

A class of stiffly stable hybrid second derivative continuous linear multistep methods for stiff IVPs in ODEs.

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Abstract

Hybrid methods incorporating one or more off-step points show promise of better stability characteristics and higher order than the conventional linear multistep methods (LMM). Infact, these type of methods provides a means of bypassing the Dahlquist order barrier for the conventional LMM. This paper is concerned with a class of stiffly stable hybrid second derivative continuous LMM with an off-step point. The methods are stiffly stable for step numbers $k \leq 8$ and of order $k + 3$. It is stiffly unstable when $k \geq 9$. The definition of stiff stability is found in Lambert [21], Enright [10] and Butcher [7]. The new methods are of comparable accuracy to that of Enright [10] and the state-of-the-art code, ODE15s in MATLAB.

Keywords: Continuous Linear Multi step Methods, off-step point, Hybrid method, Stability, Root-Locus.

1.0 Introduction

In Ikhile and Okuonghae [17, 18] and Otunta et al [23, 24], classes of stiffly stable second derivative continuous linear multistep methods (SSSDCLMM) and hybrid LMM (SSSDHCLMM) for the numerical solution of the initial value problem (IVP)

$$y' = f(x, y), \quad y(x_0) = y_0, \quad y \in R \quad (1.1)$$

was introduced. Further investigation revealed that the methods in Otunta et al [21, 22] are stiffly stable for step numbers $k \leq 12$ and $k \leq 7$ respectively, see Okuonghae [25]. In this work we shall present further stiffly stable methods in the light of the above references. The class of hybrid SDCLMM of interest in this work for the numerical solution of the IVP in (1.1) is given by

$$y_{n+k} = \sum_{j=0}^{k-1} \alpha_j(t) y_{n+j} + \alpha_v(t) y_{n+v} + h\beta_{1,k}(t) f_{n+k} + h\beta_{1,v}(t) f_{n+v} + h^2 \beta_{2,k}(t) f'_{n+k}, \quad \alpha_k \neq 0 \quad (1.2)$$

The hybrid predictor is

$$y_{n+v} = \sum_{j=0}^k \alpha_j^*(t) y_{n+j} + h\beta_{3,k}(t) f_{n+k} + h^2 \beta_{4,k}(t) f'_{n+k} \quad (1.3)$$

where t is the scaled variable given as $t = (x - x_{n+1}) / h$

Similar discrete formulations are found in Butcher [6, 7, 8], Enright [10], Gragg and Stetter [15], Kohfeld and Thompson [19], amongst other references. The way the formulas are used is that (1.3)

is used to obtain an approximation for the off-step point values of y_{n+v} and f_{n+v} at the off-step point x_{n+v} in (1.2) respectively. The value of v is taken to be $k - \frac{1}{2}$ for a fixed k . The resultant methods are found to be highly stable and give accurate results highly competitive with other existing fixed step size methods in Adeniyi and Alabi [1], Alvarez and Rojo [2], Alexander [3], Arevalo et al [4], Burrage and Tian [5], Beaudet [9], Feng-Sheng [12], Onumanyi et al [22], ODE15s in MATLAB [19] and Sirisena et al [26]. Apart from that, algorithms of this nature are often found to be good numerical tools for solving stiff initial value problem. It has been found in Gear [13, 14] that methods of these kinds can be stiffly stable. Also, see Enright [10], Fatunla [11], Lambert [21], Ikhile and Okuonghae [17, 18] and Otunta, Ikhile and Okuonghae [23, 24]. The equivalent discrete LMM to (1.2) is given as

$$y_{n+k} = \sum_{j=0}^{k-1} \alpha_j y_{n+j} + \alpha_v y_{n+v} + h\beta_{1,k} f_{n+k} + h\beta_{1,v} f_{n+v} + h^2 \beta_{2,k} f'_{n+k}, \quad \alpha_k \neq 0 \quad (1.4)$$

with the predictor

$$y_{n+v} = \sum_{j=0}^k \alpha_j y_{n+j} + h\beta_{3,k} f_{n+k} + h^2 \beta_{4,k} f'_{n+k} \quad (1.5)$$

The paper is organized into the following sections. Section 2 contains the derivation of the CLMM in (1.2). Section 3 shows the derivation of the continuous hybrid predictor in (1.3). In section 4, the stiff stability of the methods is determined, using root locus method and, in section 5, the scheme is compared with an Enright [10] scheme and ODE15s code of MATLAB [19] in some numerical experiments arising from solving a practical problem.

2.0 Derivation of the second derivative CLMM.

For the solution of the initial value problem (1.1) consider the polynomial interpolant

$$y(x) = \sum_{j=0}^{k+3} a_j x^j, \quad (2.1)$$

where a_j 's are the real parameter constants to be determined. From (2.1),

$$f(x, y) = y'(x) = \sum_{j=1}^{k+3} j a_j x^{j-1}, \quad f'(x, y) = y''(x) = \sum_{j=2}^{k+3} j(j-1) a_j x^{j-2} \quad (2.2)$$

Collocating (2.2) at $x = x_{n+j}$, $j = 0(1)k$, x_{n+v} , and interpolating (2.1) at x_{n+j} , $j = 0(1)k-1$ we obtain the linear system of equations

$$\begin{pmatrix} 1 & x_n & x_n^2 & \dots & \dots & x_n^{k+3} \\ 1 & x_{n+1} & x_{n+1}^2 & \dots & \dots & x_{n+1}^{k+3} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 1 & x_{n+k-1} & x_{n+k-1}^2 & \dots & \dots & x_{n+k-1}^{k+3} \\ 1 & x_{n+v} & x_{n+v}^2 & \dots & \dots & x_{n+v}^{k+3} \\ 0 & 1 & 2x_{n+k} & \dots & \dots & (k+3)x_{n+k}^{k+2} \\ 0 & 1 & 2x_{n+v} & \dots & \dots & (k+3)x_{n+v}^{k+2} \\ 0 & 0 & 2 & \dots & \dots & (k+3)(k+2)x_{n+k}^{k+1} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ \vdots \\ a_{k+3} \end{pmatrix} = \begin{pmatrix} y_n \\ y_{n+1} \\ \vdots \\ \vdots \\ y_{n+k-1} \\ y_{n+v} \\ f_{n+k} \\ f_{n+v} \\ f'_{n+k} \end{pmatrix} \quad (2.3)$$

Solving equation (2.3), the values of a_j 's are determined and substituting the resulting values and setting $x = x_{n+1} + th$, (where $x_{n+1} = x_n + h$) into (2.1) yield the coefficients $\{\alpha_j(t)\}_{j=0}^{k-1}$, $\alpha_v(t)$,

$\alpha_k(t) = 1$, $\beta_{1,k}(t)$, $\beta_{2,k}(t)$, and $\beta_{1,v}(t)$. For a fixed value of k with $t = k - 1$, a specific scheme is obtained with order of the method as $p = k + 3$. Table (1) in the appendix A, shows the continuous coefficients of the methods for $k \leq 8$. For a fixed $t = k - 1$, results in the discrete equivalent methods and are stiffly stable.

3.0 Derivation of the second derivative hybrid predictor CLMM.

For the hybrid method $y_{n+v} = \sum_{j=0}^k \alpha_j(t) y_{n+j} + h\beta_{1,k}(t)f_{n+k} + h^2\beta_{2,k}(t)f'_{n+k}$ the coefficients $\{\alpha_j(t)\}_{j=0}^{k-1}$, $\alpha_v(t) = 1$, $\beta_{1,k}(t)$, and $\beta_{2,k}(t)$ are obtained, using the interpolant

$$y_{n+v}(x) = \sum_{j=0}^{k+2} a_j x^j, \quad (3.1)$$

In a similar sense to the section above leads to the linear system of equations for $\{a_j\}_{j=0}^{k+2}$ given as

$$\left(\begin{array}{ccccccccc} 1 & x_n & x_n^2 & \cdot & \cdot & \cdot & x_n^{k+2} & & \\ 1 & x_{n+1} & x_{n+1}^2 & \cdot & \cdot & \cdot & x_{n+1}^{k+2} & & \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ 0 & x_{n+k} & x_{n+k}^2 & \cdot & \cdot & \cdot & x_{n+k}^{k+2} & & \\ 0 & 1 & 2x_{n+k} & \cdot & \cdot & \cdot & (k+2)x_{n+k}^{k+1} & & \\ 0 & 0 & 2 & \cdot & \cdot & \cdot & (k+2)(k+1)x_{n+k}^k & & \end{array} \right) \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \\ a_{k+1} \\ a_{k+2} \end{pmatrix} = \begin{pmatrix} y_n \\ y_{n+1} \\ \vdots \\ \vdots \\ y_{n+k} \\ f_{n+k} \\ f'_{n+k} \end{pmatrix} \quad (3.2)$$

Solving equation (3.1), the values of a_j 's are and substituting the resulting values into (3.1) and setting $x = x_{n+1} + th$, (where $x_{n+1} = x_n + h$) yield the values of the coefficients $\{\alpha_j(t)\}_{j=0}^{k-1}$, $\alpha_v(t)$, $\beta_{1,k}(t)$, and $\beta_{2,k}(t)$. See table (2) in the appendix A, for the coefficients of the methods. The order of the method (1.3) is $p = k + 2$.

4.0 The stability of the methods

Considering the stability of the methods using root locus. On substituting the hybrid solution y_{n+v} at point x_{n+v} into the LMM (1.2) for a corresponding k and applying the resultant method on the scalar test problem $y' = \lambda y$, $\text{Re}(\lambda h) < 0$, $z = \lambda h$ with an arbitrary initial value, yields the stability polynomial

$$\begin{aligned}\pi(r, z) = r^k - \sum_{j=0}^{k-1} \alpha_j r^{j-1} - \alpha_v (-\sum_{j=0}^k \alpha_j r^j + z\beta_{1,k} r^k + z^2 \beta_{2,k} r^k) - z\beta_{1,k} r^k \\ - z\beta_{1,v} (-\sum_{j=0}^k \alpha_j^* r^j + z\beta_{1,k} r^k + z^2 \beta_{2,k} r^k) - z^2 \beta_{2,k} r^k.\end{aligned}\quad (4.1)$$

Plotting $|r_j(z)|$ against z reveals the interval of absolute stability of the methods. The general graphical form of the root locus plot has been discussed in Lambert [20], Otunta, Ikhile and, Okuonghae

[23, 24]. Method (1.2) is said to be stable at z if $|r_j(z)| \leq 1$ where $r_j(z), j = 0(1)k$ are the roots of the polynomial in (4.1) with roots of $|r_j(z)| = 1$ been simple. Plotting the root locus of $\pi(r, z) = 0$, it is observed that the methods in (1.2) are stiffly stable for $k \leq 8$. The graphs below show the root loci and thus the interval of absolute / stiff stability of each method for a fixed value of $k \leq 8$. The case of $k > 9$ are stiffly unstable, see fig (4.9).

