end, we set the right hand side of (4.3) to zero and consider a parameter $\lambda^*<1$ for any arbitrary ratio Γ defined as

$$\lambda^*=\frac{\Gamma}{1+\Gamma} \tag{4.5}$$

So that $$\Gamma=\lambda^*+0(\lambda^*)^2 \tag{4.6}$$

Substituting equation (4.6) into homogenous part, of equation (4.3) yields.

$$\left[1+\lambda^*\frac{\mu}{w_p}\left(\frac{k^2\pi^2}{\lambda^2 y}\Lambda(j,j)-\Lambda^2(j,j)\right)\right]U_{tt}(j,k,t)+\left[\alpha_{j,k}^2-\frac{\mu}{w_p Dh}\left(N_x^0\Lambda^2(j,j)-N_y^o\frac{k^2\pi^2}{L_y^2}\Lambda(j,j)\right)\right]\times$$

$$U\ (j,k,t)+\ \lambda*\sum_{\substack{p=1\\p\neq j}}^{\infty}\frac{\mu}{w_p}\left\{U_{tt}(p,k,t)\left(\frac{k^2\pi^2}{L_y^2}\Lambda(p,j)-\Lambda^2(p,j)\right)\right.$$

$$\left.-\frac{1}{Dh}U\ (p,k,t)\left[N_x^0\Lambda^2(p,j)-N_y^0\frac{k^2\pi^2}{L_y^2}\Lambda(p,j)\right]\right\}=0 \tag{4.7}$$

When λ^* is set to zero in equation (4.7), we obtain a situation corresponding to the case in which effect of the cross sectional dimensions of the plate is regarded as negligible. In such a case, the solution is of the form

$$\mathrm{U}_{st}(j,k,t)=C_{sf}Cos\ (\alpha_{jk}t-\phi_{sf}) \tag{4.8}$$

where $C_{st},\alpha_{jk} and \phi_{sf}$ are constants.

Furthermore, as $\lambda^*<1,$ Stubble's techniques require that the solution of equation (4.8) can be of the form $$U(j,k,t)=A(j,k,t)Cos(\alpha_{jk}t-\phi\ (j,k,t))+\lambda^* U(j,k,t)+0(\lambda^*)^2 \tag{4.9}$$

where $A(j,k,t)$ and $\phi(j,k,t)$ are slowly varying functions of time or equivalently

Substituting equation (4.9) and its derivatives into equation (4.7) and taking into account equation (4.6) , where terms higher than ($\lambda^\bullet$) are neglected. Now, since only the terms involving $Sin(\alpha_{jk}t-\phi(j,k,t))$ and $Cos(\alpha_{jk}t-\phi(j,k,t))$contribute to the variational equations describing the behavior of A (j, k, t) and $\phi(j,k,t),$ one obtains

$$-2\alpha_{jk}\overset{o}{A}(j,k,t)Sin(\alpha_{jk}t-\phi(j,k,t))+2A(j,k,t)\overset{o}{\phi}(j,k,t)\ \ \alpha_{jk}Cos\ \ (\alpha_{jk}t-\phi(j,k,t))-$$

$$\Gamma^*\sum_{pd}^{\infty}\frac{\mu}{W_p}\left\{\alpha_{jk}^2\left(\frac{k^2\pi^2}{L_y^2}\Lambda(p,j)-\Lambda^2(p,j)\right)-\frac{1}{Dh}\left[N_x^o\Lambda^2(p,j)-N_y^o\frac{k^2\pi^2}{L_y^2}\Lambda(p,j)\right]\right\}A\ Cos(\alpha_{jk}t-\phi(j,k,t))=0 \tag{4.10}$$

The so-called variation equations of this equation (4.10) are obtained by setting the coefficients Cos $\lfloor\alpha_{jk}t-\phi(j,k,t)\rfloor$ and Sin $\lfloor\alpha_{jk}t-\phi(j,k,t)\rfloor$ to zero. Thus, one obtains

$$2\alpha_{jk}A^o(j,k,t)=0 \tag{4.11}$$

and $$2A(j,k,t)\ \ \phi^o(j,k,t)\alpha_{jk}-A(j,k,t)\Gamma^*\frac{\mu}{w_p}\left[\alpha_{jk}^2 Q_0(t)-\frac{1}{Dh}Q_1(t)\right]=0 \tag{4.12}$$

where

$$Q_0(t)=\frac{k^2\pi^2}{L^2{}_y}\Lambda(j,j)-\Lambda^2(j,j)\ ,\ Q_1(t)=N_x^o\Lambda^2(j,j)-N_y^o\frac{k^2\pi^2}{L^2{}_y}\Lambda(j,j) \qquad (4.13)$$

