Journal of the Nigerian Association of Mathematical Physics
Volume **12 (May, 2008), 379 - 386**
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Off-grid points exploitation in the development of more accurate collocation method for solving ODEs

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Abstract

In this paper, some directions to exploit grid/off-grid points for better and higher accuracy of one-step methods for solving Ordinary differential equations were suggested. Some methods were obtained from the continuous interpolation/collocation procedure. Numerical computations were done on some sample problems on a micro-computer and comparisons showed that the accuracy of the hybrid methods are better than some existing methods.

1.0 Introduction

In this paper we survey some grid/off grid points collocation methods for solving systems of ordinary differential equations (ODES) of the form:

$$y^1 = f(x, y),\ y(x_0) = y_0 \ \ over\ [x_0, x_n] \tag{1.1}$$

with y satisfying additional initial condition as in $y^{(i)}(x_0) = \alpha_i, i = 0\ (1)\ n-1\}$

Exploiting off-grid points in collocation approach in the development of linear multistep or one-step methods is now well-known. It is one of the procedures for obtaining continuous methods for ODES, (See Lamber (1973), Fatokun J., et al (2005), Yakubu (2003), Amuseghan (2004) and Ademiluyi (1987). Many areas of research for a simpler (formula), higher order accuracy and efficient methods in one-step or linear multistep methods to ease numerical solution technique are through exploiting grid/off grid points in interpolation and collocation. Onumanyi (2004), and Amuseghan (2004) showed that Hybrid linear multistep and one-step methods respectively are yielding better results for solving ODES.

As Hybrid methods retain linear multistep characteristics, it shared with Runge-Kutta methods the property of utilizing points other than grid points, we need as in this paper to explore off-grid points exploitation.

A K-step hybrid formula is defined as:

$$\sum_{j=0}^{k} \alpha_j y_{n+1} = h\sum_{j=o}^{k} \beta_j f_{n+j} + h\beta_v f_{n+v} \tag{1.2}$$

where $\alpha_k = +1,\ \alpha_0, \beta_0$ are not both zero, $v \in$ (rational number) and $f_{n+v} = f(x_{n+v}, y_{n+v})$

It should be noted that there are hybrid formulas that need a helper formula to get it started (see Amuseghan (2004) and Lambert (1973)), but today there are hybrid formulas that are self starting (i.e. which does not need a helping formula to start). Most of the past formulae could be turned to Hybrid formula that is self starting

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Many eminent scholars have given some attention at various times to solution of problems of type (1.1) by interpolation and collocation. These include:

Ademiluyi (1987), Oladele (1991), Awoyemi (1992), Amuseghan (2004), Yakubu (2003), Onumanyi (2004), to mention but few.

However in this paper we would show how exploiting grid/off-grid points collocation technique yield better results to solution of ODES.

2.0 Derivation techniques and construction of methods

Continuous methods and finite difference schemes could be obtained through:

(a) The Gaussian method which uses
  (i) Two points, x_{n+u}, x_{n+v}.
  (ii) Three points, x_{n+u}, x_{n+v}, x_{n+w}

(b) The Radue's method which uses:
  (i) Two points, x_n and x_{n+u} or x_{n+u}, x_{n+1},
  (ii) Three points, x_n, x_{n+u}, x_{n+v} or x_{n+u}, x_{n+v} and x_{n+1}
  (iii) Four points, x_n, x_{n+u}, x_{n+v}, x_{n+w} or x_{n+u}, x_{n+v}, x_{n+w} and x_{n+1}

NOTE: In Radau's method one end point is included.

(d) The Lobatto's method which uses:
  (i) Three points, x_n, x_{n+v}, x_{n+1},
  (ii) Four points, x_n, x_{n+u}, x_{n+v}, x_{n+1},
  (iii) Five points, x_n, x_{n+u}, x_{n+v}, x_{n+w}, $x_{n+1.}$

NOTE: In Lobatto's method, the two end points are included.