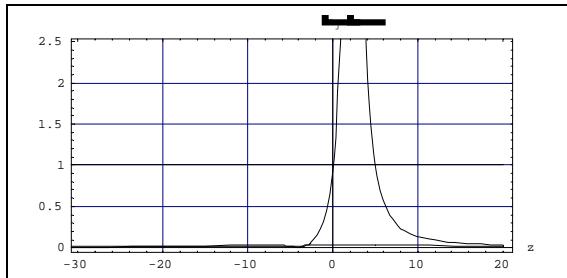


Figure 4.1: Root locus for $k = 1$

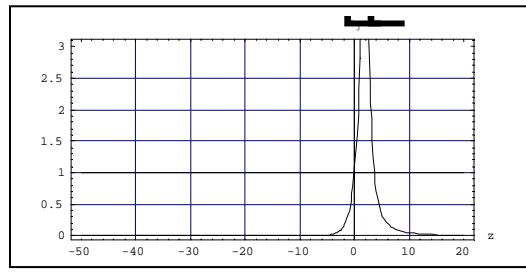


Figure 4.2: Root locus for $k = 2$

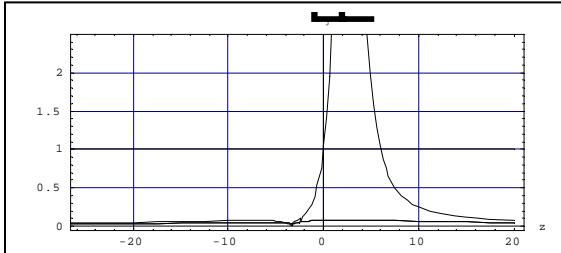


Figure 4.3: Root locus for $k = 3$

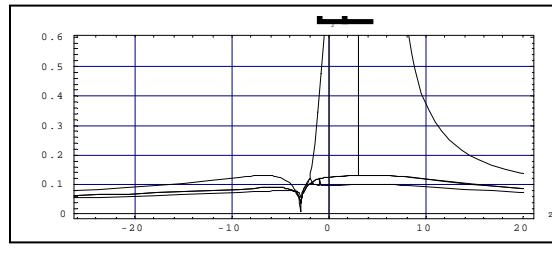


Figure 4.4: Root locus for $k = 4$

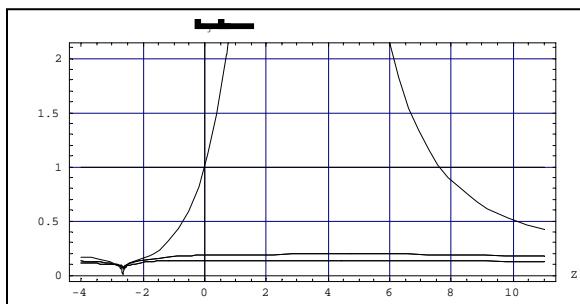


Figure 4.5: Root locus for $k = 5$

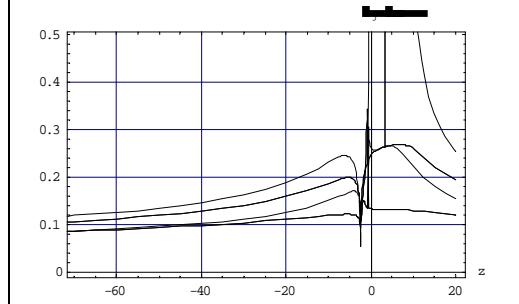


Figure 4.6: Root locus for $k = 6$

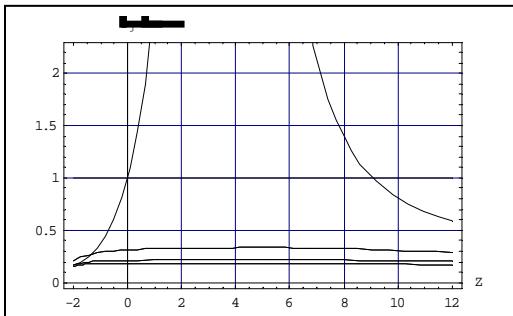


Figure 4.7: Root locus for $k = 7$

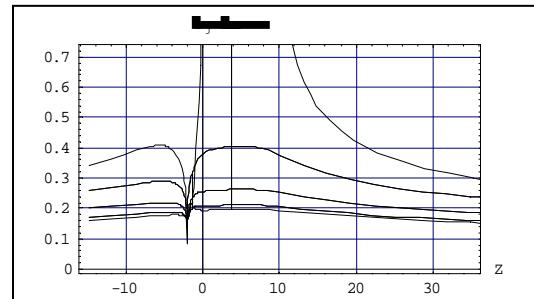


Figure 4.8: Root locus for $k = 8$

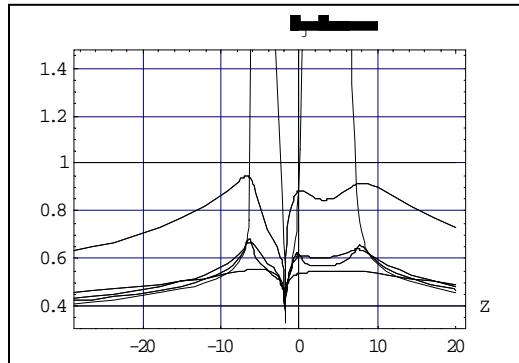


Figure 4.9: Root locus for $k = 9$

Table 4.1: The step number, interval of absolute stability, error constant and order of methods in (1.2) and (1.3) respectively.

k	Interval of Absolute Stability for z		Error Constants		Order	
	t	SDCLMM (1.4)	SDCLMM (1.4)	SDCHLMM 1.5)	P (1.4)	P ¹ (1.5)
1	0	$(-\infty, 0) \cup (3.62, \infty)$	$\frac{1}{8160}$	$-\frac{1}{384}$	4	3
2	1	$(-\infty, 0) \cup (5, \infty)$	$\frac{1}{28440}$	$-\frac{1}{1280}$	5	4
3	2	$(-\infty, 0) \cup (6, \infty)$	$\frac{3}{218680}$	$-\frac{1}{3072}$	6	5
4	3	$(-\infty, 0) \cup (6.86, \infty)$	$\frac{3}{468020}$	$-\frac{1}{6144}$	7	6
5	4	$(-\infty, -1.791) \cup (7.62, \infty)$	$\frac{25}{7400316}$	$-\frac{3}{32768}$	8	7
6	5	$(-\infty, 0) \cup (8.24, \infty)$	$\frac{5}{2575132}$	$-\frac{11}{196608}$	9	8
7	6	$(-\infty, 0) \cup (9.06, \infty)$	$\frac{245}{205595324}$	$-\frac{143}{3932160}$	10	9
8	7	$(-\infty, 0) \cup (9.8, \infty)$	$\frac{490}{636338967}$	$-\frac{13}{524288}$	11	10
9	8	Unstable	-	-	12	11

Table 4.2: The range of t for which the hybrid LMM (1.2) are stable, and the order of methods (1.2) and (1.3).

k	The range of t for which the methods are zero-stable	Order	
	SDCLMM (1.4)	P (1.4)	P ^I (1.5)
1	$\{t : t \in (-\infty, \infty)\}$	4	3
2	$\{t : t \in (-\infty, -1.24726) \cup (-0.275, \infty)\}$	5	4
3	$\{t : t \in (-\infty, -1.105) \cup (0.65, \infty)\}$	6	5
4	$\{t : t \in (1, 1.3) \cup (1.4815, 7.4)\}$	7	6
5	$\{t : t \in (-\infty, -1.05) \cup (3.395, 4.2595) \cup (4.75, \infty)\}$	8	7
6	$\{t : t \in [3, \infty]\}$	9	8

Table 4.2: The range of t for which the hybrid LMM (1.2) are stable, and the order of methods (1.2) and (1.3). (contd)

k	The range of t for which the methods are zero-stable	Order	
	SDCLMM (1.4)	P (1.4)	P ^I (1.5)
7	$\{t : t \in (-\infty, -1.22) \cup (7.38, \infty)\}$	10	9
8	$\{t : t \in (-\infty, -1.21) \cup (3, 4) \cup (5, \infty)\}$	11	10
9	Unstable	-	-

5.0 Numerical experiments

Consider the numerical solution to the initial value problems in Enright [10],

$$\text{P[1]} \quad y' = \begin{pmatrix} -0.1 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & -100 & 0 \\ 0 & 0 & 0 & -1000 \end{pmatrix} y, \quad y(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad y(x) = \begin{pmatrix} e^{-0.1x} \\ e^{-10x} \\ e^{-100x} \\ e^{-1000x} \end{pmatrix},$$

$$y = (y_a, y_b, y_c, y_d)^T$$

with $x \in [0, 1/121] \cup [2000, \infty)$. To solve this initial value problem, the continuous form of the method in (1.4) for $k = 1$, in table (1) of the appendix A is now,

$$y_{n+1} = \left(\frac{1}{17} + \frac{32t^3}{17} + \frac{48t^4}{17} \right) y_n + \left(\frac{16}{17} - \frac{32t^3}{17} - \frac{48t^4}{17} \right) y_{n+\frac{1}{2}} + h \left(\begin{aligned} & \left(\frac{4}{17} + \frac{60t^3}{17} + \frac{60t^4}{17} \right) f_{n+\frac{1}{2}} \\ & + \left(\frac{15}{17} + t - \frac{44t^3}{17} - \frac{32t^4}{17} \right) f_{n+1} \end{aligned} \right) \quad (5.1)$$

$$+ h^2 \left(\frac{-5t}{34} - \frac{25t^2}{34} - \frac{20t^3}{17} - \frac{10t^4}{17} \right) f'_{n+1}$$

of order $p = 4$. Setting $t = 0$ in (5.1) we obtain the equivalent discrete form of (5.1),

$$y_{n+1} = \frac{1}{17} y_n + \frac{16}{17} y_{n+\frac{1}{2}} + h \left(\frac{4}{17} f_{n+\frac{1}{2}} + \frac{15}{17} f_{n+1} \right) \quad (5.2)$$

Similarly, for $k = 1$ in table (2), the continuous form of (1.5) is

$$y_{n+\frac{1}{2}} = (-t^3)y_n + (1+t^3)y_{n+1} - h(t-t^3)f_{n+1} + h^2\left(\frac{t^2}{2} - \frac{t^3}{2}\right)f'_{n+1} \quad (5.3)$$

and setting $t = -\frac{1}{2}$ in (5.3) is the discrete hybrid solution formula

$$y_{n+\frac{1}{2}} = \frac{1}{8}y_n + \frac{7}{8}y_{n+1} - \frac{3h}{8}f_{n+1} + \frac{h^2}{16}f'_{n+1} \quad (5.4)$$

is of order $p = 3$. Applying the method in (5.2) with the corresponding hybrid solution method in (5.4) on the initial value problem above leads to solving implicit set of equations. This equations are thus solved using the Newton Raphson iterative scheme

$$y_{n+k}^{[s+1]} = y_{n+k}^{[s]} - F'(y_{n+k}^{[s]})^{-1}F(y_{n+k}^{[s]}), s = 0, 1, 2, \dots \quad (5.5)$$

as suggested by Enright [10], Fatunla [11] and Lambert [21]. The inverse Euler's method

$$y_{n+1}^{[0]} = y_n + \frac{hy_n y'_n}{y_n - hy'_n} \quad (5.6)$$

has been used to generate the starting values for the iterative scheme in (5.5). The graph, fig. (5.1) of the solution components $y = (y_a, y_b, y_c, y_d)^T$ below shows the numerical solutions from the scheme (5.2), this is compared with those of Enright [10] and MATLAB ODE 15s code considered in Higham et al [19]. Note that in graphs of fig (1.5) the Sshdclmm which is the same as SDCLMM (5.2) coincides with that of Enright [10] and ODE15s of MATLAB in the three components y_b, y_c, y_d showing that they are of comparable accuracy.