Solving equation (4.11), one obtains $\qquad A(j,k,t)=C_{sf} \qquad (4.14)$

where C_{st} is a constant . The first order differential equation (4.10) describing the behavior of $\phi(j,k,t)$ implies.

$$\frac{d\phi(j,k,t)}{dt}=\frac{\lambda^*\mu}{2W_\rho}\left\{\alpha_{jk}Q_0(t)-\frac{1}{Dh\alpha_{jk}}Q_1(t)\right\} \qquad (4.15)$$

hence, $$\phi(j,k,t)=\frac{\lambda^*\mu}{2w_\rho}\left\{\alpha_{jk}Q_0(t)-\frac{1}{Dh\alpha_{jk}}Q_1(t)\right\}t+\phi_{sf} \qquad (4.16)$$

Therefore when cross sectional dimension of the plate is considered, the first approximate to the homogenous system is $\qquad U(j,k,t)=C_{sf}Cos\left(\beta_{jk}t-\phi_{sf}\right) \qquad (4.17)$

Where $$\beta_{jk}=\alpha_{jk}\left(1-\frac{\lambda^*\mu}{2w_\rho}\left(Q_0(t)-\frac{1}{Dh\alpha_{jk}^2}Q_1(t)\right)\right) \qquad (4.18)$$

represents the modified frequency due to the effect of rotatory inertia of the plate. It is observed that when $\lambda^*=0,$ we recover the frequency of the moving force problem when rotatory inertia or effect is neglected. In order to solve the non homogeneous equation (4.3), the differential operator which acts on $U(j,k,t)$ and $U(p,k,t)$ is replaced by the equivalent free system operator defined by the modified frequency β_{jk} i.e. Therefore, equation (4.3) becomes

$$U_{tt}(j,k,t)+\beta_{jk}^2U(j,k,t)=P_LSin\frac{k\pi y_1}{L_y}\left[Sin\alpha_{jf}t+A_jCos\alpha_{jf}t+B_jSinh\beta_{jf}t+C_jCosh\beta_{jf}t\right] \qquad (4.19)$$

$$\alpha_{jf}=\frac{\alpha_jc}{L_x}\ ,\ \beta_{jf}=\frac{\beta_jc}{L_x} \qquad (4.20)$$

solving equation (4.19) in conjunction with the transformed initial conditions and inverting yields

$$W(x,y,t)=\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{2}{L_y}\frac{\mu}{W_j}\left\{\frac{P_LV_k(y_1)}{2(\beta_{j,k}^2-\alpha_{jf}^2)}\left(Sin\,\alpha_{jf}t-\frac{\alpha_{jf}}{\beta_{j,k}}Sin\,\beta_{jf}t+A_jCos\,\alpha t-A_jCos\,\beta_{jk}t\right)\right\}$$

$$+\frac{P_LV_k(y_1)\beta_{jf}}{2(\beta_{j,k}(\beta_{jf}^2+\beta_{j,k}^2)}\left[\frac{B_{jf}\beta_{j,k}}{\beta_j}Sinh\beta_{jf}t-B_jSin\beta_{j,k}t+\frac{C_j\beta_{jk}}{\beta_j}Cosh\beta_jt-\frac{C_j\beta_{jk}}{\beta_j}Cos\beta_{jk}t\right]\Bigg\}Sin\frac{k\pi y_1}{L_y}\frac{Sin k\pi y}{L_y}W_j(x) \qquad (4.21)$$

Equation (4.21) above represents the transverse displacement response of a rectangular plate having arbitrary edge supports along edges $x = 0$ as $x = L_x$ and simply supported along edges $y = 0$ and $y = L_y$ and traversed by a moving force.

