Journal of the Nigerian Association of Mathematical Physics
***Volume* 13 (November, 2008), 49 - 54**

Recursive backstepping design for controlling chaos of three nonlinear Bloch equations.

[1]F. Ayedun and [2]O. Sowole
Department of Physics,
Tai Solarin University of Education,
Ijebu–Ode, Nigeria.

Abstract

Recursive backstepping nonlinear control technique has become a powerful tool for controlling and synchronizing chaotic systems, because backstepping enhances global stability. The purpose of this study is to make use of recursive backstepping technique in controlling chaotic dynamics and attractors generated by dynamic states of three nonlinear modified Bloch systems described with different values of system constants and initial conditions. Numerical simulations are performed to verify that the three controllers achieve the control goals for the three dynamic states equations.

***Keywords*:** Chaos control; Nonlinear Bloch Equations; recursive backstepping control.

1.0 Introduction

In chaos theory, a control of chaos is based on the fact that any chaotic attractor contains an infinite number of unstable periodic orbits. Chaotic dynamic consists in a motion where the system state moves in the neighbourhood of one of these orbits for a while, then falls close to a different unstable periodic orbit where it remains for a limited time, and so forth. This results in a complicated and unpredictable wandering over longer periods of time.

Control of chaos and bifurcation is concerned with using some designed control input(s) to modify the characteristics of a parameterized nonlinear system. The control can be static or dynamic feedback control or open – loop control. The result is to render an otherwise chaotic motion more stable and predictable which is often an advantage (Kapitaniak, 1996 [5]; Chen et al, 1998 [2]).

Several techniques have been devised for chaos control, but most are development of two basic approaches: the OGY (Ott, Grebogi and Yorke) method (Ott, et al, 1990 [10]), closed loop feedback method, and Pyragas continuous control (Pyragas, 1992 [11]). Of recent, there have been various approaches both linear and nonlinear methods have emerged. Specifically, recursive backstepping nonlinear control scheme has been employed recently for tracking, controlling and synchronizing chaotic systems (Harb, 2004; Ge, 2000, [4]; Zhang et al, 2005; Kotkotovic, 1992 [6]; Kristic et al, 1995 [7]). Recursive backstepping is a systematic design approach and consists in a recursive procedure that skilfully interlaces the choice of a Lyapunov function with the control.

2.0 The model

Motivated by the need to interpret different anomalies that had been observed in nuclear magnetic resonance (NMR) experiments, in terms of chaos theory, Abergel (Abergel, 2002 [1]) investigated the possibility of observing chaotic solutions of the Bloch equations which has been modified to account for the presence of back action from the probe. The model consists of three nonlinear modified Bloch equations (NBE) and includes a feedback field. This is given in dimensionless unit as

$$\dot{x} = \delta y - \eta_2 yz + \gamma z\,(x \sin\psi - y\cos\psi) - \frac{x}{\tau_2},$$
$$\dot{y} = -\delta x - z + (\eta_2 - \eta_1)\,xz + \gamma z\,(y\sin\psi + x\cos\psi) - \frac{y}{\tau_2},. \qquad (2.1)$$
$$\dot{z} = y + \eta_1 xy - \gamma\left(x^2 + y^2\right)\sin\psi - \frac{z-1}{\tau_1}.$$

[1]e-mail: tofayo 2002 @ yahoo.com, [2]e-mail: segunsowole @ yahoo.com

where the dots denotes time derivatives, δ, γ and ψ are the system parameters τ_1 and τ_2 are longitudinal time and transverse relaxation time respectively. The dynamics of this system has been extensively studied by Abergel (2002 [1]) and Ucar et al (2007 [12]) for a fixed subset of the system parameters $(\delta, \gamma, \tau_1, \tau_2)$ and for a space range of the radiation damping. The feedback ψ that would admit chaotic solutions were obtained. System (2.1) exhibits chaotic behaviour for the following system parameters:

$$\delta = -0.4\pi, \quad \gamma = 30, \quad \psi = 0.173, \quad \tau_1 = 5 \text{ and } \tau_2 = 2.5$$

as shown in Figure 2.1.

