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Precision measurement of the electron/muon gyromagnetic factors

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Abstract

Clear, persuasive arguments are brought forward to motivate the need for highly precise measurements of the electron/ muon orbital g, i.e. g_L. First, we briefly review results obtained using an extended Dirac equation, which conclusively showed that, as a consequence of quantum relativistic correction arising from the time-dependence of the rest-energy, the electron gyromagnetic factors are corrected. It is next demonstrated, using the data of Kusch and Foley on the measurement of (δ_S - $2\delta_L$) together with the modern precise measurements of the electron δ_S($\delta_S \equiv g_S - 2$), that δ_L may be small (-0.6×10^{-4}), non-zero quantity, where we have assumed Russel-Saunders (LS) coupling and proposed, along with Kusch and Foley, that $g_S = 2 + \delta_S$ and $g_L = 1 + \delta_L$. Therefore, there is probable evidence from experimental data that g_L is not exactly equal to 1: the expectation that quantum effects will significantly modify the classical value of the orbital g is therefore reasonable. Finally, we show that if, as suggested by the results obtained from the modified Dirac theory, δ_s and δ_L depend linearly on a dimensionless parameter Δ such that the gyromagnetic factors are considered corrected as follows; $g_S = 2(1 + \Delta)$ and $g_L = 1 - \Delta$, then the Kusch-Foley data implies that the correction $\Delta \approx 1.0 \times 10^{-3}$; it is noteworthy that Δ is of the same order of magnitude as the measured $(g_S - 2)/2$ which, to five places of decimal, is equal to 1.12×10^{-3}. Thus, available spectroscopic data indicate that g_S and g_L may both be significantly modified, such that g_S that is increased by 2Δ, while g_L is decreased by Δ, the quantity 2Δ being equal to the precisely measured $g_S - 2$. Modern, high precision measurements of the electron and muon orbital g_L are therefore required, in order to properly determine by experiments the true value of $g_S - 1$, perhaps to about one part in a trillion as was recently done for g_S.- 2.

1.0 Introduction.

In search of new physics beyond the standard model, QED has been stringently tested to extremes of precision especially in the measurement of the Lamb shift and the anomalous magnetic moment of spin-½ particles. Measurements have been carried out on single "isolated" electrons [1, 2, 3], atomic (bound) electrons [4, 5], 'free" (unbound) electrons [6, 7], and relativistic muons [8,9]. So far QED has brilliantly survived all the tests, except for some deviations from experiments in the value of the intrinsic magnetic moment of muon [10, 11], which may perhaps be attributed to the influence of interactions other than the electromagnetic. Nevertheless, there is hope that further tests will eventually be fruitful, and that new physics will become revealed.

Departing from the traditional, high-precision measurement of the Lamb shift or the intrinsic g factor, it is hereby proposed that a qualitatively different kind of measurement (i.e., of the orbital *g*), be considered. There are reasons, both experimental and theoretical, to suggest that the orbital *g*, when measured to very high levels of precision, will be found not to be exactly equal to 1. Thus, the new question to be addressed by experiment may succinctly be put as follows; if we assume that $g_L = 1 + \delta_L$ is $|\delta_L| > 0$?

A preliminary answer to this question comes from the classic measurement by Kusch and Foley [12], of the quantity $(\delta_S - 2\delta_L) \neq 0$ considered together with the recent measurements of δ_S [1, 2, 3]. It will be shown in what follows that using experimental evidence presently available, it cannot be asserted with certainty that (on empirical grounds) $\delta_L = 0$. It is indeed highly probable that $\delta_L \neq 0$ that is, a measurable, small but finite quantity δ_L exists, such that the orbital magnetic moment of the electron differs from the classical value $g_L = 1$.

Quite apart from the above-mentioned empirical evidence, there are also theoretical reasons to expect that the orbital gyro-magnetic factor g_L when precisely measured will differ from exact unity. The electron, unlike the proton or neutron, is a point particle which, however, interacts with its own radiation field as described by the wonderfully successful QED. One of the results of this self-interaction and its interaction with the QED vacuum is a correction of the electron charge and mass which had hitherto appeared in electron theories as measurable, fixed parameters. A consequence of these interactions is the correction of the spin magnetic moment of the electron by a factor which has been astonishingly confirmed by subsequent precision measurements [1-7,12]. By considering the consequences of time-dependent correction to the rest-energy in the Dirac electron theory, it has also been shown that g_L are both subject to significant modifications; hence a non-zero g_L does not necessarily imply that the electron has a substructure.

Here we shall briefly review first the corrections δ_S and δ_L to the gyromagnetic factors, produced as a result of the interaction between an electron and a weak magnetic field, as described by an extended Dirac equation, and then follow the discussion with the result obtainable when the classic Kusch-Foley experimental measurement of $(\delta_S - 2\delta_L)$ is combined with the precise measurement of δ_S, which has since become available. Finally, conclusions are drawn regarding the significance of the various experimental and theoretical observation, and a case is made for new, high precision, direct measurements of δ_L and/or$(\delta_S - 2\delta_L)$ using modern experimental techniques, procedures and apparatus.