Respective values can be assigned to *u*, *v*, *w*, *z*.... or evaluated to obtain our finite difference schemes based on:

(a) Two Gussian values: $U = \left(3 - \sqrt{3}\right)/6$, $V = \left(3+\sqrt{3}\right)/6$

(b) Three Gussian values: $U = \left(5-\sqrt{15}\right)/10$, $V = 1/2$, $W = \left(5+\sqrt{5}\right)/10$

(c) Two arbitrary values: $U = 1/3$, $U = 2/3$

(d) Three arbitrary values: $U = 1/4$, $V = 1/2$, $W = 3/4$

(e) The zeroes of Legendre polynomial
  (*i*) Of degree 2, giving the values: $V = -\sqrt{1/3}$, $U = +\sqrt{1/3}$
  (ii) Of degree 3, giving the values $V = 0$, $U = -\sqrt{3/5}$, $W = +\sqrt{3/5}$

Some proposed points

Exploitation of grid/off-grid points could be done as suggested:

(a) A pair of block point: at x = x_n, x_{n+v}, x_{n+w}, x_{n+1} with (x_{n+v}, x_{n+w}) as off-step pair block. This is already in use, (see Onumanyi (2004),and Amuseghan (2004)),

(b) Two pairs Blocks (close to centre and to the two end points). At x = x_n, x_{n+v}, x_{n+v}, x_{n+w}, x_{n+z}, x_{n+1}, (x_{n+v} and x_{n+w}), a block close to centre. and $(x_{n+v}, and\ x_{n+z})$, a block close to the two end points.

(c) Right hand collocation (for one-step method and one point interpolation at the centre. That is, at x = x_n, x_{n+u}, x_{n+v}......x_{n+1}, but the one point collocation at the centre while all other collocation points should be at the right hand of the interpolation point.

(d) Left hand collocation (for one-step method) as in (c) above.

(e) Perfecting Gaussian values. Evaluate surd form and convert to numerical values. Then sought for better points included/around, for a better collocation.

Development of method through canonical polynomial. (As in Adeniyi (1991) and Oladele (1991)), canonical polynomial have been applied directly via collocation to solve: $y^1=f(x,y)$, $y(x_o) = y_0$

where $y, f \in R^m$, $||y|| < \infty$ for a suitable norm $||.||$ and $x \in [a,b]$ (2.1)

$a, b \in R$, we seek an approximate solution to (2.1) of the form:

$$y_n(x)=a_0\, Q(x) + a_1\, Q_1(x)+\Lambda + a_n Q_n(x) \tag{2.2}$$

where $Q_j\,(x),\ j = 0,1,\Lambda$ are the canonical polynomials generated by the operator $L \equiv \frac{d}{dx}+1$ with $Q_j(x)$ defined as

$$LQ_j\,(x) = x^j\ ,\ j = 0,1,2,\Lambda \tag{2.3}$$

and we obtain $Q_j(x)$ explicitly as

$$Lx^j = j\,x^{j-1} + x^j$$
$$Lx^j = j\,LQ_{j+1}(x) + L\,Q_j(x)$$
$$Lx^j = L\left(j\,Q_{j-1}(x) + Q_j(x)\right)$$

we assume that L^{-1} exist, we have

$$x^j = j\ Q_{j-1}(x) + Q_j(x)$$
$$Q_j(x) = x^j - j\,Q_{j-1}(x) \tag{2.4}$$

Thus, we obtain a recursive relation (2.4) with j=0, 1,2 to have

$$Q_0(x) = 1$$
$$Q_1(x) = x-1$$
$$Q_2(x) = x^2-2x+2. \tag{2.5}$$

From (2.1.2), when $n = 1$ we have $$y_1(x)=a_0\, Q_0(x) + a_1\, Q_1(x). \tag{2.6}$$

Substituting the canonical polynomial, into (2.6) we have. $$y_1(x)=a_0 + a_1(x-1) \tag{2.7}$$

Thus substituting (2.7) into (2.1) and collocating at $x=x_{i+\frac{1}{2}} = x_i + \frac{1}{2}h$ we have

$$a_1 = f\left(\left(x_{i+\frac{1}{2}}, y(x_{i+\frac{1}{2}}\right)\right) = f_{i+\frac{1}{2}} \tag{2.8}$$

from (2.7) we have

$$y_i = y_i\,(x_i) = a_0 + a_1\,(x_i - 1)$$
$$y_i = a_0 + a_1(x_i-1)$$
$$a_0 = y_i - a_1\,(x_i-1)$$
$$a_0 = y_i - (x_i-1)\ f_{i+\frac{1}{2}}.. \tag{2.9}$$

we substituted these values in (2.8) and (2.9) into equation (2.7) to give us the continuous approximation