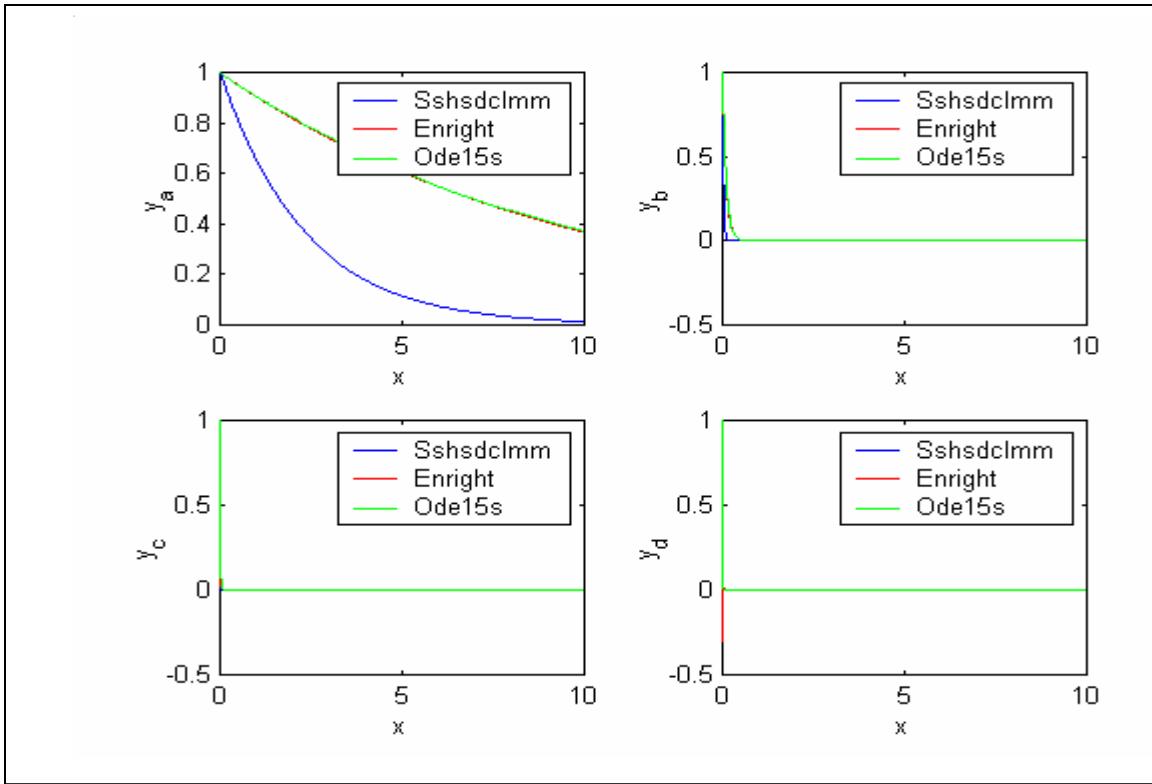


Figure 5.1: Numerical Solutions, $y = (y_a, y_b, y_c, y_d)^T$ of $P[1]$

Conclusively, in this paper, a class of SDCLMM formulas with an off-step point of step number $k \leq 8$ in (1.2), and the corresponding second derivative hybrid methods have been proposed. The orders of the methods are shown in table (4.1). The stability graphs in fig (4.1)-(4.9) shows that, the derived methods are stiffly stable like those of Enright [10]. The numerical solutions from SDCLMM compares with that of the Enright [10] and MATLAB Ode15s code in Higham et al [19] in figure (5.1) and justify the stiff stability and accuracy of the proposed methods.

APPENDIX A

Table 1: Second Derivative Continuous LMM.

k	t	j	$\alpha_j(t)$	$\alpha_j(k-1)$	$\beta_{1,j}(t)$	$\beta_{1,j}(k-1)$	$\beta_{2,k}(t)$	$\beta_{2,k}(k-1)$
1	0	0	$\frac{1}{17} + \frac{32t^3}{17} + \frac{48t^4}{17}$	$\frac{1}{17}$	0	0	0	0

		$\frac{1}{2}$	$\frac{16}{17} - \frac{32t^3}{17} - \frac{48t^4}{17}$	$\frac{16}{17}$	$\frac{4}{17} + \frac{60t^3}{17} + \frac{60t^4}{17}$	$\frac{4}{17}$	0	0
		1	1	1	$\frac{15}{17} + t - \frac{44t^3}{17} - \frac{32t^4}{17}$	$\frac{5}{17}$	$-\frac{5t}{34} - \frac{25t^2}{34} - \frac{20t^3}{17} - \frac{10t^4}{17}$	0
2	1	0	$\frac{-23t}{711} + \frac{131t^2}{711} - \frac{265t^3}{711} + \frac{224t^4}{711} - \frac{68t^5}{711}$	$-\frac{1}{711}$	0	0	0	0
		1	$1 - \frac{369t}{79} + \frac{453t^2}{79} + \frac{145t^3}{79} - \frac{58t^4}{79} + \frac{228t^5}{79}$	$\frac{8}{79}$	0	0	0	0
		$\frac{3}{2}$	$\frac{3344t}{711} - \frac{4208t^2}{711} - \frac{1040t^3}{711} + \frac{4528t^4}{711} - \frac{1984t^5}{711}$	$\frac{640}{711}$	$-\frac{424t}{237} + \frac{1096t^2}{237} - \frac{104t^3}{237} - \frac{1064t^4}{237} + \frac{5607t^5}{237}$	$\frac{64}{237}$	0	0
		2	1	1	$\frac{32t}{79} - \frac{117t^2}{79} + \frac{63t^3}{79} + \frac{128t^4}{79} - \frac{84t^5}{79}$	$\frac{22}{79}$	$-\frac{13t}{158} + \frac{25t^2}{79} - \frac{33t^3}{158} - \frac{26t^4}{79} + \frac{22t^5}{79}$	$-\frac{2}{79}$
3	2	0	$-\frac{1638t}{19525} + \frac{5343t^2}{19525} - \frac{13633t^3}{39050} + \frac{8523t^4}{39050} - \frac{1308t^5}{19525} + \frac{158t^6}{19525}$	$\frac{4}{19525}$	0	0	0	0
		1	$1 - \frac{5296t}{2343} + \frac{6304t^2}{7029} + \frac{10964t^3}{7029} - \frac{4219t^4}{2343} + \frac{4924t^5}{7029} - \frac{676t^6}{7029}$	$-\frac{3}{781}$	0	0	0	0
		2	$\frac{12006t}{781} - \frac{15063t^2}{781} - \frac{8831t^3}{1562} + \frac{28623t^4}{1562} - \frac{7200t^5}{781} + \frac{1142t^6}{781}$	$\frac{108}{781}$	0	0	0	0
		$\frac{5}{2}$	$-\frac{69376t}{5325} + \frac{289408t^2}{15975} + \frac{70976t^3}{15975} - \frac{89152t^4}{5325} + \frac{137152t^5}{15975} - \frac{21952t^6}{15975}$	$\frac{1536}{1775}$	$\frac{28096t}{3905} - \frac{132128t^2}{11715} - \frac{16336t^3}{11715} + \frac{40192t^4}{3905} - \frac{67952t^5}{11715} + \frac{11552t^6}{11715}$	$\frac{1152}{3905}$	0	0
		3	1	1	$-\frac{1647t}{781} + \frac{2862t^2}{781} + \frac{27t^3}{781} + \frac{1620t^5}{781} - \frac{296t^6}{781}$	$\frac{210}{781}$	$\frac{342t}{781} - \frac{1227t^2}{1562} + \frac{20t^3}{781} + \frac{1087t^4}{1562} - \frac{362t^5}{781} + \frac{70t^6}{781}$	$-\frac{18}{781}$

4	3	0	$-\frac{64125t}{655228} + \frac{376575t^2}{1310456} - \frac{1317725t^3}{3931368} + \frac{791383t^4}{3931368} - \frac{12385t^5}{187208} + \frac{796t^6}{70203} - \frac{781t^7}{982842}$	$\frac{9}{163807}$	0	0	0	0
		1	$1 - \frac{64719t}{33430} + \frac{40797t^2}{83575} + \frac{22866t^3}{16715} - \frac{115374t^4}{83575} + \frac{93579t^5}{167150} - \frac{8998t^6}{83575} + \frac{678t^7}{83575}$	$\frac{64}{83575}$	0	0	0	0
		2	$\frac{79725t}{13372} - \frac{149135t^2}{26744} - \frac{643177t^3}{240696} + \frac{439969t^4}{80232} - \frac{659905t^5}{240696} - \frac{5944t^6}{10029} - \frac{2905t^7}{60174}$	$\frac{-24}{3343}$	0	0	0	0
		3	$-\frac{204525t}{6686} + \frac{146535t^2}{3343} + \frac{71524t^3}{10029} - \frac{383867t^4}{10029} + \frac{153553t^5}{6686} - \frac{55738t^6}{10029} + \frac{4934t^7}{10029}$	$\frac{576}{3343}$	0	0	0	0
		$\frac{7}{2}$	$\frac{21836928}{819035} - \frac{15984428t^2}{4095175} - \frac{40486016t^3}{7371315} + \frac{417353728t^4}{12285525} - \frac{109086976t^5}{5265225} + \frac{8883008t^6}{1755075} - \frac{16622848t^7}{36856575}$	$\frac{3416064}{4095175}$	$-\frac{346176t}{23401} + \frac{2638752t^2}{117005} + \frac{145280t^3}{70203} - \frac{2269824t^4}{117005} + \frac{623488t^5}{50145} - \frac{52704t^6}{16715} + \frac{101824t^7}{351015}$	$\frac{36864}{117005}$	0	0
		4	1	1	$\frac{28475t}{6686} - \frac{90585t^2}{13372} - \frac{3543t^3}{13372} + \frac{76761t^4}{13372} - \frac{52083t^5}{13372} + \frac{3456t^6}{3343} - \frac{331t^7}{3343}$	$\frac{876}{3343}$	$-\frac{5775t}{6686} + \frac{18665t^2}{13372} + \frac{337t^3}{13372} - \frac{15697t^4}{13372} + \frac{10921t^5}{13372} - \frac{742t^6}{3343} + \frac{73t^7}{3343}$	$-\frac{72}{3343}$
		5	4	0	$-\frac{699916}{7136019} + \frac{6064429^2}{21408057} - \frac{9379553^3}{28544076} + \frac{11522039^4}{57088152} - \frac{2045435^5}{28544076} + \frac{281923^6}{19029384} - \frac{23693^7}{14272038} + \frac{3343^8}{42816114}$	$\frac{16}{792891}$	0	0
		1	$1 - \frac{3493090t}{1850079} + \frac{11964223t^2}{25901106} + \frac{44635985t^3}{345348808} - \frac{45828775t^4}{34534808} + \frac{2858785t^5}{4933544} - \frac{4630849t^6}{34534808} + \frac{208825t^7}{12950553} - \frac{20611t^8}{25901106}$	$\frac{-1125}{4316851}$	0	0	0	0