2.0 Quantum correction and the gyro-magnetic factors

The rest-energy of a relativistic electron is neither a conserved quantity nor a constant of the motion. From these and some other fundamental physical considerations, it has been shown that the rest-energy has time-dependence [13] $\quad m(t) = m - m\exp(2i\alpha.pt/\hbar);\ c = 1 \quad$ (2.1)
where m is the rest mass, $\boldsymbol{\alpha}$ is the Dirac matrix-vector, $\boldsymbol{p}$ is the momentum and t is the coordinate time.

Let it be accepted as a testable hypothesis, that the known deficiencies of the Dirac theory may be corrected by substituting for the constant rest-energy term, the time-dependent rest-energy function given above. The resulting time-dependent equation [14] is,

$$i\eta \frac{\partial \psi}{\partial t} = \left[c\alpha\ .\ p + \beta mc^2 - \beta mc^2\ \exp\left(2i\alpha\ .\ pct/\eta\right)\right]\psi \qquad (2.2)$$

which allows a consistent formal interpretation of the unusual properties of the Dirac relativistic electron theory.

Now, if the electron interacts with a slowly varying magnetic field B, the Hamiltonian in the above equation, when written in a Schroedinger-type, time-independent form [15], is

$$H = c\alpha\ .\ \pi + \beta mc^2 - \frac{i\beta mc(\eta\varepsilon)(\alpha\ .\ \pi)}{4|p|^2} \qquad (2.3)$$

where the exponential term in (2.2) has been re-expressed in the limit $t \longrightarrow \infty$. By taking the non-relativistic limit of the eigenvalue equation of the above Hamiltonian, an extended version of the Pauli equation is obtained, which may be expressed as [16],

$$E\psi_A = \left[\frac{\pi^2}{2m} - \frac{\pi^2\eta^2 m^2\varepsilon^2}{2m\ 4^2\ \ p^4} - \frac{e\eta}{2mc}\left(1 + \frac{\eta^2 m^2\varepsilon^2}{4^2\ p^4}\right)\sigma\ .\ B\right]\psi_A \qquad (2.4)$$

where in term $(\hbar^2 m^2\varepsilon^2/4^2 p^2)$, ε and p, are respectively, the frequency and momentum of the "cyclical" photon associated with the Dirac particle [14]. The second and fourth terms are new; they have arisen as a consequence of the time-dependent rest-energy term in equation (2.2). We can interpret the terms by considering the energy of a magnetic moment of a dipole in a weak, statistic magnetic field B, i.e.,

$$H_{m,S} = -\mu_S.B = \frac{eg_S}{2mc}S.B;\ \ H_{m,L} = \mu_L.B = \frac{eg_L}{2mc}L.B \qquad (2.5)$$

from which we deduce that the electron gyromagnetic factor have new values given as follows:

$$g_S = 2(1 + \Delta);\ g_L = 1 - \Delta \qquad (2.6)$$

where Δ is the correction to the g-factors due to the addition to the rest energy term in the Dirac equation. The quantity Δ is the numerical value of the term $(\hbar^2 m^2\varepsilon^2/4^2 p^4)$, contained in equation (2.4). On evaluation, it is found that $\Delta \approx 1.6 \times 10^{-3}$ which, to four places of decimal, is nearly equal to the measured anomalous part of the intrinsic g-factor, i.e., $a_e = 1.2 \times 10^{-3}$. If we adopt the suggestion by Kusch and Foley, sequel to their measurements, that the gyromagnetic factors g_s and g_L may be corrected as follows [12];

$$g_S = 2 + \delta_S;\ g_L = 1 + \delta_L \qquad (2.7)$$

then from (2.6) above, $\delta_S = 2\Delta$ and $\delta_L = -\Delta$. It is necessary however, to note that there are other possible form

s which the corrections to the g-factors can take. For example, it is not impossible that $\delta_S \neq 0$ while $\delta_L = 0$ and vice-versa. Nevertheless, all the various possibilities can be expressed as

$$g_S = 2(1 + \Delta);\ g_L = 1 + \lambda\Delta \qquad (2.8)$$

where λ is a real number, having taken into considerations the fact that it has been experimentally and theoretically shown that $g_S = 2(1 + \Delta)$ where, to four places of decimal, $\Delta = 1.2 \times 10^{-3}$. Thus the choice $\lambda = 1$ refers to the expression in equation (2.6) while $\lambda = 0$ implies that $g_L = 1$; the number λ can take any real values other than 0 or 1.

We shall, in the following sections, ascertain the compatibility of the above suggestions with available spectroscopic data and draw significant conclusions regarding the true (or probable) value of the orbital g.