Besides the study of the chaotic behaviour, Ucar et al (2007) also presented the synchronization of the NBE using the active control method. The synchronization was study in a master–slave configuration. In another development, Moukam et al (2006 [9]) studied the chaotic behaviour and chaos synchronization of a bi-axial magnet modeled by Bloch equations. In all these reports, the control of the NBE chaotic behaviour to regular dynamics has not been addressed. In this present study, we set up a new chaos control scheme which we have recently developed for the NBE.

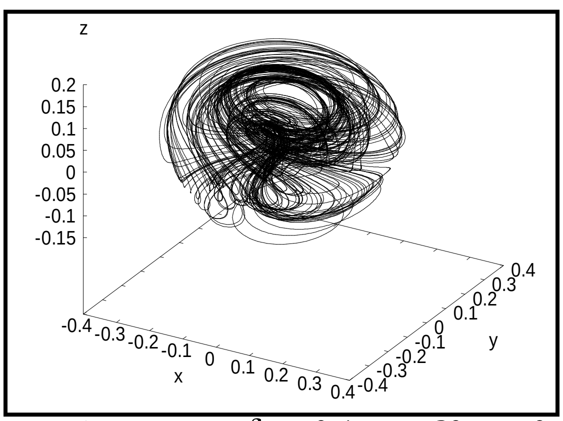


Figure 2.1: Chaotic attractor for the NBE for the parameters $\delta = -0.4\pi, \gamma = 30, \psi = 0.173, \tau_1 = 5, \tau_2 = 2.5$ and the initial conditions ($x(0), y(0), z(0) = (0.2, -0.2, 0)$)

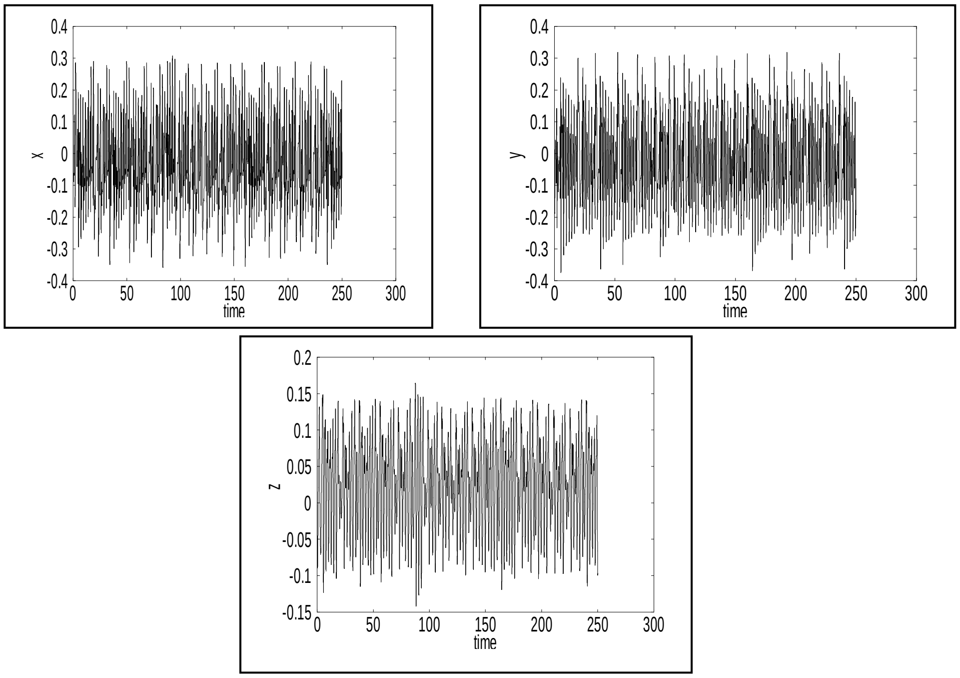


Figure 2.2: Corresponding time series for the state variables of the NBE for the same parameter set as in figure 2.1 when control is de–activated.