(b) Case II

In this case inertia term is retained and $\Gamma\neq 0$. This is termed moving mass problem. This requires the solution to the entire equation (4.1). As in case 1, and exact analytical solution to equation (4.1) does not exist and so we resort to the approximate analytical method discussed in case I to obtain the modified frequency corresponding to the frequency of the free system due to the presence of moving mass namely.

$$\omega_{jk}=\beta_{jk}\left[\frac{1+\left(S_a(j,k)+S_c(j,k)\right)}{2\beta_{jk}^2}\right] \qquad (4.22)$$

Thus equation (4.1) takes form

$$U_{tt}(j,k,t)+\omega_{jk}^2(j,k,t)=\lambda gLxLyV_k(y_1)\left[Sin\alpha_jt+A_jCos\alpha_jt+B_jSin\beta_jt+C_jCosh\beta_jt\right] \qquad (4.23)$$

Equation (4.23) is analogous to equation (4.21) and the solution after inversion becomes

$$W(x,y,t)=\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{2}{L_y}\frac{\mu}{W_j}\left\{\frac{\lambda\, gL_xL_yV_k(y_1)}{2\left(\delta_{jk}^2-\alpha_{jf}^2\right)}\left(Sin\alpha_{jf}t-\frac{\alpha_{jf}}{\omega_{jk}}Sin\omega_{jk}t+A_jCos\omega_{jk}t-A_jCos\alpha_{jf}t\right)+\right.$$

$$\left.\frac{\lambda\, gL_xL_y\beta_{jf}}{2\omega_{jk}\left(\beta_{jf}^2+\omega_{jk}^2\right)}\left[B_j\frac{\omega_{jk}}{\beta_{jf}}Sinh\beta_{jf}t-B_jSin\omega_{jk}t+C_j\frac{\omega_{jk}}{\beta_{jf}}Cosh\beta_{jf}t-C_j\frac{\omega_{jk}}{\beta_{jf}}Cos\omega_{jk}t\right]\right\}\bullet$$

$$Sin\frac{K\pi y_1}{L_y}Sin\frac{k\pi y}{L_y}W_j(x) \tag{4.24}$$

Equation (4.24) is the transverse displacement response under moving concentrated masses of our prestressed rectangular plate having simply supports along edges $y = 0$ and $y = L_y$ and arbitrary supports along edges $x = 0$ and $x = L_x$.

5.0 Applications

In this section, results of the analysis in this work are applied to our plate model by considering some examples of classical boundary conditions along the two opposite edges having arbitrary conditions

By way of illustrating the foregoing analysis, we consider a simple example of rectangular plate incorporating rotatory inertia correction factor having simple supports at all edges. Thus, along edges $x = 0$ and $x = L_x$ we have

$$W(o,y,t)=W(L_x,y,t)=0\,,\frac{\partial^2}{\partial x^2}W(x,o,t)=\frac{\partial^2}{\partial y^2}W(x,L_y,t)=0 \tag{5.1}$$

Hence for the normal modes $$W_j(0)=W_j(L_x)=0\,,\frac{\partial^2}{\partial x^2}W_j(0)=\frac{\partial^2}{\partial y^2}W_k(L_y)=0 \tag{5.2}$$

In view of (5.1) and (3.6), $A_j=0, \beta_j=0 \quad C_j=0$ and $\alpha_j = j\pi \quad W_j=\frac{\mu L_x}{2}$ (5.3)

In view of equations (5.3) equation (4.1) becomes

$$U_{tt}(j,k,t)+\alpha_{jk}^{*^2}U\ (j,k,t)=P_L^*Sin\frac{k\pi y}{L_y}Sin\frac{j\pi ct}{L_x} \tag{5.4}$$

where

$$\alpha_{jk}^{*^2}=\frac{\alpha_{jk}^2+\left(N_X\frac{j^2\pi^2}{L_x^2}+N_y\frac{k^2\pi^2}{L_y^2}\right)}{1+R_0\left(\frac{j^2\pi^2}{L_x^2}+\frac{k^2\pi^2}{L_y^2}\right)} \tag{5.5}$$

$$P_L^*=\frac{2\Gamma_\omega}{1+R_0\left(\frac{j^2\pi^2}{L_x^2}+\frac{k^2\pi^2}{L_y^2}\right)} \tag{5.6}$$

where $\Gamma_0=0$ for moving force problem of our model. Solving equation (5.4) and inverting yields

$$W(x,y,t)=\frac{4P_L^*V_k(y_1)}{L_xL_y}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{1}{\left(\alpha_{jk}^{*^2}-\alpha_{jf}^2\right)}\left[Sin\alpha_{jf}t-\frac{\alpha_{jf}}{\alpha_{jk}^*}Sin\alpha_{jk}^*t\right]Sin\frac{k\pi y}{L_y}Sin\frac{j\pi x}{L_x} \tag{5.7}$$