3.0 Atomic spectroscopy experiments on bound electrons.

We shall now consider in greater detail, the classic Kusch-Foley experiment [12] in which the quantity $(\delta_S - 2\delta_L)$ is determined from the study of the Zeeman splitting of two different atomic states which allowed the calculation of the ratio of the g_s factors corresponding to the states. It will be shown that in the light of modern, highly-precise measurement of δ_S, the Kusch-Foley data suggest that δ_L may not be exactly zero.

Using the atomic beam magnetic resonance technique, frequencies associated with Zeeman splitting of the $^2P½$ and $^2P_{3/2}$ states of Ga were measured by Kusch and Foley, in order to determine the g_s values corresponding to the states. Since each of these states in Ga may be separately subject to configuration interaction perturbations, Kusch and Foley suggested that the interpretation of the result may be rendered nuclear, thus making necessary a new determination of the ratio of the g_s value of Na in the $^2S½$ state to that of Ga $^2P½$ state. From any experiment in which the ratio of the g_j values of the states is determined, it is possible to determine only the quantity $(\delta_S - 2\delta_L)$ if LS coupling is assumed. In order to determine either δ_S or δ_L, it was therefore necessary for Kusch and Foley to make certain assumptions concerning the nature of δ_S and δ_L, some of which had no obvious experimental basis or support.

A determination of the ratio of g_j from the $^2P_{3/2}$ and the $^2P½$ states of Ga gave a value $g_{3/2}/g_{½} = 2.00344 \pm 0.00012 = 2 + A$, different from the expected value of 2. The discrepancy, if the states are assumed correctly described by Russel-Saunders (LS) coupling, can be attributed to a change, from their accepted values, in the electron intrinsic moment and/or the orbital moment. Kusch and Foley reasoned that if the intrinsic g is changed to $g_S = 2 + \delta_S$ and the orbital g becomes $g_L = 1 + \delta_L$, then $A = (3/2)\delta_S - 3\delta_L$ or $\delta_S - 2\delta_L = 0.00229 \pm 0.00008$. Since this measurement, as earlier remarked, did not permit an independent evaluation of δ_S and δ_L, an attempt was made to account for the discrepancy between the expected and measured values of g_S and g_L by either, (a) setting $\delta_L = 0$ and hence having $\delta_S = 0.00229 \pm 0.00008$, , such that $g_S = 2.00229 \pm 0.00008$ and $g_L = 1$, or (b) setting $\delta_S = 0$ and thus putting $\delta_L = 0.00114 \pm 0.00004$, such that $g_S = 2$ and $g_L = 0.99886 \pm 0.00004$.

In a subsequent version of the experiment, the determination of the ratio of the g_j values of Na $^2S_{½}$ and of Ga $^2P_{½}$ states gave the value $Nag_{½} / Gag_{½} = 3.00732 \pm 0.00018$, which differed significantly from the expected value of 3, thereby making $\delta_S - 2\delta_L = 0.00244 \pm 0.00006$. In order to find δ_S, it was therefore necessary for Kusch and Foley to reason that "if on the basis of the correspondence principle, we set δ_L equal to zero", then from the first experimental result, $g_S = 2.00229 \pm 0.00008$ and from the second experiment, $g_S = 2.00244 \pm 0.00006$.

When compared, these experimentally measured values of g_S are roughly in good agreement with the value theoretically calculated (to first order in the structure constant α) using QED, i.e., $g_S = 2(1+\alpha/2\pi)_e^{QED}$ 2.00232. Let us, however, reconsider the Kusch-Foley measurements in the light of modern experiments. Using different techniques, more precise values of δ_S have since been measured, which focused exclusively on the spin g-factor g_S; thus we have now have independent values of δ_S and can therefore determine δ_L within the limits of the accuracy of the earlier experiments, without arbitrary setting δ_L equal to zero. Recent measurements give the anomalous part of the intrinsic gyro-magnetic factor as [3, 19],

$$a_e \equiv ((g_S - 2)/2)_e = (1159652.4 \pm) \times 10^{-9} \qquad (3.1)$$

which implies that, to five places of decimal, $\delta_S = 2a_e = 0.00232$ exactly. Thus by substituting this value for δ_S in the measured (δ_S - $2\delta_L$) we can evaluate the value of δ_L.

We shall consider two special cases in which (a) the correction δ_S and δ_L are assumed independent of one another, and (b) the quantities δ_S and δ_L are assumed dependent, and related via a parameter.

3.1 Independent Correction Factors δ_S and δ_L

Assuming in consonance with Kusch and Foley, that their experiments measured (δ_S - $2\delta_L$ for the electron considered essentially free from bound state effects, then substituting $\delta_S = 0.00232$ into this expression we find that $\delta_L = 0.00002 \pm 0.00004$ or $\delta_L = (0.2 \pm 0.4) \times 10^{-4}$, from the first measurement. Thus it

is observed that δ_L could lie between -0.2×10^{-4} and $+0.6 \times 10^{-4}$. Noting the relative magnitude of the quantity and that of its error, it does not appear feasible from this result, to assert with certainty that δ_L.= 0.