3.0. Backstepping design for chaos control in the NBE

To control the chaotic behaviour of the NBE depicted by the chaotic attractor of system (2.1), we introduce the control functions $u_i\,(i=1,2,3)$ as follows:

$$\begin{aligned}\dot{x} &= \delta y-\eta_2 yz+\gamma z(x\sin\psi-y\cos\psi)-\frac{x}{\tau_2}+u_1,\\ \dot{y} &= -\delta x-z+(\eta_2-\eta_1)xz+\gamma z(y\sin\psi+x\cos\psi)-\frac{y}{\tau_2}+u_2,\\ \dot{z} &= y+\eta_1 xy-\gamma\left(x^2+y^2\right)\sin\psi-\frac{z-1}{\tau_1}+u_3.\end{aligned} \qquad (3.1)$$

where $u_i\,(i=1,2,3)$ are the control functions to be determined.

To design the control functions $u_i\,(i=1,2,3)$ that will suppress the chaotic behaviour, the error signals are defined as follows:

$$\begin{aligned}e_1&=x-x_d\\ e_2&=y-y_d\\ e_3&=z-z_d\end{aligned} \qquad (3.2)$$

where x_d , y_d and z_d are desired states of the state variables x, y and z.

For simplicity, let

$$x_d = 0,$$
$$y_d = c_1 e_1 \tag{3.3}$$
$$z_d = c_2 e_1 + c_3 e_2$$

where the $c_i^{'} s \, (i = 1,2,3)$ are arbitrary chosen parameters; x_d is the reference output; y_d and z_d are recursively introduced control inputs.

Now differentiating equations (3.2) and (3.3) with time we have:

$$\dot{e}_1 = \dot{x} - \dot{x}_d$$
$$\dot{e}_2 = \dot{y} - \dot{y}_d \tag{3.4}$$
$$\dot{e}_3 = \dot{z} - \dot{z}_d$$

and

$$\dot{x}_d = 0,$$
$$\dot{y}_d = c_1 \dot{e}_1, \tag{3.5}$$
$$\dot{z}_d = c_2 \dot{e}_1 + c_3 \dot{e}_2,$$

Substituting equation (3,1) into equation (3.4) and using the error signals definition (3.2), we get the following error dynamic systems:

$$\dot{e}_1 = \delta(e_2 + c_1 e_1) - \eta_2 (e_2 + c_1 e_1)(e_3 + c_2 e_1 + c_3 e_2)$$
$$+ \gamma(e_3 + c_2 e_1 + c_3 e_2)[e_1 \sin\psi - (e_2 + c_1 e_1)\cos\psi] - \frac{e_1}{\tau_2} + u_1,$$
$$\dot{e}_2 = -\delta e_1 - (e_3 + c_2 e_1 + c_3 e_2) + (\eta_2 - \eta_1) e_1 (e_3 + c_2 e_1 + c_3 e_2) +$$
$$\gamma(e_3 + c_2 e_1 + c_3 e_2)[(e_2 + c_1 e_1)\sin\psi + e_1 \cos\psi] - \frac{e_2 + c_1 e_1}{\tau_2}$$

$$-c_1 \left\{ \begin{matrix} \delta(e_2 + c_1 e_1) - \eta_2 (e_2 + c_1 e_1)(e_3 + c_2 e_1 + c_3 e_2) \\ + \gamma(e_3 + c_2 e_1 + c_3 e_2)[e_1 \sin\psi - (e_2 + c_1 e_1)\cos\psi] - \dfrac{e_1}{\tau_2} + u_2 \end{matrix} \right\},$$

$$\dot{e}_3 = e_2 + c_1 e_1 + \eta_1 e_1 (e_2 + c_1 e_1) - \gamma\left(e_1^{\,2} + (e_2 + c_1 e_1)^2\right)\sin\psi - \frac{e_3 + c_2 e_1 + c_3 e_2 - 1}{\tau_1}$$
$$-c_2 \left[\begin{matrix} \delta(e_2 + c_1 e_1) - \eta_2 (e_2 + c_1 e_1)(e_3 + c_2 e_1 + c_3 e_2) \\ + \gamma(e_3 + c_2 e_1 + c_3 e_2)[e_1 \sin\psi - (e_2 + c_1 e_1)\cos\psi] - \dfrac{e_1}{\tau_2} \end{matrix} \right]$$
$$-c_3[-\delta e_1 - (e_3 + c_2 e_1 + c_3 e_2) + (\eta_2 - \eta_1) e_1 (e_3 + c_2 e_1 + c_3 e_2) + \tag{3.6}$$
$$\gamma(e_3 + c_2 e_1 + c_3 e_2)[(e_2 + c_1 e_1)\sin\psi + e_1 \cos\psi] - \frac{e_2 + c_1 e_1}{\tau_2}$$
$$-c_1 \left\{ \begin{matrix} \delta(e_2 + c_1 e_1) - \eta_2 (e_2 + c_1 e_1)(e_3 + c_2 e_1 + c_3 e_2) \\ + \gamma(e_3 + c_2 e_1 + c_3 e_2)[e_1 \sin\psi - (e_2 + c_1 e_1)\cos\psi] - \dfrac{e_1}{\tau_2} \end{matrix} \right\}] + u_3$$