Using arguments similar to what we had in last section, the modified frequency for the moving mass problem is obtained as

$$\gamma_{jk}=\alpha_{jk}^*\left\{1-\Gamma_oSin^2\frac{k\pi y_1}{L_y}\left(1+\frac{2c^2\pi^2j^2}{\alpha_{jk}^*L_y^2}\right)\right\} \tag{5.8}$$

Thus, the moving mass problem takes the form

$$\frac{d^2}{dt^2}U\ (j,k,t)+\gamma_{jk}^2U\ (j,k,t)=\in L_xL_ygSin\frac{k\pi y_1}{L_y}\frac{Sinj\pi ct}{L_x} \tag{5.9}$$

which when solved and inverted gives

$$W(x,y,t)=\frac{\in L_xL_ygV_k}{\left(\gamma_{jk}^2-\alpha_{jf}^2\right)}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left[Sin\alpha_{jf}t-\frac{\alpha_{jf}}{\gamma_{jk}}Sin\gamma_{jk}t\right]\frac{Sink\pi y}{L_y}\frac{Sinj\pi x}{L_x} \tag{5.10}$$

Next, as second example, we consider a rectangular plate clamped at edges $x=0$, $x=l_x$ with simple supports at edges $y=0$, $y=l_y$, the boundary conditions at such opposite edges are

$$W(0,y,t)=W(l_x,y,t)=0\,,W(x,0,t)=0, W(x,l_y,t)=0 \tag{5.11}$$

$$\frac{\partial W(o,y,t)}{\partial x} = \frac{\partial W(l_x,y,t)}{\partial x} = 0, \ \frac{\partial^2 W(x,o,t)}{\partial y^2} = \frac{\partial^2 W(x,l_y,t)}{\partial y^2} = 0 \quad (5.12)$$

while for have the normal modes we have

$$W_j(o) = W_j(l_x) = 0, \ \frac{W_j(o)}{\partial x} = \frac{W_j(l_x)}{\partial x} = 0, \ \frac{\partial^2 W_k(o)}{\partial y^2} = \frac{\partial^2 W_k(l_x)}{\partial y^2} = 0 \quad (5.13)$$

Applications of equation (5.13) to equation (3.6) yields

$$A_j = \frac{\frac{\alpha_i}{\beta_j}\sinh\beta_j - \sin\alpha_j}{\cos\alpha_j - \cosh\beta_j}, B_j = -\frac{\alpha_j}{\beta_j}, \ A_j = -C_j \quad (5.14)$$

The frequency equation of the clamped edges

$$2 - 2\cos\alpha_j \cosh\beta_j + \left(\frac{\alpha_j}{\beta_j} - \frac{\beta_j}{\alpha_j}\right)\sin\alpha_j \sinh\beta_j = 0 \quad (5.15)$$

The corresponding general solution of the associated moving force and moving mass problems are obtained by substituting results in (5.14) into (4.21) and (4.22) respectively. Thus, solution for any chosen classical boundary conditions along x=0 and x=L_x requires only to obtain the corresponding constants A_j, B_j and C_j the proof of the convergence of solutions in both examples are similar to those in [1].

6.0 Discussion of the analytical solution

In an undamped system such as this, it is necessary to examine the phenomenon of resonance. Equation (5.7) clearly shows that the simply supported elastic rectangular plate traversed by a moving force will be in state of resonance whenever

$$\alpha_{jk}^* = \frac{j\pi c}{L_x} \quad (6.1)$$

While equation (5.10) shows that the plate under the action of a moving mass encounters a resonance effects at

$$\gamma_{jk} = \frac{j\pi c}{L_x} \text{ or } \gamma_{jk} = \alpha_{jk}^*\left\{1 - \Gamma_o Sin^2 \frac{k\pi y_1}{L_y}\left(1 + \frac{2c^2\pi^2 j^2}{\alpha_{jk}^* L_y^2}\right)\right\} \quad (6.2)$$

It is obvious from equation (6.1) and (6.2) that for the same natural frequency, the critical speed for the system of simply supported elastic rectangular plate incorporating rotatory inertia correction factor, and traversed by a moving force is greater than that traversed by a moving mass. Thus, resonance is reached earlier in the moving mass than in the moving force system.