Likewise, substituting $\delta_S = 0.00232$ into the second measurement gives $\delta_L = -0.00006 \pm 0.00003$ which may similarly be re-expressed as $\delta_L = (-0.6 \pm 0.3) \times 10^{-4}$. Here we observe that, though close to edge of precision, the data shows that δ_L is a probable, small but finite quantity which, with modern techniques, may be precisely measured with greater assurance. It is therefore not unreasonable to expect that g_L may deviate from the classical value 1.0.

We remark, nevertheless, that it can indeed be rationally argued that when δ_L is set equal to zero, the difference $\xi = \delta_S - \alpha_{SL}$ between the precise δ_S and the measured α_{SL} is in fact the residual bound-state effects on the electron in a complex atom, which have herein been equated to $2\delta_L$. Arguments of this sort however, leave unexplained the origin of the bound-state effects which make ξ in the $^2P_{1/2}$ of Na^{23} as much as four times its magnitude in the $^2P_{3/2}$ of Ga^{69}. Moreover, we note that residual bound-state effects are typically much smaller than δ_L by several orders of magnitude. The electron g factor, which includes the Dirac value g_D, is given as follows [20]:

$$g = g_D + \Delta g_{QED} + \Delta g_{int} + \Delta g_{nuc} + \Delta g_{SQED} \quad (3.2)$$

Various corrections to g_D arise due o inter-electronic interaction (Δg_{int}), one-electron QED effects (Δg_{QED}), the screened QED effects (Δg_{SQED}), and nuclear effect (Δg_{nuc}). The first two quantities in the above equation are already measured in the Kusch-Foley experiment, while the next two are factored out in the analysis of their data. The other QED bound-state effects and other postulated contributions are two to five orders of magnitude smaller than the residual $\xi = \delta_S - \alpha_{SL}$, i.e., they are about 1×10^{-6} to 1×10^{-9} times the g-factor, as calculated for Li-like ion. Similar quantities, which have been calculated/measured for hydrogen-like $^{12}C^{5+}$, $^{16}O^{7+}$ and $^{40}Ca^{19+}$, are of the same order of magnitude [21].

3.2 Parameter-dependent Correction Factors δ_S and δ_L

More important results follows if it is observed that the two choices of δ_S and δ_L listed above are not exhaustive. It is reasonable to assume that δ_S and δ_L inter-related and, as will be shown below, approximate results compatible (to an order of magnitude) with available measurements follow if we accept as shown [16, 17], that both g_s and g_L are modified by a small quantity Δ such that, $g_S = 2(1 + \Delta)$ and $g_L = 1 - \Delta$ which implies that $\delta_S = +2\Delta$ and $\delta_L = -\Delta$. Substituting these into (δ_S **-** $2\delta_L$**)** measured in the first Kusch & Foley experiment gives the expression $4\Delta = 0.00229 \pm 0.00008$, from which it follows that $\Delta = 0.00057 \pm 0.00002$, and similarly, substituting them into the same quantity measured in the second experiment implies that $4\Delta = 0.00244 \pm 0.00006$ or $\Delta = 0.00061 \pm 0.00002$.

Both values of Δ obtained above may be rounded up to the nearest order of magnitude to give $\Delta \approx 0.00100$. It is interesting to note that the correction factors Δ obtained from the two experiments reported by Kusch and Foley, is nearly equal while, on the contrary, there is an unexplained discrepancy between the values of g_S deduced from the experiments [12] when δ_S and δ_L are assumed to be independent and δ_L is set equal to zero. Thus we write, $\Delta \approx 1.0 \times 10^{-3} = 1.0 \times 10^{-3}$.

We note from the above comments that Δ above is of the same order of magnitude as $a_e = 1.12 \times 10^{-3}$ which, to five places of decimal, is to the measured anomalous part of the intrinsic g-factor. Thus, we may write

$$\Delta \approx \left(\frac{g_S - 2}{2}\right) = a_e \quad (3.3)$$

Hence, it is not unreasonable to assume that both g_S and g_L are modified as suggested in equation (2.6), and thus precise measurements of the g_L and g_J are crucial. Using (2.6), the effect of bound-state influences, if any, has been naturally included in both δ_S and δ_L.