$$y(x)\approx y_1(x) = y_i + (x - x_i)\ f_{i+\frac{1}{2}}\quad x_i \le x \le x_{i+1} \tag{2.10}$$

at $x = x_{i+1}$ (2.10)) yields the discrete approximation formula

$$y_1(x_{i+1})=y_i+(x_{i+1}-x_i)\,f_{i+\frac{1}{2}}$$
$$y_{i+1} = y_i + hf_{i+\frac{1}{2}}. \tag{2.11}$$

where $h = x_{i+1} - x_i$

we apply the backward Euler method with a step length ½h to obtain a formula for approximating the off-grid function value $f_{i+½}$

$$y_{i+1/2} = y_i + \frac{h}{2} f_{i+1/2}.$$

$$2y_{i+1/2} = 2y_i + hf_{i+1/2} \qquad (2.12)$$

substituting (2.12) in (2.11) for $hf_{i+1/2}$ we get

$$y_{i+1} = -y_i + 2y_{i+1/2} \qquad (2.13a)$$

where
$$y_{i+1/2} = y_i + 1/2\, hf_{i+1/2} \qquad (2.13b)$$

The continuous scheme (2.10) with (2.13b) becomes

$$y(x) = y_1(x) = y_i + \frac{2(x - x_i)(y_{i+1/2} - y_i)}{h} \qquad (2.14)$$

where $y_{i+½}$ *is obtained from* (2.13b). We now express (2.13) by an equivalent semi-implicit method, as follows:
$$y_{i+1} = y_i + hk_1$$

where
$$k_1 = f_{i+1/2} = f\left(x_{i+1/2}, y(x_{i+1/2})\right) = f\left(x_{i+\frac{1}{2}h}, y_{i+1/2}\right) \qquad (2.14b)$$

but from Backward Euler with step length ½h we have

$$y_{i+1} = y_i + hk_1, \text{ where } k_1 = f\left(x_{i+\frac{1}{2}h}, y_{i+1/2} + hk_1\right) \qquad (2.15)$$

Note that, If $x = x_{i+1/2}$, is the zero of the Legendre polynomial $p_1(x),\ x \in [x_i, x_{i+1}]$. That is

$$p_1(x) = \frac{2x(x_{i+1} + x_i)}{h} = 0$$

$$x = \frac{x_{i+1} + x_i}{2}$$

where $x_i = x_o + ih$, $x_{i+1} = x_o + ih + h$

$x = (x_0 + ih) + ½h$

$x = x_{i+½}$ is a zero of $p_1(x)$.

Discrete scheme like (2.15) and continuous scheme like (2.10) are already well known. They can also be obtained through a real polynomial function of a single variable x, as a basis function. We then obtained our methods through as in Kayode (2004),Olorunfemi (2005)and Amuseghan (2004), we find a real polynomial function of a single variable x as a basis function in the form:

$$y(x) = \sum_{j=0}^{m} x_j$$

yielding
$$y(x) = \sum_{j=0}^{m} a_j x^j \qquad (2.16)$$

as our approximate solution to equation (1.1), where all a_j's are m real coefficients. The first derivative is given as

$$y^1(x)=\sum_{j=0}^{m} j \ a_j x^{j-1} \tag{2.17}$$

Two different methods developed by collocation and interpolation will be considered in this paper.

In this case, equations (2.16) and (2.17), for m = 2, j = 0, 1, 2 is considered. Where x_n is the only interpolation point, and the two collocation points are at x_{n+u} and x_{n+v}.

From (2.16) we will have:

$$y(x)=ao+a_1 x+a_2 x^2 \tag{2.18}$$

and from (2.17) we will have:

$$y^1(x)=a_1+2a_2 x \tag{2.19}$$

and from (1.1) and (2.19) we have $a_1+2a_2 x=f(x,y).$ (2.20)

and we have the following non-linear system of equations as a result of collocating and interpolating as required.