Consider the Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{3} K_i e_i^{\,2} = K_1 e_1 + K_2 e_2 + K_3 e_3 \tag{3.7}$$

Its time derivative along the error dynamic system (3.6) is:

$$\dot{V} = \sum_{i=1}^{3} K_i e_i \dot{e}_i = K_1 e_1 \dot{e}_1 + K_2 e_2 \dot{e}_2 + K_3 e_3 \dot{e}_3 \tag{3.8}$$

Here, we fix $c_1 = c_2 = c_3 = 0$ and $K_1 = K_2 = K_3 = 1$.

Substitution of equation (3.6) into equation (3.8) gives:

$$\begin{aligned} \dot{V} = {} & e_1[\delta e_2 - \eta_2 e_2 e_3 + \gamma e_3 (e_1 \sin\psi - e_2 \cos\psi) - \frac{e_1}{\tau_2} + u_1] \\ & + e_2[-\delta e_1 - e_3 + (\eta_2 - \eta_1) e_1 e_3 + \gamma e_3 (e_2 \sin\psi + e_1 \cos\psi) - \frac{e_2}{\tau_2} + u_2] \\ & + e_3\left[e_2 + \eta_1 e_1 e_2 - \gamma\left(e_1^{\,2} + e_2^{\,2}\right)\sin\psi - \frac{e_3 - 1}{\tau_1} + u_3\right] \end{aligned} \tag{3.9}$$

Suppose;

$$\begin{aligned} & \delta e_2 - \eta_2 e_2 e_3 + \gamma e_3 (e_1 \sin\psi - e_2 \cos\psi) - \frac{e_1}{\tau_2} + u_1 = -e_1 \\ & -\delta e_1 - e_3 + (\eta_2 - \eta_1) e_1 e_3 + \gamma e_3 (e_2 \sin\psi + e_1 \cos\psi) - \frac{e_2}{\tau_2} + u_2 = -e_2 \\ & e_2 + \eta_1 e_1 e_2 - \gamma\left(e_1^{\,2} + e_2^{\,2}\right)\sin\psi - \frac{e_3 - 1}{\tau_1} + u_3 = -e_3. \end{aligned} \tag{3.10}$$

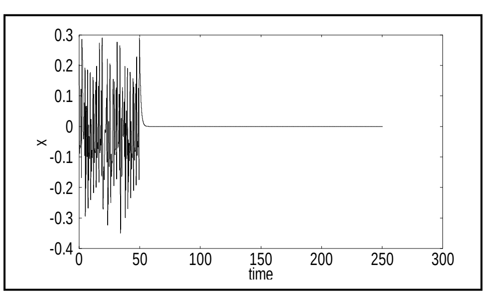
then

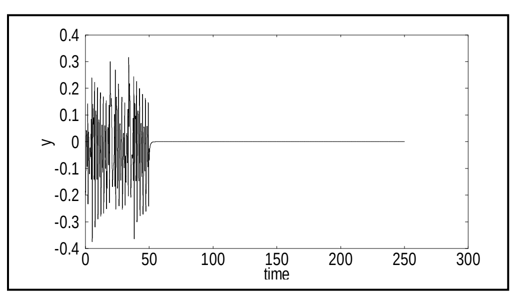
$$\dot{V} = -e_1^{\,2} - e_2^{\,2} - e_3^{\,2} = -\left(e_1^{\,2} + e_2^{\,2} + e_3^{\,2}\right) \tag{3.11}$$

hence $\dot{V}$ is negative definite, and thus satisfying the Lasalle–Yoshizawa stability criteria (Lasalle, 1968 [8]; Yoshizawa, 1996 [14]).