We can further the discussion by considering (2.8) where $g_S = 2(1 + \Delta)$ and $g_L = 1 - \Delta\lambda$ and $\boldsymbol{\lambda}$ is a real number. Putting $\delta_S = 2\Delta$ and $\delta_L = -\Delta$ into $\delta_S - 2\delta_L = \alpha_{SL}$, where $\alpha_{SL} = 0.00229$ or $\alpha_{SL} = 0.00244$ we have, $2(1 + \Delta) = \alpha_{SL}$. Thus assuming independent δ_S and δ_L**,** the value of λ compatible with the Kusch-Foley data is $\lambda =$

0.05172; this is another was of stating the observed fact that the data implies that $\delta_L \neq 0$ and thus g_L cannot be exactly equal to its classical value 1.0. A more precisely measured value of α_{SL} will no doubt help confirm (or refute) the various points of view discussed in the preceding sections. Moreover, experiments designed to directly measure δ_L will be very significant for the quantum theories of fields and particles.

4.0 Conclusions and discussion

4.1 Needs for high precision measurements.

By bringing together the recent high-precision measurements of the intrinsic g-factors and the classic experiment of Kusch and Foley which determined (δ_S - 2δ_L) from the ratio of the g_J factors of Na and Ga atomic states (obtained from their Zeeman spectra), it has been possible to conclude that the orbital g of the electron deviates from the classical value $g_L = 1$. A probable value of the orbital g is $g_L = 1 - \delta_L$ where $\delta_L = (0.6 \pm 0.3) \times 10^{-4}$. However, it is more probable that the true value of the orbital g is $g_L = 1 - a_e$, where $2a_e$ is the anomalous part of the intrinsic gyro-magnetic factor g_S which has, in recent times, been measured with unparalleled precision. [1, 2, 3].

It has been shown that an extended Dirac equation which includes the fluctuation of the rest energy term leads to a modified Pauli equation; an interpretation of the new terms in the extended Pauli equation indicates that g_S and g_L are corrected by finite measurable quantities. A complementary view described in the Appendix utilizes a simple heuristic argument in which quantum interactions, by analogy with the radiative processes in QED, are assumed to produce corrections to the electronic charge and mass, leading to the modifications of the gyro-magnetic factors g_S, g_L and g_L.. The corrections δe and δm to the charge and mass respectively, unlike the radiative corrections are, however, assumed to be finite. Further analysis indicate that corrections to both the spin and orbital g-factors are linearly related, and that an expression $\delta_S - 2\delta_L = \alpha_{SL}$, similar to the Kusch and Foley measurement results, follows when δ_S and δ_L are assumed to be non-vanishing. Their interdependence can be understood from a physical point of view as follows; if the quantum interactions, then the orbital and spin magnetic moments may not be independent, since mutual interactions between the spins and the orbital motions of the electrons are possible. Assuming that the corrections δ_S and δ_L depend on parameter **Δ,** then the Kusch-Foley experimental data implies that $g_S = 2(1 + \Delta)$ and $g_L = 1 - \Delta$, where Δ is of the order of magnitude of the measured $(g_S - 2)/2$

The measurement of (δ_S - 2δ_L) as described by Kusch and Foley may be subject to some limitations due to the possible perturbations of the energy levels since Ga and Na are many-electron atoms, and the effect of the system of electrons and nuclei, may not be completely taken into account or totally eliminated. Despite these limitations however, it has been possible to consistently infer that δ_L may not vanish. One must therefore avoid assuming a *priori* that $\delta_L = 0$ and thus ascribing to the influence of systematic error in measurement of α_{SL}, the observed fact that $\delta_S - \alpha_{SL} \neq 0$. Although there are well known methods for the elimination or the reduction of systematic error [22], the suggestion can conceivable be made that the residual, systematic errors in the measurement of α_{SL} are what we have here called 2δ_L. That this reasoning is untenable may be shown as follows: let α_{SL} be subject to a systematic error γ, such that $\alpha_{SL} \pm \gamma$ is the true value of (δ_S - 2δ_L). The systematic shift γ in the measurement would have to increase in magnitude three-fold and change sign in the two experiments in order for δ_L to be equal to zero, which is contrary to the behaviour of systematic errors. It therefore seems highly improbable that any shift γ in the measurement of (δ_S - 2δ_L), as determined from Zeeman spectra displayed by two pairs of atomic systems, would act in such a peculiar way, just so that δ_L could vanish. We may also wish to note that it could be conversely argued that the effect of systematic errors, if any, may be such that their elimination would increase α_{SL} to its proper value, which will make δ_L higher than the approximate value herein calculated, i.e. closer to the expected true value. Thus, only further experiments and precise, refined measurements of (δ_S - 2δ_L) and or δ_L can firmly establish, otherwise repudiate, these suppositions.