$$a_1+2a_2 x_{n+u}=f_{n+u} \tag{2.21}$$

$$a_1+2a_2 x_{n+v}=f_{n+v} \tag{2.22}$$

$$a_0+a_1 x_n+a_2 x_n^{\ 2}=y_n \tag{2.23}$$

Solving for the a_j's and substituting into (2.18) and simplify to obtain a continuous method of the form

$$y(x)=y_n+(x-x_n)f_{n+u}-2k(x-x_n)(x_{n+u}) \ +k(x-x_n)(x+x_n)$$

$$=y_n+(x-x_n)f_{n+u}-2k(x-x_n)(x_n+uh)+k(x-x_n)(x+x_n) \tag{2.24}$$

where $k=\dfrac{1}{2h(v-u)}[f_{n+v}-f_{n+u}]$. From (2.24) substitute for k, we have

$$y(x)=y_n+\frac{(x-x_n)}{2h(v-u)}[2hv-(x-x_n)F_{n+u}+((x-x_n)-2uh)f_{n+v}] \tag{2.25}$$

Evaluating (2.25) at $x=x_{n+1}$, x_{n+u} and x_{n+v} we obtained the following three finite difference schemes.

$$y(x_{n+1})-y_n=\frac{h}{2(v-u)}[(2v-1)f_{n+u}+(1-2u)f_{n+v}] \tag{2.26a}$$

$$y(x_{n+u})-y_n=\frac{uh}{2(v-u)}[(2v-u)f_{n+u}+uf_{n+v}] \tag{2.26b}$$

$$y(x_{n+v})-y_n=\frac{vh}{2(v-u)}[vf_{n+u}+(v-2u)f_{n+v}] \tag{2.26c}$$

setting $u={}^1\!/_3$, $v={}^2\!/_3$ in equation 2.26 (a to c) we get:

$$y_{n+1}-y_n=\frac{h}{2}[f_{n+u}+f_{n+v}] \tag{2.27a}$$

$$y_{n+u}-y_n=\frac{h}{6}[3f_{n+u}-f_{n+v}] \tag{2.27b}$$

$$y_{n+v}-y_n=\frac{h}{3}[2f_{n+u}] \tag{2.27c}$$

Considering equations (2.16) and (2.17) with m=3, j=0, 1, 2, 3, and x_n is the only interpolation point while collocation is done at x_{n+u}, x_{n+v} and x_{n+w}. Thus from (2.16) and (2.7) we have:

$y(x) = a_o + a_1x + a_2x^2 + a_3x^3$ (2.28

and $y^1(x) = a_1 + 2a_2x + 3a_3x^2$ (2.29)

Also collocating and interpolating at points x_{n+u}, x_{n+v} and x_n we have

$$a_1 + 2a_2x_{n+u} + 3a_3x^2_{n+u} = fn_{+u} \quad (2.30a)$$

$$a_1 + 2a_2\ x_{n+v} + 3a_3x^2_{n+v} = f_{n+v} \quad (2.30b)$$

$$a_1 + 2a_2\ x_{n+w} + 3a_3x^2_{n+w} = f_{n+w} \quad (2.30c)$$

$$a_o + a_1\ x_n + a_2x^2_n + a_3x_n^3 = y_n \quad (2.30d)$$

Solving for the *aj's* and substitute in (2.28b) we obtain the continuous method:

$$y(x)=\frac{(x-x_n)}{6h^2(v-u)(w-u)}\left[+6h^2vw-3hw(x-x_n)-3hv(x-x_n)+2(x-x_n)^2\right]f_{n+u}$$

$$+\frac{(x-x_n)}{6h^2(v-u)(w-v)}\left[-6h^2uw+3hw(x-x_n)+3hu(x-x_n)-2(x-x_n)^2\right]f_{n+v}$$

$$+\frac{(x-x_n)}{6h^2(w-u)(w-v)}\left[6h^2uv-3hv(x-x_n)-3hu(x-x_n)+2(x-x_n)^2\right]f_{n+w} \quad (2.31)$$

Evaluating ***(2.31)*** at $x = x_{n+1}, x_{n+u}, x_{n+v}$ *and* x_{n+w}, we obtained the following four finite difference schemes.