Thus, from equation (3.10), the control functions are:

$$\begin{aligned} u_1 &= -e_1 - \delta e_2 + \eta_2 e_2 e_3 - \gamma e_3 (e_1 \sin\psi - e_2 \cos\psi) + \frac{e_1}{\tau_2} \\ u_2 &= -e_2 + \delta e_1 + e_3 - (\eta_2 - \eta_1) e_1 e_3 - \gamma e_3 (e_2 \sin\psi + e_1 \cos\psi) + \frac{e_2}{\tau_2} \\ u_3 &= -e_3 - e_2 - \eta_1 e_1 e_2 + \gamma\left(e_1^{\,2} + e_2^{\,2}\right)\sin\psi + \frac{e_3 - 1}{\tau_1} \end{aligned} \tag{3.12}$$





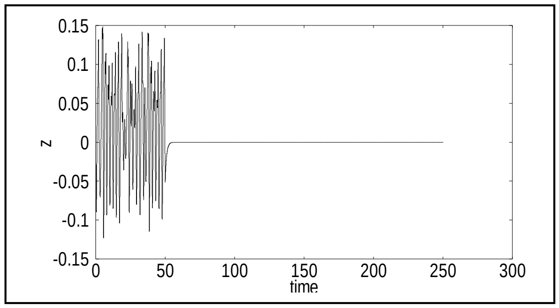


Figure 3.1: Corresponding time series for the state variables of the NBE when control is given by equation (13) is activated at time (t) = 50s

4.0 Conclusion

In this paper, we used three control signals simultaneously in suppressing the chaotic dynamic states of nonlinear Bloch equations studied by Ucar et al [13] by applying recursive backstepping technique. The proposed recursive backstepping is easy to implement. Numerical simulations have been employed to confirm our results.

References

[1] Abergel, D. (2002) Chaotic solutions of the feedback driven Bloch equations. Phys. Lett.A 302 **pg** 17.

[2] Chen, G and Dong X (1998). From Chaos to Order: Methodologies, Perspectives and Applications. (World Scientific Singapore).

[3] Ge, S.S; Wang, C and Lee T.H. (2000). International Journal. Bifurcation and Chaos 10 **pg** 1149.

[4] Harb, A. M. (2004). Nonlinear chaos control in a permanent magnet reluctance machine. Chaos, Solitons and Fractals 19 **pg** 1217.

[5] Kapitaniak, T. (1996). Controlling Chaos: Theoretical and practical methods in nonlinear dynamics. (Academic Press, London).

[6] Kokotovic, P.V. (1992). IEEE Control System. Mag. 6 **pg** 7 **.**

[7] Kristic, M; Kanellakopouus,I and Kokotovic, P.V.(1995). Nonlinear and Adaptive Control (John Wiley and Sons Incorporation).

[8] Lasallel, J.P. (1968). Stability theory for ordinary differential equations. J. Diff. Equations, 4 **pg** 51.

[9] Moukam- Kakmeni, F.M; Nguenang, J. P. and Kofane, T.C. (2006). Chaos synchronization in bi– axial magnets modeled by Bloch equation. Chaos, Solitons and Fractals 30 **pg** 690.

[10] Ott, E., Grebogi, C., and Yorke, (1990). J.A. Controlling Chaos, Phys. Rev Lett.64 **pg** 1164.

[11] Pyragas, K. (1992). Controlling chaos using continuous control. Phys. Lett.A **170 pg** 421.

[12] Ucar, A., Lonngren, K.E. and Bai, E. W. (2007). Synchronization of chaos in RCL Josephson junction using active control. Chaos, Solitons & Fractals 31 **pg** 105.

[13] Ucar, A., Lonngren, K.E. and Bai, E. W. (2003). Synchronization of chaotic behaviour in nonlinear Bloch equations. Phys.Lett. A. 314 **pg** 96.

[14] Yoshizawa, T. (1996). Stability theory by Lyapunov's second method. The Mathematical Society of Japan.