		2	$\begin{aligned} & \frac{1125549t}{2202475} - \frac{9993914t^2}{2202475} - \frac{9595517t^3}{4404950} \\ & + \frac{3899987t^4}{8809900} - \frac{518405t^5}{2202475} + \frac{534493t^6}{8809900} \\ & - \frac{172449t^7}{2202475} + \frac{895t^8}{2202475} \end{aligned}$	$\frac{160}{88099}$	0	0	0	
		3	$\begin{aligned} & \frac{8057462t}{792891} + \frac{67081609t^2}{4757346} + \frac{1666607t^3}{704792} \\ & - \frac{7581540t^4}{6343128} + \frac{15863179t^5}{2114376} - \frac{13524359t^6}{6343128} \\ & + \frac{233837t^7}{792891} - \frac{76789t^8}{475734} \end{aligned}$	$\frac{-1000}{88099}$	0	0	0	0
		4	$\begin{aligned} & \frac{113809964t}{264297} - \frac{21446327t^2}{352396} - \frac{1079821t^3}{704792} + \frac{46881091}{352396} \\ & - \frac{16578689t^5}{704792} + \frac{1022180t^6}{528594} - \frac{1132163t^7}{528594} + \frac{65479t^8}{528594} \end{aligned}$	$\frac{18000}{88099}$	0	0	0	0
		$\frac{9}{2}$	$\begin{aligned} & - \frac{5646312243.2t}{1248803325} + \frac{1857679259.64t^2}{2622486982.5} \\ & + \frac{1673148262.4t^3}{8741623275} - \frac{5058504578.56t^4}{8741623275} \\ & + \frac{5168675379.2t^5}{1248803325} - \frac{3746831667.2t^6}{2913874425} \\ & + \frac{1670309785.6t^7}{8741623275} - \frac{2913036032.t^8}{2622486982.5} \end{aligned}$	$\frac{3129344}{3885165}$	$\begin{aligned} & \frac{96131072}{3964455} - \frac{3222169088^2}{83253555} \\ & - \frac{2897024^3}{27751185} + \frac{869098816^4}{27751185} \\ & - \frac{91312832^5}{3964455} + \frac{67709632^6}{9250395} \\ & - \frac{30830656^7}{27751185} + \frac{5485952^8}{83253555} \end{aligned}$	$\frac{20480}{61669}$	0	0
		5	1	1	$\begin{aligned} & - \frac{584913t}{88099} + \frac{954037t^2}{88099} \\ & - \frac{99215t^3}{352396} - \frac{3050551t^4}{352396} \\ & + \frac{2320587t^5}{352396} - \frac{758393t^6}{352396} \\ & + \frac{29570t^7}{88099} - \frac{1801t^8}{88099} \end{aligned}$	$\frac{22620}{88099}$	$\begin{aligned} & \frac{115248t}{88099} \\ & - \frac{379211t^2}{176198} \\ & + \frac{28693t^3}{352396} \\ & + \frac{602857t^4}{352396} \\ & - \frac{465317t^5}{352396} \\ & + \frac{154057t^6}{352396} \\ & - \frac{6092t^7}{88099} \\ & + \frac{377t^8}{88099} \end{aligned}$	$\frac{-1800}{88099}$
6	5	0	$\begin{aligned} & - \frac{4174515}{44512996} + \frac{48714977^2}{178051984} - \frac{472230937^3}{1456788960} \\ & + \frac{4986212328^4}{2403701780} - \frac{119232242^5}{1502313615} + \frac{11265493^6}{600925446} \\ & - \frac{11739689^7}{4370366880} + \frac{2577491^8}{1201850890} - \frac{8009^9}{1092591720} \end{aligned}$	$\frac{-100}{11128249}$	0	0	0	0
		1	$\begin{aligned} & 1 - \frac{6348175}{3310884} + \frac{32125055^2}{59595912} + \frac{222518389^3}{178787736} \\ & - \frac{122076245^4}{89393868} + \frac{14444686^5}{22348467} - \frac{30615595^6}{178787736} \\ & + \frac{4711601t^7}{178787736} - \frac{49352t^8}{22348467} + \frac{3493^9}{44696934} \end{aligned}$	$\frac{32}{275907}$	0	0	0	0

		2	$\begin{aligned} & \frac{45953325t}{9012962} - \frac{168613335t^2}{36051848} - \frac{139094827t^3}{72103696} \\ & + \frac{119420843t^4}{27038886} - \frac{91773929t^5}{36051848} + \frac{81060031t^6}{108155544} \\ & - \frac{26710037t^7}{216311088} + \frac{195729t^8}{18025924} - \frac{21541t^9}{54077772} \end{aligned}$	$\frac{-3375}{4506481}$	0	0	0	0
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		3	$\begin{aligned} & -\frac{798993t}{91969} + \frac{22646777t^2}{1839380} + \frac{122121299t^3}{82772100} \\ & -\frac{1250912407t^4}{124158150} + \frac{847316207t^5}{124158150} \\ & -\frac{109328437t^6}{49663260} + \frac{95947873t^7}{248316300} - \frac{2211974t^8}{62079075} \\ & + \frac{84229t^9}{62079075} \end{aligned}$	$\frac{320}{91969}$	0	0	0	0
		4	$\begin{aligned} & \frac{6359025t}{367876} - \frac{120846385t^2}{4414512} + \frac{14454047t^3}{79461216} \\ & + \frac{857214137t^4}{39730608} - \frac{325896509t^5}{19865304} + \frac{113171269t^6}{19865304} \\ & - \frac{84095975t^7}{76461216} + \frac{2030395t^8}{19865304} - \frac{80359t^9}{19865304} \end{aligned}$	$\frac{-1500}{91969}$	0	0	0	0
		5	$\begin{aligned} & -\frac{29490885}{367876} + \frac{98275959^2}{735752} - \frac{35540633^3}{3678760} \\ & -\frac{563988053^4}{5518140} + \frac{77072927t^5}{919690} - \frac{67896979^6}{2207256} \\ & + \frac{66198329^7}{11036280} - \frac{278266t^8}{459845} + \frac{68749t^9}{2759070} \end{aligned}$	$\frac{21600}{91969}$	0	0	0	0
		$\frac{11}{2}$	$\begin{aligned} & \frac{336068098048t}{4907557809} - \frac{25318254336512t^2}{220840101405} \\ & + \frac{2714044850432t^3}{301145592825} + \frac{289677752926976t^4}{3312601521075} \\ & - \frac{239314596080976t^5}{3312601521075} - \frac{17668282651648t^6}{662520304215} \\ & - \frac{1573937491712t^7}{301145592825} + \frac{1754648862464t^8}{3312601521075} \\ & - \frac{6595554304t^9}{301145592825} \end{aligned}$	$\frac{1273692160}{1635852603}$	$\begin{aligned} & -\frac{250250240}{7081613} \\ & + \frac{3809158912^2}{63734517} \\ & - \frac{480860032^3}{86910705} \\ & - \frac{43247414650t^4}{956017755} \\ & + \frac{3635649689t^5}{956017755} \\ & - \frac{2722393088^6}{191203551} \\ & + \frac{245917312^7}{86910705} \\ & - \frac{278003584^8}{956017755} \\ & + \frac{1059584^9}{86910705} \end{aligned}$	$\frac{2457600}{7081613}$	0	0

		6	1	1	$\begin{aligned} & \frac{842994t}{91969} - \frac{2887935t^2}{183938} \\ & + \frac{190829t^3}{1103628} \\ & + \frac{3894754t^4}{3310884} \\ & - \frac{33479269t^5}{3310884} \\ & + \frac{12764765t^6}{3310884} \\ & \frac{1291721t^7}{1655442} + \frac{67631t^8}{827721} \\ & - \frac{2890t^9}{827721} \end{aligned}$	$\begin{aligned} & \frac{23220}{91969} - \frac{161595t}{91969} \\ & + \frac{1112913t^2}{367876} \\ & - \frac{261435t^3}{735752} \\ & - \frac{415043t^4}{183938} \\ & + \frac{720327t^5}{367876} \\ & - \frac{276853t^6}{367876} \\ & + \frac{113025t^7}{735752} \\ & - \frac{2987t^8}{183938} \\ & + \frac{129t^9}{183938} \end{aligned}$	- 1800 91969	
7	6	0	$\begin{aligned} & \frac{69651714t}{789672949} + \frac{2070907443t^2}{7896729490} - \frac{15164280563t^3}{47380376940} \\ & + \frac{6091987139t^4}{284282261640} - \frac{49931536517t^5}{568564523280} \\ & + \frac{1007900801t^6}{43735732560} - \frac{2216052119t^7}{568564523280} \\ & + \frac{234104657t^8}{568564523280} - \frac{1759021t^9}{71070565410} + \frac{91969t^{10}}{142141130820} \end{aligned}$	$\frac{3600}{789672949}$	0	0	0	0

		1	$\begin{aligned} & 1 - \frac{2028418449}{1027976620} + \frac{8892392353t^2}{1356929138} \\ & + \frac{146433548118t^3}{122123622460} - \frac{350216793059t^4}{2442472449120} \\ & + \frac{7473585649t^5}{10176968580} - \frac{8836692091t^6}{40707874120} \\ & + \frac{1608722050t^7}{40707874120} - \frac{3571898357t^8}{81415748340} \\ & + \frac{15168077t^9}{55510737480} - \frac{4478791t^{10}}{610618112280} \end{aligned}$	$\frac{-34300}{565387141}$	0	0	0	0
		2	$\begin{aligned} & \frac{224231150}{42053589} - \frac{436438585t^2}{84107178} - \frac{2566215265t^3}{1513929204} \\ & + \frac{4267832653t^4}{9083575224} - \frac{1784511694t^5}{6055716816} + \frac{651769019t^6}{672857424} \\ & - \frac{114286419t^7}{6055716816} + \frac{133060867t^8}{6055716816} - \frac{59618t^9}{42053589} \\ & + \frac{17760t^{10}}{4541787612} \end{aligned}$	$\frac{5488}{14017863}$	0	0	0	0
		3	$\begin{aligned} & - \frac{2075114910t}{228958429} + \frac{6113910429t^2}{457916858} \\ & + \frac{144711823t^3}{196250082} - \frac{346149934597t^4}{32970013776} \\ & + \frac{9120083039t^5}{1177500492} - \frac{6552495679t^6}{2355000984} \\ & + \frac{2373518893t^7}{4121251722} - \frac{2312294909t^8}{32970013776} \\ & + \frac{5517305t^9}{1177500492} - \frac{1095469t^{10}}{8242503444} \end{aligned}$	$\frac{-7875}{4672621}$	0	0	0	0