One analysis of the Kusch-Foley experiment assumes the independence of δ_S and δ_L in the expression $g_S = 2 + \delta_S$ and $g_L = 1 + \delta_L$; there is, however, no reason to assume, α *prior*, that the corrections to g_S and g_L are independent. As shown above, they could have a common origin and thus be related by a parameter as suggested in equation (2.8) which, with λ = 0.05172 adequately fits the data, giving $g_L = 0.99986$ and $g_S = 2.00232$ with the latter in good agreement with the currently accepted measured and calculated values to five places of decimal. Hitherto, the expression $\delta_S - 2\delta_L = \alpha_{SL}$ had been used to determine δ_S by arbitrary setting $\delta_L = 0$, on the basis of the "correspondence principle" [23] which seems inappropriate to the situation. This procedure gave an acceptable value of δ_S sufficiently close to the value calculated at that time. Since we now have available to us, more precise values of δ_S measured by several different techniques (quantum jump spectroscopy, polarization procession method, magnetic resonance method etc) which are reliable and in agreement with one another, we can determine the true value of δ_L from the equation $\delta_S - 2\delta_L = \alpha_{SL}$. Substituting the known, precisely measured values of δ_S into the expression and assuming that δ_S and δ_L are independent, we see that δ_L is either $(+0.2 \pm 0.4) \times 10^{-3}$ or $(-0.6 \pm 0.2) \times 10^{-3}$. If, on the other hand, δ_S and δ_L related**,** then a larger value of δ_L can be obtained. For example, if $\delta_S = 2\Delta$ and $\delta_L = -\Delta$, it is found that $\Delta \approx 1.0 \times 10^{-3}$**,** which is of the same order of magnitude as $(g_S - 2)/2$, i.e., the anomalous part of g_S. However, there is, as yet, no independent precise measurement (direct or indirect) of the electron g_L and/or of δ_L.

Hence, the need for high precision experiments whereby δ_L may be determined, given that δ_S has been measured to extra-ordinary levels of accuracy and precision by the above-listed methods. The precise measurement of $(\delta_S - 2\delta_L)$ employing novel techniques, other that used by Kusch & Foley, will help determine δ_L and hence show whether or not $g_L = 1$ to the same high level of precision to which g_S has been measured. There are currently available new methods of spectroscopy, viz Quantum beats, Doppler-free saturation spectroscopy, Level-crossing spectroscopy etc., which permit higher resolution of neighbouring levels. More accurate data and precise measurement may possibly be obtained from them. From the preceding analysis, we see that it is not sufficiently clear or absolutely certain that $\delta_L = 0$, and that $g_L = 1$. The available evidence does not allow an incontrovertible assertion that $\delta_L = 0$, but rather supports the view that $g_L = 1$. The situation will

greatly benefit from new high precision experiments, which will help clarify unambiguously the true value of g_L.

Appendix

Expressions Relating the Gyromagnetic-Factor Corrections

Assuming that the gyromagnetic factors have non-zero, finite values we show, using a simple heuristic argument, that δ_S and δ_L are linearly related; furthermore we derive an expression $\delta_S - 2\delta_L = \alpha_{SL}$ which, in form, is similar to the result of the Kusch & Foley experiment. Finally, the correction δ_J to the factor g_J is expressed in terms of δ_S and δ_L, and the change in the magnetic dipole moment μ_j, as a consequence of δ_J is discussed.

The value of the intrinsic gyro-magnetic factor $g_S = 2$ is modified by a small quantity δ_S due to quantum corrections (e.g. QED radiative corrections), such that the corrected factor $g_S = 2 + \delta_S$. However, the possible modification of the orbital g by δ_L, and that of the total angular momentum by δ_J, has received less consideration.

A heuristic argument for the modification of all the gyromagnetic factors corresponding to the spin **S,** the orbital **L** and total angular momentum **J = L + S,** may be understood by considering the following: Let the electron magnetic moment μ_K corresponding to the angular momentum **K** be written as

$$\mu_K = \frac{e g_K}{2mc} K \qquad \text{(A1)}$$

where **K** is a genetic symbol for the angular momenta **L, S & J**, e/m is the measured charge-to-mass ratio, and g_K is the gyro-magnetic factor corresponding to the angular momentum **K**. Let the charge and mass of the particle be respectively modified by a quantum interaction, such that $e \to e' = e + \delta e$ and $m \to m' = m + \delta m$, where δe and δm are, in general, finite function of **p, r** and of the angular momenta, including the spin-orbit term coupling terms.

Considering the effect on the magnetic moment, of the quantum corrections of the mass and charge, we write

$$\mu'_K = \frac{e' g_K}{2m'c} K \qquad \text{(A2)}$$

The charge-to-mass ratio e'/m' can be expressed as follows:

$$\frac{e'}{m'} = \frac{e + \delta e}{m + \delta m} = \frac{e[1 + \delta e/e]}{m[1 + \delta m/m]} = \frac{e}{m}\left[\frac{1 + \delta e/e}{1 + \delta m/m}\right] \equiv \frac{e}{m} Z_K \qquad \text{(A3)}$$

where $Z_K = \{[1 + \delta e/e]/[1 + \delta m/m]\}$ is a function. Substituting (2.3) into (2.2) gives