Similarly, equation (4.21) and (4.24) show that the resonance conditions associated with the simple-clamped elastic rectangular plate traversed by moving force and moving mass are respectively.

$$\alpha_{e,j,k}^{sv} = \frac{j\pi c}{L_x} \text{ and } \beta_{p,j,k}^{sv} = \frac{j\pi c}{L_x} \quad (6.3)$$

Consequently,

$$\beta_{p,j,k}^{sv} = \alpha_{e,j,k}^{sv}\left[1 + \left(\frac{S_a(j,k) + S_c(j,k)}{2\alpha_{e,j,k}^{sv^2}}\right)\right] \quad (6.4)$$

Thus, from equation (6.3) and (6.4) it is evident that the same results and analysis similar to those of the simply supported plate are also obtained for simple-clamped plate.

7.0 Numerical result and discussion of results

In order to illustrate the analytical results, the rectangular plate length L_y = 0.914m, and height L_x = 0.457m the mass travels at the constant velocity 1.5m/s. Furthermore, E, y and λ are chosen to be 2.109 × $10^9 kg/m^2$, 0.4m and 0.2 respectively. The transverse deflection of the plate are calculated and plotted against time for both illustrative examples for values of rotatory inertia R_0, axial force along x-axis N_x axial force along y-axis N_y, foundation stiffness K.

Figure 7.1 displays the effect of Rotatory inertia R_0 on the transverse deflection of the simply supported plate for moving mass for fixed values of K, N_x and N_y (K=2 × 10^6N/m^3 and axial forces N_x =2 × 10^6N and N_y = 2.5 × 10^6 N). The graphs show that the response amplitude decreases as R_0 increases. The values of R_0 used are 10, 20 and 30.

Figure 7.2 depicts the transverse displacement response of the simply supported plate for moving mass for fixed values of N_x, N_y and R_0 for various values of foundation stiffness K. It is evident that as K increases, the response amplitude decreases.

Figure 7.3 shows the deflection profile of the simply supported plate under moving mass for fixed value of K, R_0 and N_y (K = 2 × 10^6 N/m^3, R_0 = 10 and N_y =2.5 × 10^6 N) for various values of axial force along x-axis N_x. The analyses show that as N_x increases, response amplitudes decreases.

Figure 7.4 display the deflection profile of the plate under moving mass for fixed values of K and N_x

(K=2 × 10^6 N/m^3 and N_x = 2 × 10^6 N) for various values of axial force along y-axis N_y. The analyses show that as N_y increases, response amplitude decreases.

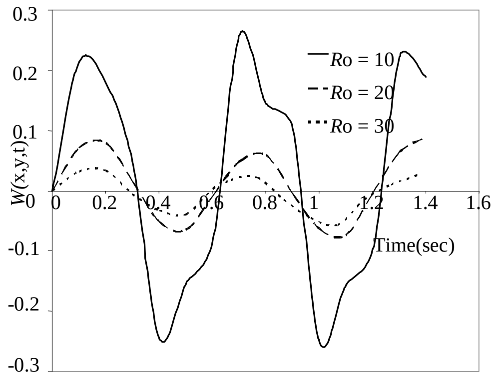

Figure 7.1: Deflection profile of simply-suppoted rectangular plate transversed by moving mass for fixed $Nx = 2$M, Ny = 2.5M, K = 2M for various values of Ro.

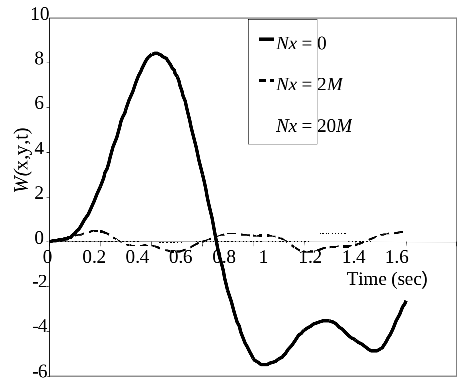

Figure 7.3: Deflection profile of simply-supported rectangular plate traversed by moving mass for fixed R_0 = 10, Nx = 2.5M, K = 2M for various values of Nx