		4	$\frac{360893874}{23363105} - \frac{17753260129t^2}{700893150} + \frac{2114680617t^3}{1261607670}$ $+ \frac{48146798431t^4}{25232153400} - \frac{265543641403t^5}{16821435600}$ $+ \frac{102262906053t^6}{16821435600} - \frac{22299965633t^7}{16821435600}$ $+ \frac{2830967531t^8}{16821435600} - \frac{73396427t^9}{6308038350} + \frac{4284961t^{10}}{12616076700}$	$\frac{27440}{4672621}$	0	0	0	0
		5	$- \frac{51729448t}{18690484} + \frac{8912661197t^2}{186904840} - \frac{20866448911t^3}{3364287120}$ $- \frac{701242273427t^4}{20185722720} + \frac{724029882t^5}{23363105} - \frac{4228215772t^6}{3364287120}$ $+ \frac{320947702t^7}{1121429040} - \frac{253860832t^8}{6728574240} + \frac{4536070t^9}{1682143560}$ $- \frac{409113t^{10}}{5046430680}$	$\frac{-102900}{4672621}$	0	0	0	0
		6	$\frac{2670932946t}{23363105} - \frac{1893595275t^2}{9345242} + \frac{9596087059t^3}{280357260}$ $+ \frac{242157260177t^4}{1682143560} - \frac{45423148765t^5}{3364287120}$ $+ \frac{191222338001t^6}{3364287120} - \frac{45082720913t^7}{3364287120}$ $+ \frac{6143402081t^8}{210267945} - \frac{28323773t^9}{841071780} + \frac{3509641t^{10}}{841071780}$	$\frac{1234800}{4672621}$	0	0	0	0
		13	$- \frac{1844489485}{1915351737} \frac{254656t}{7995} + \frac{1803395320}{1053443455} \frac{16377856t^2}{789725}$ $- \frac{4012250937}{1354427300} \frac{3648896t^3}{301075} + \frac{3449548462}{2844297330} \frac{951897088t^4}{6322575}$ $+ \frac{1549009447}{1354427300} \frac{04951296t^5}{301075} - \frac{5596276203}{1157630171} \frac{79648t^6}{1975}$ $+ \frac{1084748165}{9480991102} \frac{42881792t^7}{107525} - \frac{1483713574}{9480991102} \frac{1599744t^8}{107525}$ $+ \frac{1761153341}{1520120426} \frac{44t^9}{825} - \frac{1024309927}{2844297330} \frac{10656t^{10}}{6322575}$	$\frac{215916216320}{286651280487}$	$\frac{1222209208}{425208511} \frac{32t}{2338646810} - \frac{1202443148}{5} \frac{0832t^2}{9373817507}$ $+ \frac{7617687121}{1002277204} \frac{2032t^4}{2104782129} - \frac{3468180905}{3468180905} \frac{3184t^5}{45} + \frac{3787207577}{100227720} \frac{6t^6}{256994155}$ $- \frac{2469952979}{7015940431} \frac{968t^7}{5} + \frac{3411263580}{7015940431} \frac{16t^8}{5}$ $- \frac{110423552}{303720365} \frac{t^9}{2104782129} + \frac{2402980864}{45} \frac{t^{10}}{2104782129}$	$\frac{1445068800}{668184803}$	0	0

		7	1	1	$- \frac{330700260t}{4672621} + \frac{596419527t^2}{4672621}$ $- \frac{692882905t^3}{28035726} + \frac{2496961583t^4}{28035726}$ $+ \frac{403553843t^5}{4672621} - \frac{174341678t^6}{4672621}$ $+ \frac{84358565t^7}{9345242} - \frac{11806649t^8}{9345242}$ $+ \frac{1342856t^9}{14017863} - \frac{42782t^{10}}{14017863}$	$\frac{6977880}{4672621}$	$\frac{371000520t}{4672621}$ $- \frac{48140136t^2}{4672621}$ $+ \frac{133232976t^3}{4672621}$ $+ \frac{933341571t^4}{9345242}$ $+ \frac{455575806t^5}{4672621}$ $+ \frac{197868825t^6}{4672621}$ $+ \frac{48140136t^7}{4672621}$ $+ \frac{13558419t^8}{9345242}$ $- \frac{517554t^9}{4672621}$ $+ \frac{16614t^{10}}{4672621}$	$\frac{-3175200}{4672621}$
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8	7	0	$ \begin{aligned} & -\frac{5728513063t}{69418796400} + \frac{348097999327t^2}{1388375928000} \\ & -\frac{690498053107t^3}{2186692086600} + \frac{25774987057601t^4}{116623577952000} \\ & -\frac{3371445103849t^5}{349870733856000} + \frac{76612177829t^6}{2776751856000} \\ & -\frac{44055758353t^7}{8330255568000} + \frac{3740852749t^8}{5553503712000} \\ & -\frac{19166948201t^9}{349870733856000} + \frac{18787399t^{10}}{7288973622000} \\ & -\frac{4672621t^{11}}{87467683464000} \end{aligned} $	$-\frac{49}{19282999}$	0	0	0
		1	$ \begin{aligned} & 1 - \frac{10219095383t}{5013579740} + \frac{462385680589t^2}{586588829580} \\ & + \frac{269440887747t^3}{2346355318320} - \frac{136312549889t^4}{90244435320} \\ & + \frac{391078507189t^5}{469271063664} - \frac{52842126229t^6}{195529609860} \\ & + \frac{43739129483t^7}{782118439440} - \frac{8797232213t^8}{1173177659160} \\ & + \frac{1484308259t^9}{2346355318320} - \frac{17941097t^{10}}{586588829580} \\ & + \frac{379711t^{11}}{586588829580} \end{aligned} $	$\frac{115200}{3258826831}$	0	0	0
		2	$ \begin{aligned} & \frac{158728046659t}{27998914548} - \frac{1098684094417t^2}{186659430320} \\ & - \frac{3571848978769t^3}{2519902309320} + \frac{102664403939231t^4}{20159218474560} \\ & - \frac{13991590065943t^5}{4031843694912} + \frac{842720196463t^6}{671973949152} \\ & - \frac{311272092349t^7}{1119956581920} + \frac{262686800993t^8}{6719739491520} \\ & - \frac{1252192057t^9}{366531244992} + \frac{427649777t^{10}}{2519902309320} \\ & - \frac{18492979t^{11}}{5039804618640} \end{aligned} $	$-\frac{548800}{2333242879}$	0	0	0

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		$ \begin{aligned} & -\frac{31478358275t}{3123845838} + \frac{193686581725t^2}{12495383352} \\ & -\frac{43236487013t^3}{224916900336} - \frac{18224491267t^4}{1561922919} \\ & + \frac{2111168354045t^5}{224916900336} - \frac{46284220133t^6}{12495383352} \\ & + \frac{65301807755t^7}{74972300112} - \frac{29612159t^8}{231395988} \\ & + \frac{2601571907t^9}{224916900336} - \frac{409715t^{10}}{694187964} \\ & + \frac{733399t^{11}}{56229225084} \end{aligned} $	$\frac{175616}{173546991}$	0	0	0	0
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		4	$ \begin{aligned} & \frac{15855858005 t}{925583952} - \frac{2270730111949 t^2}{77749051968} \\ & + \frac{256217569087 t^3}{68030420472} + \frac{45559576974799 t^4}{2176973455104} \\ & - \frac{41141119093469 t^5}{2176973455104} + \frac{413389396811 t^6}{51832701312} \\ & - \frac{102133456805 t^7}{51832701312} + \frac{93615428041 t^8}{310996207872} \\ & - \frac{61143567389 t^9}{2176973455104} + \frac{200218589 t^{10}}{136060840944} \\ & - \frac{18097969 t^{11}}{544243363776} \end{aligned} $	$\frac{-63000}{19282999}$	0	0	0	0
		5	$ \begin{aligned} & - \frac{152046770239 t}{5784899700} + \frac{113285949322 t^2}{2410374875} \\ & - \frac{1866601952993 t^3}{208256389200} - \frac{16881668927851 t^4}{520640973000} \\ & + \frac{32895044096441 t^5}{1041281946000} - \frac{607518394409 t^6}{43386747750} \\ & + \frac{416594317361 t^7}{115697994000} - \frac{98781309493 t^8}{173546991000} \\ & - \frac{56964643459 t^9}{1041281946000} - \frac{765732157 t^{10}}{260320486500} \\ & + \frac{17702959 t^{11}}{260320486500} \end{aligned} $	$\frac{175616}{19282999}$	0	0	0	0
		6	$ \begin{aligned} & \frac{145365493819 t}{3470939820} - \frac{1068768729119 t^2}{13883759280} \\ & + \frac{1099609342969 t^3}{62476916760} + \frac{8599890819337 t^4}{166605111360} \\ & - \frac{5290600070239 t^5}{99963066816} + \frac{674900703829 t^6}{27767518560} \\ & - \frac{537431918551 t^7}{83302555680} + \frac{19441859197 t^8}{18511679040} \\ & - \frac{51846339431 t^9}{499815334080} + \frac{119121793 t^{10}}{20825638920} \\ & - \frac{16912939 t^{11}}{124953833520} \end{aligned} $	$\frac{-548800}{19282999}$	0	0	0	0
		7	$ \begin{aligned} & - \frac{17903094937 t}{115697994} + \frac{2009830862429 t^2}{6941879640} \\ & - \frac{7147975382213 t^3}{97186314960} - \frac{4616370858079 t^4}{24296578740} \\ & + \frac{3925422356641 t^5}{19437262992} - \frac{44046597715 t^6}{462791976} \\ & + \frac{39950753199 t^7}{1542639920} - \frac{7499320559 t^8}{1735469910} \\ & + \frac{8501544607 t^9}{19437262992} - \frac{600292559 t^{10}}{24296578740} \\ & + \frac{14542879 t^{11}}{24296578740} \end{aligned} $	$\frac{5644800}{19282999}$	0	0	0	0