$$y(x_{n+1})-y_n=\frac{h}{6(v-u)(w-u)}[6vw-3w-3v+2]f_{n+u}+\frac{h}{6(v-u)(w-v)}[-6uw+3w+3u-2]f_{n+v}$$
$$+\frac{h}{6(w-u)(w-v)}[6uv-3v-3u+2]f_{n+w}, \quad (2.32a)$$

$$y(x_{n+u})-y_n=\frac{uh}{6(v-u)(w-u)}\left[6vw-3uw-3vu+2u^2\right]f_{n+u}+\frac{uh}{6(v-u)(w-v)}\left[-3uw+u^2\right]f_{n+v}$$
$$+\frac{uh}{6(w-u)(w-v)}\left[3uv-u^2\right]f_{n+w} \quad (2.32b)$$

$$y(x_{n+v})-y_n=\frac{vh}{6(v-u)(w-u)}\left[3vw-v^2\right]f_{n+u}+\frac{vh}{6(v-u)(w-v)}\left[-6uw+3wv+3uv-2v^2\right]f_{n+v}$$
$$+\frac{uh}{6(w-u)(w-v)}\left[3uv-u^2\right]f_{n+w} \quad (2.32c)$$

$$y(x_{n+w})-y_n=\frac{wh}{6(v-u)(w-u)}\left[3vw-w^2\right]f_{n+u}+\frac{wh}{6(v-u)(w-v)}\left[-3uw+w^2\right]f_{n+v}$$
$$+\frac{wh}{6(w-u)(w-v)}\left[6uv-3uw-3uw+2w^2\right]f_{n+w} \quad 2.32d)$$

Setting $u={}^1\!/_4, v=\frac{1}{2}, w={}^3\!/_4$. Our four finite differences schemes becomes

$$y_{n+1}-y_n = \frac{h}{3}\left[2f_{n+u}-f_{n+v}+2f_{n+w}\right]\text{order 4, } C_5=\frac{7}{23040} \quad (2.33a)$$

$$y_{n+u}-y_n = \frac{h}{48}\left[23f_{n+u}-16f_{n+v}+5f_{n+w}\right]\text{order 3, } C_4=\frac{-3}{2048} \quad (2.33b)$$

$$y_{n+v}-y_n = \frac{h}{12}\left[7f_{n+u}-2f_{n+v}+f_{n+w}\right]\text{order 3, } C_4=\frac{-1}{768} \quad (2.33c)$$

$$y_{n+w} - y_n = \frac{h}{16}\left[9 f_{n+u} + 3 f_{n+w}\right] \text{order } 3,\ C_4 = \frac{-3}{2048} \tag{2.33d}$$

where u = ¼ , v = ½ , w = ¾ . At Gaussian points, with $u = \frac{(5-\sqrt{15})}{10}, v = 1/2,\ w = \frac{(5+\sqrt{15})}{10}$

Equations (2.32a), (2.32b), (2.32c) and (2.32d) become

$$y_{n+1} - y_n = \frac{h}{18}\left[5 f_{n+v} + 8 f_{n+v}\ \ 5 f_{n+w}\right] order\, 6,. \tag{2.34a}$$

$$y_{n+v} - y_n = \frac{h}{180}\left[25 f_{n+u} + (40 - 12\sqrt{15}) f_{n+v} + (25 - 6\sqrt{15}) f_{n+w}\right], order\, 4, \tag{2.34b}$$

$$y_{n+w} - y_n = \frac{h}{72}\left[(10 + 3\sqrt{15}) f_{n+u} + 16 f_{n+v} + (10 - 3\sqrt{15}) f_{n+v}\right], order\, 4 \tag{2.34c}$$

$$y_{n+w} - y_n = \frac{h}{180}\left[(25 + 6\sqrt{15}) f_{n+u} + 16 f_{n+v} + (10 + 12\sqrt{15}) f_{n+v} + 25 f_{n+w}\right], order\, 4 \tag{2.34d}$$

where u = ¼ , v = ½ , w = ¾

Analyzing the basic properties of the methods using Dahlquist (1963) stability theorems and Lambert (197) approach, we find that the methods are constant, zero-stable and convergent.

3.0 Some existing one-step related methods for comparison

There are many one-step related method to the one derived. However we are considering the followings:

(a) Butcher's implicit Runge-Kutta Method $y_{n+1} = y_n + \frac{h}{2}[k_1 + k_2]$

where, $k_1 = f\left[x_n + \left(\frac{1}{2} - \frac{\sqrt{3}}{b}\right)h, y_n + \frac{1}{4}hk_1 + \left(\frac{1}{4} - \frac{\sqrt{3}}{6}\right)hk_2\right],$

$$k_2 = f\left[x_n + \left(\frac{1}{2} + \frac{\sqrt{3}}{b}\right)h, y_n + \frac{1}{4} + \left(\frac{1}{4} - \frac{\sqrt{3}}{6}\right)hk_1 + \frac{1}{4}hk_2\right]$$