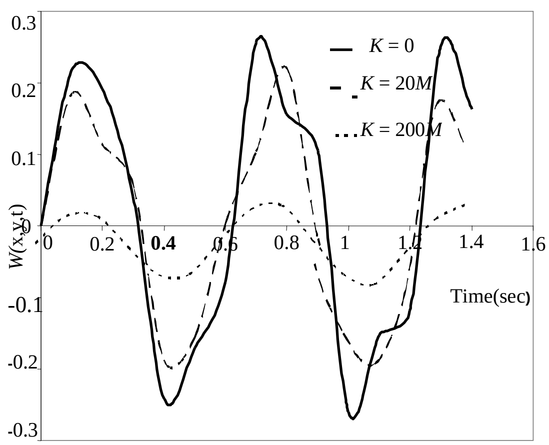

Figure 7.2: Deflection profile of simply-suppoted rectangular plate transversed by moving mass for fixed Nx = 2M, Ny = 2.5M, for various value of K

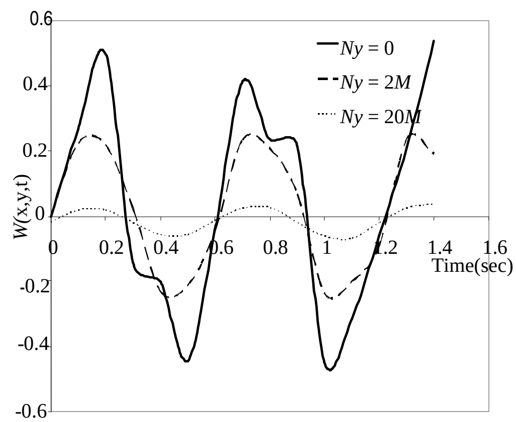

Figure 7.4: Deflection Profile of Simply-Suppoted Rectangular Plate Trasversed by Moving Mass for fixed Ro=10,Nx=2M,K=2M for various values of Ny

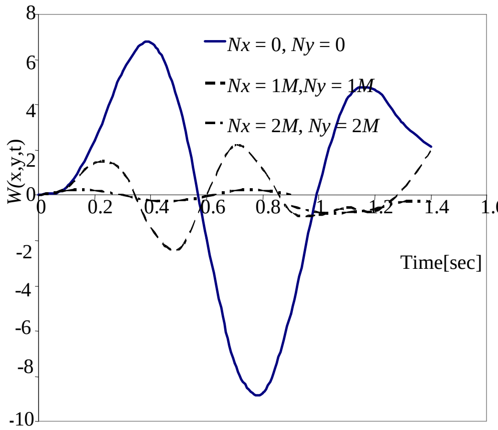

Figure 7.5: Deflection profile of simply-supported rectangular plate trasversed by moving Mass for fixed *R*o=10, *K* = 2*M* when values of *Nx* and *Ny* are increase simultaneously

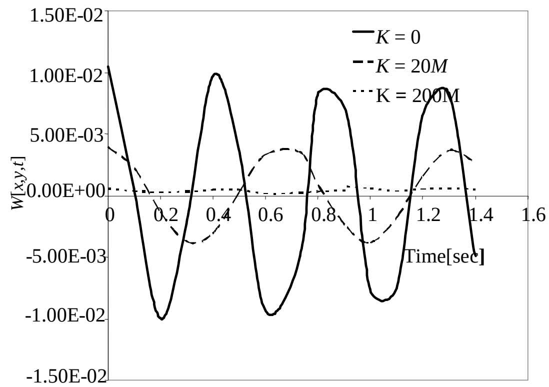

Figure 7.8: Deflection profile of simple-clamped rectangular plate traversed by moving mass for fixed *R*o = 10, *Nx* =2*M*, Ny = 2.5 for various values of *K*.