		$\frac{15}{2}$	$\begin{aligned} & \frac{7895289105498112t}{61422618789675} + \frac{6735012603521953792t^2}{27947291549302125} \\ & + \frac{21805251819953106944t^3}{352135873521206775} + \frac{2373818122399192064t^4}{15048541603470375} \\ & - \frac{296452068118243784704t^5}{1760679367606033875} + \frac{2224325219511231488t^6}{27947291549302125} \\ & - \frac{1821050791441248256t^7}{83841874647906375} + \frac{11287084995347456t^8}{3105254616589125} \\ & - \frac{59062934823643136t^9}{160061760691457625} + \frac{454415797629952t^{10}}{21736782316123875} \\ & - \frac{894287298842624t^{11}}{1760679367606033875} \end{aligned}$	$\begin{aligned} & \frac{2587390640128}{3548862418959} \\ & \frac{590225678336t}{954508505} + \frac{72223799379968t^2}{620430492825} \\ & - \frac{47864172763136t^3}{1563484841919} \\ & - \frac{8438634783232t^4}{111359319225} + \frac{3189561287161856t^5}{39087121047975} \\ & - \frac{24087207880192t^6}{620430493825} \\ & + \frac{19848831386624t^7}{1861291478475} \\ & - \frac{371611146752t^8}{206810164275} \\ & + \frac{6528880687104t^9}{3553374640725} \\ & - \frac{45549870592t^{10}}{4343013449775} \\ & + \frac{10038025216t^{11}}{39087121047975} \end{aligned}$	$\frac{1027604480}{2757468857}$	0	0
		8	1	1	$\begin{aligned} & \frac{1118578045t}{77131996} - \frac{8470954401t^2}{308527984} \\ & + \frac{2585419489t^3}{347093982} + \frac{196979622565t^4}{11107007424} \\ & - \frac{214732330783t^5}{11107007424} + \frac{17180758567t^6}{1851167904} \\ & - \frac{1587411571t^7}{617055968} + \frac{1620401335t^8}{3702335808} \\ & - \frac{502205327t^9}{11107007424} + \frac{1811279t^{10}}{694187964} \\ & - \frac{179435t^{11}}{2776751856} \end{aligned}$	$\begin{aligned} & \frac{4740120}{19282999} \\ & - \frac{102264435t}{38565998} \\ & + \frac{776217663t^2}{154263992} \\ & - \frac{53305177t^3}{38565998} \\ & - \frac{1999001667t^4}{617055968} \\ & + \frac{219022426t^5}{617055968} \\ & - \frac{527695931t^6}{308527984} \\ & + \frac{146837089t^7}{308527984} \\ & - \frac{50175043t^8}{617055968} \\ & + \frac{5207829t^9}{617055968} \\ & - \frac{9440t^{10}}{19282999} \\ & + \frac{1881t^{11}}{154263992} \end{aligned}$	$\frac{-35200}{19282999}$
9	8	0	$\begin{aligned} & - \frac{3977307800t}{51547632679} + \frac{259395552715t^2}{1082500286259} \\ & - \frac{898433494423t^3}{2886667430024} + \frac{353986853167391t^4}{1558800412212960} \\ & - \frac{489461560602739t^5}{4676401236638880} + \frac{603751818319751t^6}{1870560494655520} \\ & - \frac{1523247352019t^7}{3122107536407} + \frac{3117600824425920}{4464580877t^{10}} \\ & - \frac{222685773173280}{15365435327t^9} + \frac{4464580877t^{10}}{692800183205760} \\ & - \frac{1558800412212966t^{11}}{1142437061t^{11}} + \frac{19282999t^{12}}{4676401236638880} \end{aligned}$	$\frac{78400}{51547632679}$	0	0	0
		1	$\begin{aligned} & 1 - \frac{394355096991}{18728378655} + \frac{695254053047t^2}{749135146200} \\ & + \frac{42016561120657t^3}{38526950376000} + \frac{695254053047t^2}{749135146200} \\ & + \frac{42016561120657t^3}{38526950376000} - \frac{1283703209820779t^4}{809065957896000} \\ & + \frac{607249172339371t^5}{647252766316800} - \frac{213373119169561t^6}{647252766316800} \\ & + \frac{40816491516329t^7}{2101260773831t^8} + \frac{539377305264000}{179792435088000} \\ & + \frac{37238973839t^9}{30821560300800} - \frac{52007748497t^{10}}{647252766316800} \\ & + \frac{1259170511t^{11}}{404532978948000} - \frac{43236169t^{12}}{809065957896000} \end{aligned}$	$\begin{aligned} & - \frac{3969}{178365511} \end{aligned}$	0	0	0

		2	$\begin{aligned} & \frac{182650557600t}{30143771359} - \frac{201947242980t^2}{30143771359} \\ & - \frac{4990203021t^3}{4637503286} + \frac{20011062758929t^4}{3617252563080} \\ & - \frac{44295052299527t^5}{10851757689240} + \frac{23192687115899t^6}{14469010252320} \\ & - \frac{8563266459529t^7}{21703515378480} + \frac{173524276711t^8}{2712939422310} \\ & - \frac{148720293451t^9}{21703515378480} + \frac{20328332743t^{10}}{43407030756960} \\ & - \frac{67048741t^{11}}{3617252563080} + \frac{20791t^{12}}{64211583960} \end{aligned}$	$\frac{4665600}{30143771359}$	0	0	0	0
		3	$\begin{aligned} & - \frac{247135687400t}{21582226831} + \frac{1190431652035t^2}{64746680493} \\ & - \frac{247576475847t^3}{172657814648} + \frac{1231185180824491t^4}{93235219909920} \\ & + \frac{6470001231222019t^5}{559411319459520} - \frac{2774691552732563t^6}{559411319459520} \\ & + \frac{120715323381511t^7}{93235219909920} - \frac{1021448632489t^8}{4661760995496} \\ & + \frac{4524374518777t^9}{186470439819840} - \frac{874486183t^{10}}{513692671680} \\ & + \frac{601626326t^{11}}{8740801866555} - \frac{171137731t^{12}}{139852829864880} \end{aligned}$	$\frac{-14817600}{21582226831}$	0	0	0	0
		4	$\begin{aligned} & \frac{98029946000t}{4815868797} - \frac{521212859150t^2}{14447606391} \\ & + \frac{1190185518235t^3}{173371276692} + \frac{50946349637729t^4}{2080455320304} \\ & - \frac{50209227786083t^5}{2080455320304} + \frac{91973096933315t^6}{8321821281216} \\ & - \frac{1049221361063t^7}{346742553384} + \frac{245764331855t^8}{462323404512} \\ & - \frac{259736485t^9}{4280772264} + \frac{36326262031t^{10}}{8321821281216} \\ & - \frac{375006649t^{11}}{2080455320304} + \frac{6787687t^{12}}{2080455320304} \end{aligned}$	$\frac{395136}{178365511}$	0	0	0	0
		5	$\begin{aligned} & - \frac{38890226475t}{1248558577} + \frac{4041231101805t^2}{69919280312} \\ & - \frac{8085447571263t^3}{559354242496} + \frac{315343221962389t^4}{8390313637440} \\ & + \frac{4029679816089839t^5}{100683763649280} - \frac{644875525033063t^6}{33561254549760} \\ & + \frac{39290624250901t^7}{7191697403520} - \frac{49882312995359t^8}{50341881824640} \\ & + \frac{11703517902911t^9}{100683763649280} - \frac{862251865261t^{10}}{100683763649280} \\ & + \frac{504892583t^{11}}{1398385606240} - \frac{167523841t^{12}}{25170940912320} \end{aligned}$	$\frac{-1020600}{178365511}$	0	0	0	0

	6	$\begin{aligned} & \frac{37893052064t}{891827555} - \frac{1083116309276t^2}{13377413325} \\ & + \frac{1035972620783t^3}{44591377750} + \frac{245545571851997t^4}{4815868797000} \\ & - \frac{165320154462011t^5}{2889521278200} + \frac{328423392281833t^6}{11558085112800} \\ & - \frac{80147022385981t^7}{9631737594000} + \frac{7465545675811t^8}{4815868797000} \\ & - \frac{359005093891t^9}{1926347518800} + \frac{18031748659t^{10}}{1284231679200} \\ & - \frac{8728477283t^{11}}{14447606391000} + \frac{163909951t^{12}}{14447606391000} \end{aligned}$	$\begin{aligned} & \frac{2370816}{178365511} \\ & 0 \end{aligned}$	0	0	
	7	$\begin{aligned} & - \frac{32361066200t}{535096533} + \frac{439196582945t^2}{3745675731} \\ & - \frac{3287058594341t^3}{89896217544} + \frac{1165957857915271t^4}{16181319157920} \\ & + \frac{2716349225604767t^5}{32362638315840} - \frac{1383040371713363}{32362638315840} \\ & + \frac{9368013733941t^7}{770539007520} - \frac{243690103301t^8}{99884686160} \\ & + \frac{3233274883661t^9}{10787546105280} - \frac{746168902687t^{10}}{32362638315840} \\ & + \frac{2046916093t^{11}}{2022664894740} - \frac{156682171t^{12}}{8090659578960} \end{aligned}$	$\begin{aligned} & - \frac{6350400}{178365511} \\ & 0 \end{aligned}$	0	0	0
	8	$\begin{aligned} & - \frac{251505163800t}{1248558577} + \frac{49354108801t^2}{1248558577} \\ & + \frac{185927675127t^3}{1426924088} + \frac{142849367892463t^4}{599308116960} \\ & - \frac{513220176351851t^5}{118325736972607t^6} + \frac{118325736972607t^6}{1797924350880} \\ & - \frac{81235885676669t^7}{7197924350880} + \frac{6305319761005t^8}{719169740352} \\ & - \frac{281903220977t^9}{256846335840} + \frac{618881898007t^{10}}{7191697403520} \\ & - \frac{2307318887t^{11}}{599308116960} + \frac{134998831t^{12}}{1797924350880} \end{aligned}$	$\begin{aligned} & \frac{57153600}{178365511} \\ & 0 \end{aligned}$	0	0	0
	$\frac{17}{2}$	$\begin{aligned} & - \frac{328830623207582336t}{19324435183392619} + \frac{338323986645425756992t^2}{10459228471281124975} \\ & - \frac{12991216407563812385792t^3}{120684032851478221125} - \frac{9186522558909845309504}{4706677281207650623875} \\ & + \frac{220765424273862010973304t^5}{94133546241530124775} - \frac{11476637500537259762073}{379298546083998281728t^8} \\ & + \frac{8357992391291797446656t^7}{2241274868131669375} - \frac{52298414256406624875}{556498262417354752t^{10}} \\ & - \frac{10576822812062069216t^9}{11621425385697902775} - \frac{7779631869764711775}{1746850659454976t^{12}} \\ & + \frac{15099794308618330112t^{11}}{4706677281207650623875} - \frac{9539767307264t^{11}}{27850161427264204875} \end{aligned}$	$\begin{aligned} & \frac{744130565111808}{1054097540652871} \\ & \frac{100206757543936t}{1300819671723} - \frac{4147483176730624t^2}{27317213106183} \\ & + \frac{8043109721882624t^3}{157599306381825} + \frac{558637507507389568t^4}{6146372948891175} \\ & - \frac{135194774702686208t^5}{1229274589778235} + \frac{14125088824167424t^6}{245854917955647} \\ & - \frac{5166642475154432t^7}{292684426137675} + \frac{2356371398168576t^8}{682930327654575} \\ & - \frac{59451184211968t^9}{136586065530915} + \frac{768934024192t^{10}}{22350447086877} \\ & - \frac{9539767307264t^{11}}{6146372948891175} + \frac{14435588096t^{12}}{472797919145475} \end{aligned}$	$\begin{aligned} & \frac{168471925760}{433606557241} \\ & 0 \end{aligned}$	0	0

	9	1	1	$ \begin{aligned} & -\frac{3080125125t}{178365511} + \frac{12191283075t^2}{356731022} \\ & -\frac{33386168145t^3}{2853848176} - \frac{173925340427t^4}{8561544528} \\ & +\frac{2550677095249t^5}{102738534336} - \frac{1341466076827t^6}{102738534336} \\ & +\frac{69159049351t^7}{17123089056} - \frac{13615162555t^8}{17123089056} \\ & +\frac{3461688907t^9}{34246178112} - \frac{275880161t^{10}}{34246178112} \\ & +\frac{4708393t^{11}}{12842316792} - \frac{186959t^{12}}{25684633584} \end{aligned} $	$ \begin{aligned} & \frac{43366680}{178365511} \\ & -\frac{552006000t}{178365511} \\ & -\frac{2188457475t^2}{356731022} \\ & +\frac{3018370455t^3}{1426924088} \\ & +\frac{3891358847t^4}{1070193066} \\ & -\frac{2291968585t^5}{51369267168} \\ & +\frac{120873142805t^6}{51369267168} \\ & -\frac{6249061525t^7}{8561544528} \\ & +\frac{1233983248t^8}{8561544528} \\ & -\frac{314794945t^9}{1712308905} \\ & +\frac{8393405t^{10}}{5707696352} \\ & -\frac{4314853t^{11}}{6421158396} \\ & +\frac{17209t^{12}}{12842316792} \end{aligned} $	$\frac{-317520}{17836551}$
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Table 2: Second Derivative Hybrid Predictor Continuous Linear Multi- step Method of (1.5).

