Journal of the Nigerian Association of Mathematical Physics
***Volume* 12 (May, 2008), 439 - 442**

Mathematical modelling of mortality rate of broiler chickens

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Abstract

The poultry industry is one of the most successful agricultural ventures in a nation's economy. A good and stable economy allows for an improved animal husbandry which shall lead to better poultry production. Unfortunately this is not entirely so in the Nigerian experience. Our poultry industry still depends largely on governmental support for its sustenance. Mortality in broiler flocks represents lost income to growers. Though mortality is an everyday occurrence in broiler production, growers need to device management programmes that will reduce its overall effect on flock performance. We formulate, by adopting some epidemic model approach, a mathematical model to help farmers initiate management programmes that will reduce the mortality rate and its overall effect on poultry production, and thus help the poultry business to be more successful.

1.0 Introduction

Poultry is a general term for birds of several species such as chickens or domestic fowls, turkeys, ducks, geese, guinea fowls, pen fowls, ostriches, pheasants and other game birds. Domestication of birds is thought to have occurred about 8000 yeas ago, first in India and China, then spreading along trade routes (Wood – Gush, 1959, in European Commission 2000 [5]). It is one of the most successful industries in any nation's economy. For instance, the poultry industry is one of the largest businesses in the state of Virginia in USA, consisting of nearly 500 million dollars in profit per year (Katie Clayton, *et al* 2002 [2]).

To understand the underlying causes of the welfare issues that relate specifically to commercial broiler production, one must be aware of the differences in biology and behaviour between modern broiler birds and other strains of domestic fowl. These have arisen as a consequence of intensive selection for faster and more efficient production of chicken meat. Each recognized breed or stock of chicken has a characteristic shape and form. The configuration of the head is a good indication of health and vitality. Birds that have strong and moderately long heads with bright eyes are often good egg produces. A flat top is often preferred to a rounded skull. Smooth, soft-textured, skin and bright red comb are attributes of a standard chicken (Nwosu, *et al* in Youdeowi, A. 1986). Large abdominal or internal capacity is necessary of efficient conversion of feed and therefore for higher egg production.

The performance of the modern day boiler chickens represents one of the most marked increases in livestock productivity achieved by selective breeding. In the last 30 years the time taken to produce a broiler chicken weighing 2kg has been halved from more than ten weeks to less than six weeks. Initially selection was for greater growth rate and meat yield, but as excessive carcass fat became a problem, the emphasis shifted to improving food conversion efficiency as well. In recent years there has also been selection against susceptibility to certain types of diseases.

Mortality in broiler production is due to certain factors including welfare, medication and physiological and feeding patterns of the birds. (Elrom, 2000 [3], Clayton *et al*, 2002 [2]; Vest, 1999 [12]; Butcher and Miles, 1995 [1]).

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Poor welfare conditions could lead to reduced life expectancy, reduced ability to grow, body damage (e.g. inability to have abundance of feathers), disease and immunosuppression, and self narcotizing. Vaccines are substances introduced into the blood to protect the body from diseases by causing a very mild form of it. When these are not administered on schedule and appropriately, the population can be adversely affected. The quality of the feeds also contributes to high mortality rate.

The mortality rate is dependent on many variables like the number of birds, the state of the birds when they get to the farm, the vaccines given against viral infections, the environmental welfare conditions, the lightings in the barn, the period of rearing of the birds before they are killed and the initial mortality rate of the birds before they are two weeks of age. However, the variables of relevance are the number of birds, the number of weeks they are reared before being killed and the initial mortality rate assuming that the welfare and medical conditions are under control. It has been noted that flocks that lost the most birds early, tended to loose the most birds late. (Tabler *et al*, 2004 [11]). Working on broiler enterprises in Cross River State of Nigeria, Etuk, E. A. *et. al* (2007 [4]), observed that the mortality rate of birds during the first four weeks was higher than that of the second four weeks.