$$\mu'_K = \frac{e}{2mc} Z_K g_K K \qquad \text{(A4)}$$

We may therefore suppose that the effect on the μ'_K of the quantum correction contained in Z_K is to modify the gyromagnetic factor g_K such that $g_K \to g'_K$. As a result, the gyromagnetic factor g'_K in the presence of the quantum corrections will differ from the g_K observed when the interactions considered above are absent; i.e., there are interesting consequences when $Z_K \neq 1$. Thus we may infer from equation (A4) the expression,

$$g'_K = g_K Z_K \qquad \text{(A5)}$$

Hence for **K = S,** $g_S \to g'_S$. and for **K = L**, $g_L \to g'_L$; likewise for **K = J**, $g_J \to g'_J$. The modification of the intrinsic g by non-classical interactions is well-known; experiments have shown that $Z_S = 1 + a_e$ where a_e gives the anomalous part of the intrinsic magnetic moment. It is not unreasonable therefore to expect that the quantum interactions which couple spin and orbital degree of freedom, will give rise to the mutual modifications of both the g_L and g_S that, in high precision experiments, may be detected without ambiguity; indeed further considerations indicate that $Z_L = 1 - a_e$. The inter-dependence of the spin and the orbital gyro-magnetic factor will be demonstrated below.

In QED, the interaction of the electron with its own radiation field, leads to a correction of the mass as follows; $m \rightarrow m' = m + Z_1$, where the self-energy term is expressed as $Z_1 = (3m\alpha/2\pi)\ln(\Lambda/m) + \dots$. Similarly, vacuum polarization requires that the electronic charge e be corrected as follows; $e \rightarrow e' = eZ_3^{1/2}$, the term Z_3 being given as $Z_3 = 1 + (\alpha/3\pi)\ln(\Lambda^2/m) + \dots$. The factors Z_1 and $(Z_3 - 1)$ in QED are analogous to δm and δe above respectively and, as is well known, the radiative corrections in QED lead to a modification of g_S by δ_S [12.24]; it should likewise be possible to calculate from QED a finite correction of the g-factor g_L by δ_L. The factor Z_1 and Z_3, as they occur in QED, are infinite; the theory can, nevertheless, be renormalized to accommodate the divergencies.

We shall here be concerned, however with interactions which produce small, finite corrections (or corrections which can be made small and finite), and therefore write, from equation (A3),

$$\frac{e'}{m'} = \frac{e[1+\delta e/e]}{m[1+\delta m/m]} = \frac{e}{m}\left[\left(1+\frac{\delta e}{e}\right)\left(1+\frac{\delta m}{m}\right)^{-1}\right]_K \qquad \text{(A6)}$$

Assuming that the terms $\delta e/e$ and $\delta m/m$ are small compared to 1 (i.e. $\delta m/m \ll 1$), and linearly expanding the expression $[1 + \delta m/m]^{-1}$ in a convergent binomial series, we observe that

$$\frac{e'}{m'} = \frac{e[1+\delta e/e]}{m[1+\delta m/m]} = \frac{e}{m}\left[1+\frac{\delta e}{e}-\frac{\delta m}{m}-\frac{\delta e\,\delta m}{em}+\dots\right]_K \equiv \frac{e}{m}[1+Z_K] \qquad \text{(A7)}$$

from which, when substituted into eqn (A6), we can infer that $g'_K = g_K + g_K Z_K$ (A8)

since $Z_K = 1 + Z_K$ from (A3) and (A7). Hence,

$$\delta_S = g'_S - g_S = g_S Z_S \text{ and } \delta_L = g'_L - g_L = g_L Z_L \qquad \text{(A9)}$$

If $Z_L = Z_S$, then from (A9) $g_L\delta_S = g_S\delta_L$ which, with $g_S = 2$ and $g_L = 1$, implies that $\delta_S - 2\delta_L = 0$. contrary to the experimental results of Kusch and Foley. If, however, $Z_S \neq Z_L$ then, because $Z_S/Z_L = Z_{SL} \neq 1$ is finite, we see that

$$\delta_S = 2\delta_L Z_{SL} \qquad \text{(A10)}$$

from which it follows that δ_S and δ_L are mutually dependent. Furthermore, equation (A10) can also be expressed as

$$\delta_S - 2\delta_L = \alpha_{SL} \qquad \text{(A11)}$$

where $\alpha_{SL} \equiv 2\delta_L(Z_{SL} - 1)$, which agrees, in form, with the result of the Kusch and Foley experiment.