of order $p = 4$

(b) Yakubu's New continuous implicit Runge-Kutta method

$$y_{n+1} = y_n + \frac{h}{18}[5K_1 + 8k_2 + 5k_3]$$

where $k_1 = f\left[x_n + \left(\frac{1}{2} - \frac{\sqrt{15}}{10}\right)h, y_n + \frac{h}{2}\left(\frac{5}{18}k_1 + \left(\frac{4}{9} - \frac{2\sqrt{15}}{15}\right)k_2 + \left(\frac{5}{18} - \frac{\sqrt{15}}{15}\right)k_3\right)\right]$

$$k_2 = f\left[x_n + \frac{1}{2}h, yn + \frac{h}{2}\left(\frac{5}{18} - \frac{\sqrt{15}}{12}\right)k_1 + \frac{4}{9}k_2 + \left(\frac{5}{18} - \frac{\sqrt{15}}{12}\right)k_3\right]$$

$$k_3 = f\left[x_n + \left(\frac{1}{2} - \frac{\sqrt{15}}{10}\right)h, y_n + \frac{h}{2}\left(\frac{5}{18} + \frac{\sqrt{15}}{15}\right)k_1 + \left(\frac{4}{9} + \frac{2\sqrt{15}}{15}\right)k_2 + \frac{5}{18}k_3\right]$$

of order p = 6 in which collocation was done at 3 points and interpolation only at x_n (one point).

4.0 Comparing two numerical examples with previous methods, at Gaussian points

We compare two of the numerical results with Butcher's 4th order method and Yakubu's order 6 methods as shown in the tables below.

The errors of numerical solutions for example I and example II with $h = 0.1$ are as below:

***Example* I**

$$y^1 = 3x^2 y,\ y(0)=1,\ 0 \le x \le 0.5, \qquad y(x) = \exp\left(x^3\right)$$

Table 4.1: **Comparing results of errors of Numerical solutions with $h = 0.1$**

Mesh value x	Butcher's 4th order method (now referred to as general order four method)	Yakubu's order six method	Our new order 4/order 6 method
0.1	-5.9997 x 10^{-4}	-1.9988 x 10^{-4}	5.6838 x 10^{-2}
0.2	-1.8107 x 10^{-3}	-1.8097 x 10^{-3}	3.7366 x 10^{-2}
0.3	-3.6776 x 10^{-2}	-3.6722 x 10^{-2}	7.7129 x 10^{-2}
0.4	-6.3088 x 10^{-2}	-6.2901 x 10^{-2}	6.0392 x 10^{-2}
0.5	-9.9115 x 10^{-2}	9.8583 x 10^{-2}	3.6903 x 10^{-2}

***Example* II**

$$y^1 = x + y,\ y(0)=1,\ 0 \le x \le 0.5,\ y(x) = 2e^x - x - 1$$

Table 4.2: Comparing results of errors of numerical solutions with $h = 0.1$

Mesh value x	Butcher's 4th order method (now referred to as general order four method)	Yakubu's order six method	Our new order 4/order 6 method
0.1	-4.0169 x 10^{-2}	4.7377 x 10^{-3}	2.8333 x 10^{-2}
0.2	1.9151 x 10^{-2}	1.8538 x 10^{-2}	2.0407 x 10^{-2}
0.3	1.8454 x 10^{-2}	8.7056 x 10^{-3}	3.1229 x 10^{-2}
0.4	2.6395 x 10^{-2}	3.5820 x 10^{-2}	2.1215 x 10^{-2}
0.5	4.07991 x 10^{-2}	5.0848 x 10^{-3}	1.1199 x 10^{-2}

From the results above the newly proposed methods are very much comparable with the earlier developed methods.

5.0 Conclusion

This paper is about exploiting off grid points for development of more accurate methods for numerical solution of ODES.

Consequently, two off-grid points collocation methods are derived for solution of initial value problems of ODEs.

One of the methods was used to solve some sample initial value problems and compared with some existing methods. The results show that the method compared favourably.

It is believed that better collocation/hybrid formulas can be exploited more off-grid points. More useful general purpose code for the solution of ODES may be discovered, which is what this paper is all about.

Acknowledgement

The authors are grateful to Dr. E. A. Adebile for his useful discussions, questions and suggestions that helped to improve this paper.

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