2.0 Model formulation

We shall adopt the epidemic model (Murray, J. D 1990 [9] and Katiyar, 1996 [6]) for the purpose of formulating this model. Epidemic models are basically two types. In one, the total population is taken to be approximately constant with for example the population divided into susceptible, infected and immune groups. In the other, the population size is affected by the disease mortality and so on.

The population is divided into two groups – the *susceptible* and the *infected*. The population of broiler chickens which can catch any disease or are prone to any poor welfare conditions is regarded as the susceptible. The infected are those that are undergoing stress factors, infection, bad weather conditions. We shall adopt the general deterministic model with removal, that is, *S I R* model, since those infected have to be removed from the population.

The effective rate of population growth is negative due to the constraints of disease, weather and other adverse conditions. The model then assumes decay model character. On the other hand, if the disease is not prevented in time through vaccination and improvement of the environmental conditions as well as removal of the infected, then the rate of infection will increase.

We shall let I represent the infective; N, the population of birds and S the susceptible population. Thus at any time t

$$S(t) + I(t) + R(t) = N = I_0 + S_0 \tag{2.1}$$

where R is the removed population after infection, and $S_0 = S(0) > 0$, $I_0 = I(0) > 0$.
Let the infected birds be removed from the system at a rate of α. We obtain the system of equation as.

$$\frac{dS}{dt} = -\gamma SI \tag{2.2}$$

$$\frac{dI}{dt} = \gamma SI - \alpha I \tag{2.3}$$

$$\frac{dR}{dt} = \alpha I \tag{2.4}$$

$$\Rightarrow \qquad \frac{dI}{dt} = \gamma I\left(S - \frac{\alpha}{\gamma}\right) = \gamma I\left(S - \rho\right) \tag{2.5}$$

where γ is a constant and ρ = α /γ is the effective mortality rate. i.e. the ratio of the rate at which broiler chickens are dying from the infective to the rate at which they are added to the infective.

$$\frac{dS}{dR} = -\frac{\gamma S}{\alpha} = -\frac{1}{\rho}S$$

$\Rightarrow$ $$S = S_0\, e^{-R/\rho} \tag{2.6}$$

$\Rightarrow$ $$\frac{dR}{dt} = \alpha\,(N - S_0 e^{-R/\rho} - R)$$

$$t = \frac{1}{\alpha}\int_0^R \frac{dR}{N - S_0 e^{-R/\rho} - R} \tag{2.7}$$

In general this equation (2.7) has to be integrated numerically if the values γ, α, S_0 and N are known. We shall, however, consider two cases in order to provide analytic solutions.

Case1

ρ, the effective mortality rate is small. Then, as ρ → 0, (2.7) yields $t = \frac{1}{\alpha}\int_0^R \frac{dR}{N-R}$

This yields $$-\alpha t = \lambda n \frac{N-R}{R}$$

$\Rightarrow$ $$R = N(1 - e^{-\alpha t}) \tag{2.8}$$

Case 2

ρ is large $\Rightarrow$ R/ρ is small. If the epidemic is large, this implies $R/\rho < 1$. We can expand $e^{-R/\rho}$ by Taylor series thus

$$e^{-\frac{R}{\rho}} = 1 - \frac{R}{P} + \frac{R^2}{2\rho^2} + \Lambda \tag{2.9}$$

$$\Rightarrow t = \frac{1}{\alpha}\int_0^R \frac{dR}{\mathrm{N} - S_0(1 - R/\rho + R^2/2\rho^2 + \ldots) - R} = \frac{1}{\alpha}\int_0^{\mathrm{R}} \frac{dR}{(N - S_0) + (S_0/\rho - 1)R - (S_0/2\rho^2)R^2} \tag{2.10}$$