Let **K = J** in eqn (A2) where **J = L + S,** then the magnetic moment vector is

$$\mu'_J = \frac{e'}{2m'c} g_J J \qquad \text{(A12)}$$

Assume, based on the vector model [15], that $\mu'_j = \mu'_L + \mu'_S$ (A13)

then,

$$\mu'_J = \frac{e}{2mc} g_J Z_J J = \frac{e}{2mc}(g_L Z_L L + g_S Z_S S);\ \mu'_J \frac{e}{2mc} g'_J J \qquad \text{(A14)}$$

Taking the components of the magnetic dipole moment vectors in the direction of **J** in (A14) gives,

$$g_J Z_J |J|_J = g_L Z_L |L| \cos(L,J) + g_S Z_S |S| \cos(S,J) \qquad \text{(A15)}$$

Noting that cos(L,J) and cos (S,J) are $\cos(L,J) = \frac{|J|^2+|L|^2-|S|^2}{2|J||L|}$ and $\cos(S,L) = \frac{|J|^2+|S|^2-|L|^2}{2|J||S|}$

Equation (A15) becomes,

$$Z_J = Z_L \frac{g_L}{g_J}\frac{|J|^2+|L|^2-|S|^2}{2|J|^2} + Z_S \frac{g_S}{g_J}\frac{|J|^2+|S|^2-|L|^2}{2|J|^2} \qquad \text{(A16)}$$

where $|L|^2 = l(l+1), |S|^2 = s(s+1)$ *and* $|J|^2 = j(j+1)$. Putting

$$\alpha \equiv \frac{g_L}{g_J}\frac{|J|^2+|L|^2-|S|^2}{2|J|^2} \text{ and } \beta \equiv \frac{g_S}{g_J}\frac{|J|^2+|S|^2-|L|^2}{2|J|^2} \qquad \text{(A17)}$$

and recalling that, in general, $Z_L = 1 - \lambda\Delta$ and $Z_S = 1 + \Delta$, where $\Delta \approx 1.12 \times 10^{-3}$, we have

$$Z_J = \alpha' - \beta'\Delta \;;\; \alpha + \beta = \alpha',\; \lambda\alpha - \beta = \beta' \tag{A18a}$$

or

$$Z_J = \alpha'(1 - \gamma'\Delta) \;;\; \gamma' = (\lambda\alpha - \beta)/(\alpha + \beta) \tag{A18b}$$

Hence, as defined in (A5), $$g'_J = g_J Z_J = g_J \alpha'(1 - \gamma'\Delta) \tag{A19}$$

We may. As was done for δ_S and δ_L, define δ_J using Z_J such that $\delta_J = g_J (Z_J - 1)$ which, upon substituting for Z_J from (A18) gives $\delta_J [(\alpha - 1) - \alpha'\gamma'\Delta]$. For a singlet state (S = 0), $\alpha' = g_J = \gamma' = 1$, hence $\delta_J - \Delta = \delta_L$ and for L = 0, the correction $\delta_J = +2\Delta = \delta_S$, as should be expected for $\lambda = 1$. It also follows from (A9) and (A16) or from (A5) and (A15) that

$$\delta_J = \delta_L \frac{|L|}{|J|}\cos(L, J) + \delta_S \frac{|S|}{|J|}\cos(S, J) \tag{A20}$$

Thus the contribution to δ_J, from δ_S and δ_L can be reduced or enhanced by the proper choice of L, S and J. Hence the empirical consequence of δ_L may be clearly resolved in carefully controlled experiments.

It is however necessary for completeness, to make clear the difference between g'_J in (A19), and that calculate/measured in H-like [21, 26] & Li-like ions [20]. High precision measurements of the factor g_J of an electron bound in highly-charged ions have recently come into prominence, making it possible to investigate QED effects for electrons confined by strong Coulomb fields [25-30]. These measurements, being mainly concerned with the 1S½ and 2S½ states (in which L = 0), are in essence calculations/measurements of the gs,

i.e., the spin g-factor for bound-state electrons in strong fields. The corrected g-factors $g'_L = g_L + \delta_L$, $g'_S = g_S + \delta_S$ and $g'_J = g_J + \delta_J$ described above refer to the situation in which quantum effects are significant and the electron is weakly coupled, i.e. relatively "free"; thus, for example, the orbital g-factor for a weakly-bound atomic electron will be different from the classical value $g_L = 1$, as a consequence of the quantum-relativistic fluctuations of the rest energy. Also, g'_J for such electrons (assuming LS coupling) will be different from that to be expected in the absence of quantum relativistic effect, but will nevertheless be measured for weakly-bound electrons after extraneous factors have been appropriately eliminated or taken into account. Hence δ_J described by (A19) will differ significantly from that arising from atomic bound-state effects due to the size & shape of the nuclear charge, nuclear-recoil corrections, electron self-energy etc. It will therefore be instructive to compare with experimental values measured for states other than the nS, the theoritcal values of g_J predicted by bound-state QED; electron states with non-zero orbital angular momentum are very significant for the complete determination of g'_J.

Finally, the substitution of (A19) into (A14) shows also that the magnetic dipole moment vector μ_J is significantly modified when δ_J and δ_L have non-zero values. The effect of the calculated correction δ_J may therefore be observed in high-precision spectroscopic or magnetic resonance measurements.

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