k	t	j	$\alpha_j(t)$	$\alpha_j(t_{value})$	$\beta_{3,j}(t)$	$\beta_{3,j}(t_{value})$	$\beta_{4,k}(t)$	$\beta_{4,j}(t_{value})$
1	$\frac{-1}{2}$	0	$-t^3$	$\frac{1}{8}$	0	0	0	0
		$\frac{1}{2}$	1	1	0	0	0	0
		1	$1+t^3$	$\frac{7}{8}$	$t-t^3$	$-\frac{3}{8}$	$\frac{t^2}{2}+\frac{t^3}{2}$	$\frac{1}{16}$
2	$\frac{1}{2}$	0	$-\frac{t}{8}+\frac{3t^2}{8}-\frac{3t^3}{8}+\frac{t^4}{8}$	$-\frac{1}{128}$	0	0	0	0
		1	$1-2t+2t^3-t^4$	$\frac{3}{16}$	0	0	0	0
		$\frac{3}{2}$	1	1	0	0	0	0
		2	$\frac{17t}{8}-\frac{3t^2}{8}-\frac{13t^3}{8}+\frac{7t^4}{8}$	$\frac{105}{128}$	$-\frac{5t}{4}+\frac{3t^2}{4}+\frac{5t^3}{4}-\frac{3t^4}{4}$	$-\frac{21}{64}$	$\frac{t}{4}-\frac{t^2}{4}-\frac{t^3}{4}+\frac{t^4}{4}$	$\frac{3}{64}$
3	$\frac{3}{2}$	0	$-\frac{4t}{27}+\frac{10t^2}{27}-\frac{t^3}{3}$ $+\frac{7t^4}{54}-\frac{t^5}{54}$	$\frac{1}{576}$	0	0	0	0
		1	$1-\frac{3t}{2}-\frac{t^2}{4}+\frac{11t^3}{8}$ $-\frac{3t^4}{4}+\frac{t^5}{8}$	$-\frac{5}{256}$	0	0	0	0

		2	$4t - 2t^2 - 3t^3 + \frac{5t^4}{2} - \frac{t^5}{2}$	$\frac{15}{64}$	0	0	0	0
		$\frac{5}{2}$	1	1	0	0	0	0
		3	$-\frac{127t}{54} + \frac{203t^2}{108} + \frac{47t^3}{24}$ $-\frac{203t^4}{108} + \frac{85t^5}{216}$	$\frac{1805}{2304}$	$\frac{14t}{9} - \frac{25t^2}{18} - \frac{5t^3}{4}$ $+\frac{25t^4}{18} - \frac{11t^5}{36}$	$\frac{-115}{384}$	$-\frac{t}{3} + \frac{t^2}{3}$ $+\frac{t^3}{4} - \frac{t^4}{3}$ $+\frac{t^5}{12}$	$\frac{5}{128}$
4	$\frac{5}{2}$	0	$-\frac{9t}{64} + \frac{45t^2}{128} - \frac{21t^3}{128}$ $+\frac{7t^4}{48} - \frac{t^5}{32} + \frac{t^6}{384}$	$-\frac{5}{8192}$	0	0	0	0
		1	$1 - \frac{3t}{2} - \frac{t^2}{6} + \frac{35t^3}{27} - \frac{22t^4}{27}$ $+\frac{11t^5}{54} - \frac{t^6}{54}$	$\frac{7}{1152}$	0	0	0	0
		2	$\frac{27t}{8} - \frac{27t^2}{16} - \frac{9t^3}{4}$ $+\frac{17t^4}{8} - \frac{5t^5}{8} + \frac{t^6}{16}$	$-\frac{35}{1024}$	0	0	0	0

		3	$-\frac{9t}{2} + \frac{9t^2}{2} + 3t^3 - \frac{13t^4}{3}$ $+ \frac{3t^5}{2} - \frac{t^6}{2}$	$\frac{576}{3343}$	0	0	0	0
		$\frac{7}{2}$	1	1	0	0	0	0
		4	$\frac{177t}{64} - \frac{9529t^2}{3456} - \frac{7183t^3}{3456}$ $+\frac{9529t^4}{3456} - \frac{2375t^5}{3456}$	$-\frac{81095}{36864}$	$-\frac{29t}{16} + \frac{65t^2}{32} + \frac{155t^3}{144}$ $-\frac{35t^4}{18} + \frac{53t^5}{72} - \frac{25t^6}{288}$	$-\frac{1715}{6144}$	$\frac{3t}{8} - \frac{7t^2}{16}$ $-\frac{5t^3}{24} + \frac{5t^4}{12}$ $-\frac{t^5}{6} + \frac{t^6}{48}$	$\frac{35}{1024}$
5	$\frac{7}{2}$	0	$-\frac{16t}{125} + \frac{124t^2}{375} - \frac{41t^3}{125} + \frac{49t^4}{300}$ $-\frac{131t^5}{3000} + \frac{3t^6}{500} - \frac{t^7}{3000}$	$\frac{7}{25600}$	0	0	0	0
		1	$1 - \frac{19t}{12} - \frac{t^2}{48} + \frac{247t^3}{192}$ $-\frac{359t^4}{384} + \frac{113t^5}{384}$ $-\frac{17t^6}{384} + \frac{t^7}{384}$	$-\frac{45}{16384}$	0	0	0	0
		2	$\frac{32t}{9} - \frac{56t^2}{27} - \frac{58t^3}{27} + \frac{131t^4}{54}$ $-\frac{97t^5}{108} + \frac{4t^6}{27} - \frac{t^7}{108}$	$\frac{7}{512}$	0	0	0	0

		3	$-4t + \frac{13t^2}{2} + \frac{9t^3}{4}$ $-\frac{193t^4}{48} + \frac{83t^5}{48} - \frac{5t^6}{16} + \frac{t^7}{48}$	$\frac{-105}{2048}$	0	0	0	0
		4	$\frac{16t}{3} - \frac{20t^2}{3} - \frac{7t^3}{3} + \frac{73t^4}{12}$ $-\frac{71t^5}{24} + \frac{7t^6}{12} - \frac{t^7}{24}$	$\frac{315}{1024}$	0	0	0	0
		$\frac{9}{2}$	1	1	0	0	0	0
		5	$-\frac{14299t}{4500} + \frac{221269t^2}{54000}$ $+\frac{274973t^3}{216000}$ $-\frac{321137t^4}{86400} + \frac{810739t^5}{432000}$ $-\frac{164467t^6}{432000} + \frac{12019t^7}{432000}$	$\frac{300013}{409600}$	$-\frac{76t}{75} + \frac{59t^2}{72}$ $+\frac{5729t^3}{7200} - \frac{1921t^4}{2400}$ $+\frac{1567t^5}{7200} - \frac{137t^6}{7200}$	$\frac{-1799}{10240}$	$-\frac{2t}{5} + \frac{8t^2}{15}$ $+\frac{17t^3}{120} - \frac{23t^4}{48}$ $+\frac{61t^5}{240} - \frac{13t^6}{240}$ $+\frac{t^7}{240}$	$\frac{63}{2048}$
6	$\frac{9}{2}$	0	$\frac{5t}{216} - \frac{-149t^2}{2592} + \frac{1399t^3}{25920}$ $-\frac{65t^4}{2592} + \frac{t^5}{162}$ $-\frac{t^6}{1296} + \frac{t^7}{25920}$	$\frac{7}{24576}$	0	0	0	0
		1	$1 - \frac{101t}{60} + \frac{29t^2}{200} + \frac{959t^3}{750}$ $-\frac{3199t^4}{3000} + \frac{119t^5}{300} - \frac{47t^6}{600}$ $+\frac{t^7}{125} - \frac{t^8}{3000}$	$\frac{77}{51200}$	0	0	0	0

		2	$\frac{125t}{32} - \frac{1025t^2}{384} - \frac{1615t^3}{768}$ $+\frac{2221t^4}{768} - \frac{163t^5}{128} + \frac{53t^6}{192}$ $-\frac{23t^7}{768} + \frac{t^8}{768}$	$\frac{-495}{65536}$	0	0	0	0
		3	$-\frac{125t}{27} + \frac{1775t^2}{324} + \frac{335t^3}{162}$ $-\frac{1583t^4}{324} + \frac{202t^5}{81} - \frac{191t^6}{324}$ $+\frac{11t^7}{162} - \frac{t^8}{324}$	$\frac{77}{3072}$	0	0	0	0
		4	$\frac{125t}{24} - \frac{22t^2}{32} - \frac{305t^3}{192}$ $+\frac{1177t^4}{192} - \frac{337t^5}{96}$ $+\frac{43t^6}{48} - \frac{7t^7}{64} + \frac{t^8}{192}$	$\frac{-1155}{16384}$	0	0	0	0
		5	$-\frac{25t}{4} + \frac{215t^2}{24} + \frac{4t^3}{3} - \frac{919t^4}{120}$ $+\frac{19t^5}{4} - \frac{31t^6}{24} + \frac{t^7}{6} - \frac{t^8}{120}$	$\frac{693}{2048}$	0	0	0	0
		$\frac{11}{2}$	1	1	0	0	0	0

		6	$\begin{aligned} & \frac{15397}{4320}t - \frac{1345709}{259200}t^2 \\ & - \frac{1713679}{2592000}t^3 + \frac{11428061}{259200}t^4 \\ & - \frac{725941}{259200}t^5 + \frac{100777}{129600}t^6 \\ & - \frac{265111}{259200}t^7 + \frac{13489}{259200}t^8 \end{aligned}$	$\frac{3505733}{4915200}$	$\begin{aligned} & \frac{-53t}{24} + \frac{4669t^2}{1440} + \frac{1813t^3}{4800} \\ & - \frac{4389t^4}{1600} + \frac{847t^5}{480} - \frac{119t^6}{240} \\ & + \frac{317t^7}{4800} - \frac{49t^8}{14400} \end{aligned}$	$\frac{20559}{81920}$	$\begin{aligned} & \frac{5t}{12} - \frac{89t^2}{144} \\ & - \frac{91t^3}{1440} + \frac{749t^4}{1440} \\ & - \frac{49t^5}{144} + \frac{7t^6}{72} \\ & - \frac{19t^7}{1440} + \frac{t^8}{1440} \end{aligned}$	$\frac{231}{8192}$
7	$\frac{11}{2}$	0	$\begin{aligned} & -\frac{36t}{343} + \frac{501t^2}{1715} - \frac{558t^3}{1715} \\ & + \frac{142t^4}{735} - \frac{299t^5}{4410} \\ & + \frac{57t^6}{3920} - \frac{463t^7}{246960} \\ & + \frac{11t^8}{82320} - \frac{t^9}{246960} \end{aligned}$	$\frac{33}{401408}$	0	0	0	0
		1	$\begin{aligned} & 1 - \frac{107t}{60} + \frac{19t^2}{60} + \frac{34t^3}{27} \\ & - \frac{3871t^4}{3240} + \frac{13153t^5}{25920} \\ & - \frac{391t^6}{3240} + \frac{43t^7}{2592} \\ & - \frac{t^8}{810} + \frac{t^9}{25920} \end{aligned}$	$\frac{-91}{98304}$	0	0	0	0
		2	$\begin{aligned} & \frac{108t}{25} - \frac{432t^2}{125} - \frac{252t^3}{125} + \frac{428t^4}{125} \\ & - \frac{1303t^5}{750} + \frac{2729t^6}{6000} - \frac{133t^7}{2000} \\ & + \frac{31t^8}{6000} - \frac{t^9}{6000} \end{aligned}$	$\frac{1001}{204800}$	0	0	0	0
		3	$\begin{aligned} & -\frac{5t}{192} + \frac{47t^2}{2304} + \frac{t^3}{48} \\ & - \frac{23t^4}{1152} + \frac{t^5}{192} - \frac{t^6}{2304} \end{aligned}$	$\frac{-2145}{16384}$	0	0	0	0