Let $\alpha t \equiv \int_0^{\mathrm{R}} \frac{dR}{a + bR - cR^2}$

where $$a = N - S_0,\ b = S_0/\rho - 1,\ c = S_0/2\rho^2, \tag{2.11}$$

i.e. $\alpha t = \frac{1}{c}\int_0^{\mathrm{R}} \frac{dR}{(R - b/2c)^2 - b^2 + 4ac/4c^2}$

and if we let $= \sqrt{\frac{b^2 + 4ac}{4c^2}}$, then

$$\alpha t = \frac{1}{c}\int_{-\frac{b}{2c}}^{\mathrm{R}-\frac{b}{2c}} \frac{d\zeta}{\beta^2 - \zeta^2} = \frac{1}{2\beta c}\lambda n \left.\frac{\beta + \zeta}{\beta - \zeta}\right|_{-\frac{b}{2c}}^{R-\frac{b}{2c}}$$

$\Rightarrow$ $$2\alpha\beta c t = \lambda n \frac{\beta^2 + R\beta + (b/2c)(R - b/2c)}{\beta^2 - R\beta + (b/2c)(R - b/2c)} \tag{2.12}$$

Simplifying we obtain

$$R = \frac{(\beta^2 - b^2/4c^2)(e^{2\alpha\beta ct} - 1)}{(\beta - b/2c)e^{2\alpha\beta ct} + \beta + (b/2c)} = \frac{(\beta^2 - b^2/4c^2)(1 - e^{-2\alpha\beta ct})}{\beta - b/2c + (\beta + b/2c)e^{-2\alpha\beta ct}} \tag{2.13}$$

As $t \to \infty$, this equation yields $$R_\infty = \beta + (b/2c) \tag{2.14}$$

and using (2.11) etc, we obtain $$R_\infty = \frac{\rho^2}{S_0}\sqrt{\left(\frac{S_0}{\rho}-1\right)^2 + \frac{2S_0}{\rho^2}(N-S_0)} + \frac{\rho^2}{S_0}\left(\frac{S_0}{\rho}-1\right)$$

$$\therefore \quad R_\infty = \frac{\rho^2}{S_0}\left(\frac{S_0}{\rho}-1+\mu\right) \tag{2.15}$$

where $\mu = \left[\left(\frac{S_0}{\rho}-1\right)^2 + \frac{2S_0(N-S_0)}{\rho^2}\right]^{1/2}$. We obtain $$R_\infty \cong 2\rho(1 - \rho/S_0), \tag{2.16}$$

Since $I_0 \cong 0$ as $2S_0I_0/\rho \to 0$, where $I_0 = N - S_0$, but $R = 0$ at ∞. Hence $2\rho(1-\rho/S_0) = 0$

$$\Rightarrow \quad \rho = S_0 \tag{2.17}$$

Thus the effective mortality rate is equivalent to the initial susceptible population of the birds.

3.0 Discussion of results and conclusion

In the first case, with ρ small we obtained the result that

$$R = N(1 - e^{-\alpha t}) \tag{3.1}$$

This is an exponential decay model which has been validated by results of mortality data that were gathered from October 1996 – June 2003 by a research carried out at the University of Arkansas, USA (Tabler, G. and Berry, L (2004 [10])). This result shows that as $t \to \infty$, R also approaches N which means that as the chickens are kept for longer periods during the infection, the mortality rate increases until all the chickens have died of the infection.

In case 2:

We obtained the result that for large mortality rate, $\rho = S_0$.

This confirms that once a flock of birds is brought into the farm with very high mortality rate, the broiler chickens will die off in no time.

Case 3:

If $S_0 < \rho$ then equation (2.3) implies that dI/dt is initially negative and the epidemic does not build up. This means that the epidemic builds up only if $S_0 > \rho$, that is, only when the effective mortality rate is less than the initial number of susceptible and in this case all the birds are immediately removed. A further implication of this is that an epidemic builds up only when the density of susceptibles is high owing to overcrowding, and the rate of removal is low due to inadequate isolation conditions. Where the conditions are good and the density of the susceptibles is low, the epidemic fades out.

Thus for successful rearing of broiler chickens, good welfare conditions must be observed to avoid the extreme case of the effective mortality rate being large. Good management programmes must be developed in broiler production to combat early mortality.

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