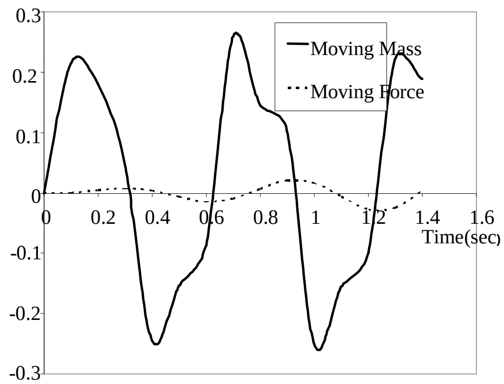

Figure 7.6: Comparison of the deflection of moving force and moving mass cases for simple supported rectangular plate for fixed *R*o = 10, *K* = 2M, *Nx* = 2M and *Ny* = 2.5*M*

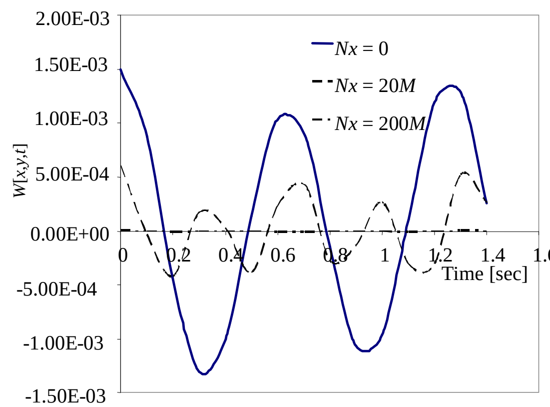

Figure 7.9: Deflection profile of simply-clamped rectangular plate traversed by Moving Mass for fixed *R*o = 10, *Ny* = 2.5, *K* = 2*M* for various values of *Nx*

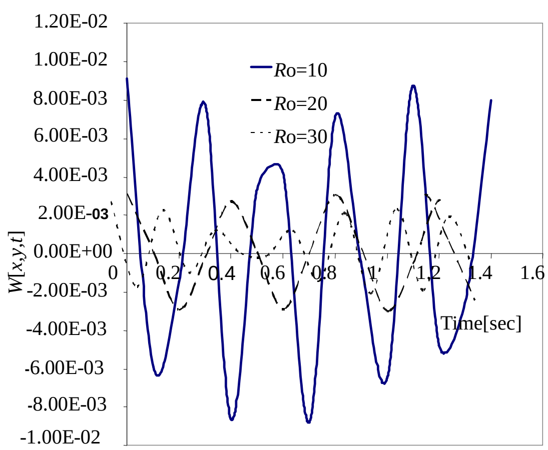

Figure 7.7: Deflection profile of simply-clamped rectangular plate trasversed by moving mass for fixed *Nx* = 2M, *Ny* = 2.5M, *K* = 2*M* for various values of *R*o

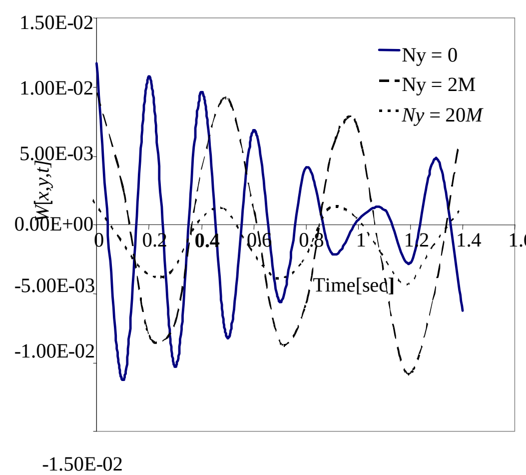

Figure 7.10: Deflection profile of simply-clamped rectangular plate traversed by moving mass for fixed *R*o = 10, *Nx* = 2*M*, *K* = 2*M* for various values of *Ny*

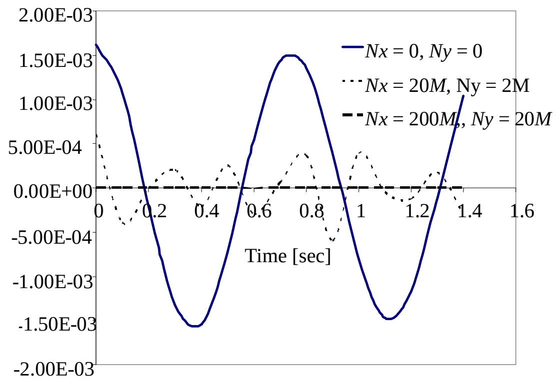

Figure 7.11: Deflection profile of simple-clamped rectangular plate traversed by moving mass for fixed *R*o=10 and *K*=2*M* when values of *Nx* and *Ny* are increase simultaneously.

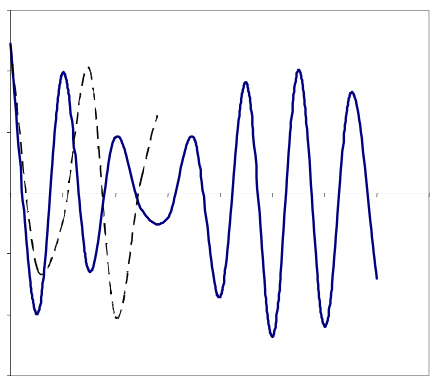

Figure 7.12: Comparison of the deflection of moving force and moving mass cases for simple-clamped plate for fixed *R*o =3 0, *Ny* = 2.5*M* and *Nx* = 2*M*

Figure 7.5 displays the deflection profile of the plate under moving load when N_x N_x and N_y are increase simultaneously while other parameters K and R_0 (K=2 ×10^6 N/m^3 and R_0=10) are fixed for moving mass of the rectangular plate. the response amplitude decreases as N_x and N_y are increased simultaneously. Also Figure 7.6 compares the displacement curves of the moving force and moving mass for the plate for fixed R_0=20, K=2 × 10^6 N/m^3, N_x = 2 × 10^6 N, and N_y =2.5 × 10^6 N. Obviously, the response amplitude of moving mass is greater than that of moving force problem. This result shows the moving force solution is not always an upper bound moving mass solution.

Figure 7.7 display the effect of Rotatory inertia R_0 on the transverse deflection of the simple-clamped plate under the action of moving mass for fixed values of K, N_x and N_x (K = 2 × 10^6 N/m^3, N_x = 2 × 10^6 N and N_y =2.5 ×10^6 N). The graph shows that the response amplitude decreases as the R_0 increases. The values of R_0 which are used are 10, 20 and 30.

Figure 7.8 depicts the transverse displacement response of the simple-clamped rectangular plate under the action of moving mass for fixed values of N_x, N_y and R_0 (N_x =2 × 10^6 N, N_y = 2.5 × 10^6 N and R_0 = 10) for various values of foundation stiffness K. The graph shows that as K increases, the response amplitude decreases. Figure 7.9 shows deflection profile of rectangular plate under the action of and moving mass for various values of axial force along x-axis, N_x and for fixed values of K, N_y and R_0 (K=2 ×10^6 N/m^3, N_y = 2.5 × 10^6N and R_0 = 10).It shows that higher values of axial force along x-axis, N_x reduce the deflection profiles of the plate in both cases

Figure 7.10 depicts the transverse deflection of simple-clamped plate under moving mass respectively for various values of N_y for fixed values K, R_0 and N_x(K = 2 × 10^6 N/m ^{3}and N_x = 2 × 10^6 N). As N_y increases, the maximum amplitude of the plate decreases.

Figure 7.11 displays the response amplitude when axial forces N_x and N_y are increased simultaneously for fixed values of K and R_0 (K = 2 × 10^6 N/m ^{3}and R_0 = 10) for simple-clamped rectangular plate under the action of moving mass. Evidently, the response amplitudes decrease when values of N_x and N_y are increases simultaneously. Finally, figure 7.12 depicts the comparison of transverse displacement of moving force and moving mass cases for simple-clamped rectangular plate traversed by a moving load for fixed values of R_0, N_y, N_x and K(R_0 = 10, N_y =2.5 × $10^6$$N$ and N_x = 2 × 10^6 N and K=2 × 10^6 N/m^3). Clearly, the response amplitude of moving mass is higher than that of the moving force. This important result show that relying on moving force solution as an approximation to moving mass solution is seriously misleading

8.0 Conclusion.

This paper presents transverse displacement of a prestressed rectangular plate incorporating rotatory inertia correction factor under concentrated masses. The solution technique is suitable for all variants of classical boundary conditions of practical interest. The effects of axial force, foundation moduli and rotatory inertia on transverse vibration of the plate are investigated. It is observed for both simply supported and simple-clamped rectangular plate that the critical speed for the system traversed by a moving mass is smaller than that traversed by a moving force for both simply supported and simple-clamped end conditions. The results show that as the axial force, foundation moduli and rotatory inertia increase, the response amplitudes of the dynamical system decrease for both illustrative examples. Finally, it is observed from all of the above results that the moving force solution is not an upper bound for the accurate solution. Consequently, it will be misleading to rely on the moving force solution as an approximation to the moving mass problem.

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