		4	$\begin{aligned} & \frac{5t}{162} - \frac{19t^2}{648} - \frac{29t^3}{1296} + \frac{37t^4}{1296} \\ & - \frac{11t^5}{1296} + \frac{t^6}{1296} \end{aligned}$	$\frac{1001}{3072}$	0	0	0	0
		5	$\begin{aligned} & -\frac{t}{32} + \frac{31t^2}{960} + \frac{t^3}{48} - \frac{t^4}{32} \\ & + \frac{t^5}{96} - \frac{t^6}{960} \end{aligned}$	$-\frac{3003}{4096}$	0	0	0	0
		6	$\begin{aligned} & -\frac{t}{30} - \frac{13t^2}{360} - \frac{t^3}{48} + \frac{5t^4}{144} \\ & - \frac{t^5}{80} + \frac{t^6}{720} \end{aligned}$	$\frac{3003}{1024}$	0	0	0	0
		$\frac{13}{2}$	1	1	0	0	0	0

		7	$\begin{aligned} & -\frac{807431t}{205800} + \frac{8606723t^2}{1372000} \\ & -\frac{6290717t^3}{74088000} - \frac{314791321t^4}{63504000} \\ & -\frac{479219107t^5}{127008000} - \frac{41097743t^6}{31752000} \\ & +\frac{104165237t^7}{444528000} - \frac{9670603t^8}{444528000} \\ & +\frac{726301t^9}{889056000} \end{aligned}$	$\frac{335572523}{481689600}$	$\begin{aligned} & \frac{1159t}{490} - \frac{37241t^2}{9800} \\ & +\frac{13613t^3}{176400} + \frac{150823t^4}{50400} \\ & -\frac{23154t^5}{10080} + \frac{20009t^6}{25200} \\ & -\frac{51131t^7}{352800} + \frac{4789t^8}{352800} \\ & -\frac{121t^9}{235200} \end{aligned}$	$\begin{aligned} & -\frac{275847}{1146880} \\ & -\frac{3t}{7} + \frac{97t^2}{140} \\ & -\frac{17t^3}{840} - \frac{391t^4}{720} \\ & +\frac{607t^5}{1440} - \frac{53t^6}{360} \\ & +\frac{137t^7}{5040} - \frac{13t^8}{5040} \\ & +\frac{t^9}{10080} \end{aligned}$	$\frac{429}{16384}$	
8	$\frac{13}{2}$	0	$\begin{aligned} & -\frac{49t}{512} + \frac{2821t^2}{10240} - \frac{5939t^3}{18432} \\ & +\frac{58733t^4}{286720} - \frac{14567t^5}{184320} + \frac{7141t^6}{368640} \\ & -\frac{7t^7}{2304} + \frac{109t^8}{368640} - \frac{t^9}{61440} + \frac{t^{10}}{2580480} \end{aligned}$	$\begin{aligned} & -\frac{429}{8388608} \end{aligned}$	0	0	0	0
		1	$\begin{aligned} & 1 - \frac{263t}{140} + \frac{2153t^2}{4410} + \frac{302863t^3}{246960} \\ & -\frac{324659t^4}{246960} + \frac{7331t^5}{11760} - \frac{669t^6}{3920} \\ & +\frac{337t^7}{11760} - \frac{103t^8}{35280} + \frac{41t^9}{246960} \\ & -\frac{t^{10}}{246960} \end{aligned}$	$\begin{aligned} & \frac{495}{802816} \end{aligned}$	0	0	0	0
		2	$\begin{aligned} & \frac{343t}{72} - \frac{2009t^2}{480} - \frac{24101t^3}{12960} \\ & +\frac{206459t^4}{51840} - \frac{985t^5}{432} + \frac{11917t^6}{17280} \\ & -\frac{533t^7}{4320} + \frac{227t^8}{17280} - \frac{t^9}{1296} + \frac{t^{10}}{51840} \end{aligned}$	$\begin{aligned} & -\frac{455}{131072} \end{aligned}$	0	0	0	0
		3	$\begin{aligned} & \frac{343t}{50} + \frac{9457t^2}{1000} + \frac{24829t^3}{18000} \\ & -\frac{46339t^4}{6000} + \frac{92821t^5}{18000} - \frac{6113t^6}{3600} \\ & +\frac{5791t^7}{18000} - \frac{643t^8}{18000} + \frac{13t^9}{6000} - \frac{t^{10}}{18000} \end{aligned}$	$\begin{aligned} & \frac{1001}{81920} \end{aligned}$	0	0	0	0
		4	$\begin{aligned} & \frac{1715t}{192} - \frac{31801t^2}{2304} - \frac{553t^3}{2304} \\ & +\frac{100217t^4}{9216} - \frac{12469t^5}{1536} + \frac{2931t^6}{1024} \\ & -\frac{73t^7}{128} + \frac{607t^8}{9216} - \frac{19t^9}{4608} + \frac{t^{10}}{9216} \end{aligned}$	$\begin{aligned} & -\frac{32175}{1048576} \end{aligned}$	0	0	0	0

		5	$\begin{aligned} & -\frac{343t}{36} + \frac{931t^2}{60} - \frac{1001t^3}{1296} \\ & -\frac{76967t^4}{6480} + \frac{20641t^5}{2160} - \frac{7669t^6}{2160} \\ & +\frac{319t^7}{432} - \frac{191t^8}{2160} + \frac{37t^9}{6480} - \frac{t^{10}}{6480} \end{aligned}$	$\frac{1001}{16384}$	0	0	0	0
		6	$\begin{aligned} & \frac{343t}{40} - \frac{2303t^2}{160} + \frac{1883t^3}{1440} \\ & +\frac{20693t^4}{1920} - \frac{13129t^5}{1440} + \frac{20287t^6}{5760} \\ & -\frac{1093t^7}{1440} + \frac{541t^8}{5760} - \frac{t^9}{160} + \frac{t^{10}}{5760} \end{aligned}$	$\begin{aligned} & -\frac{15015}{131072} \end{aligned}$	0	0	0	0

		7	$\begin{aligned} & -\frac{49t}{6} + \frac{5033t^2}{360} - \frac{1193t^3}{720} \\ & -\frac{51869t^4}{5040} + \frac{433t^5}{48} - \frac{287t^6}{80} \\ & +\frac{191t^7}{240} - \frac{73t^8}{720} + \frac{t^9}{144} - \frac{t^{10}}{5040} \end{aligned}$	$\frac{6435}{16384}$	0	0	0	0
		$\frac{15}{2}$	1	1	0	0	0	0
		8	$\begin{aligned} & \frac{3433279t}{806400} - \frac{92025733t^2}{12544000} \\ & +\frac{6670070881t^3}{7112448000} + \frac{152706293123t^4}{28449792000} \\ & -\frac{3226094747t^5}{677376000} + \frac{518413757t^6}{270950400} \\ & -\frac{18157057t^7}{42336000} + \frac{74832377t^8}{1354752000} \\ & -\frac{54422483t^9}{14224896000} + \frac{3144919t^{10}}{28449792000} \end{aligned}$	$\frac{1401794537}{2055208960}$	$\begin{aligned} & -\frac{801t}{320} + \frac{193843t^2}{44800} \\ & -\frac{179671t^3}{313600} + \frac{35628637t^4}{11289600} \\ & +\frac{252431t^5}{89600} - \frac{122323t^6}{107520} \\ & -\frac{359t^7}{1400} + \frac{17863t^8}{537600} \\ & +\frac{1453t^9}{627200} - \frac{761t^{10}}{11289600} \end{aligned}$	$\begin{aligned} & -\frac{1700127}{7340032} \\ & \frac{7t}{16} \frac{28t^2}{30} \frac{219t^3}{2080} \\ & +\frac{44477t^4}{80960} \frac{955t^5}{1920} \frac{155t^6}{768} \\ & +\frac{11t^7}{240} \frac{28t^8}{380} \frac{17t^9}{40320} \\ & +\frac{10}{80960} \end{aligned}$	$\frac{6435}{262144}$	
9	$\frac{15}{2}$	0	$\begin{aligned} & -\frac{64t}{729} + \frac{2216t^2}{8505} - \frac{14615t^3}{45927} \\ & +\frac{394109t^4}{1837080} - \frac{657851t^5}{7348320} + \frac{8533t^6}{349920} \\ & -\frac{2063t^7}{466560} + \frac{649t^8}{1224720} - \frac{593t^9}{14696640} \\ & +\frac{13t^{10}}{7348320} - \frac{t^{11}}{29393280} \end{aligned}$	$\frac{715}{21233664}$	0	0	0	0
		1	$\begin{aligned} & -\frac{551t}{280} + \frac{13243t^2}{20160} \\ & +\frac{9071t^3}{7680} - \frac{20413t^4}{14336} + \frac{239329t^5}{322560} \\ & -\frac{27943t^6}{122880} + \frac{38131t^7}{860160} - \frac{1439t^8}{258048} \\ & +\frac{9t^9}{20480} - \frac{17t^{10}}{860160} + \frac{t^{11}}{2580480} \end{aligned}$	$\frac{7293}{16777216}$	0	0	0	0
		2	$\begin{aligned} & -\frac{256t}{49} + \frac{8672t^2}{1715} - \frac{25076t^3}{15435} \\ & +\frac{140339t^4}{30870} - \frac{357229t^5}{123480} + \frac{17347t^6}{17640} \\ & -\frac{2063t^7}{10080} + \frac{6653t^8}{246960} - \frac{271t^9}{123480} \\ & +\frac{5t^{10}}{49392} - \frac{t^{11}}{493920} \end{aligned}$	$\frac{8415}{3211264}$	0	0	0	0
		3	$\begin{aligned} & -\frac{224t}{27} + \frac{548t^2}{45} + \frac{1553t^3}{2430} \\ & -\frac{184279t^4}{19440} + \frac{550027t^5}{77760} - \frac{135763t^6}{51840} \\ & +\frac{9973t^7}{17280} - \frac{2053t^8}{25920} + \frac{259t^9}{38880} \\ & -\frac{49t^{10}}{155520} + \frac{t^{11}}{155520} \end{aligned}$	$\frac{7735}{786432}$	0	0	0	0

		4	$\begin{aligned} & -\frac{896t}{75} + \frac{21968t^2}{1125} + \frac{502t^3}{375} \\ & +\frac{4367t^4}{300} - \frac{43771t^5}{3600} + \frac{28867t^6}{6000} \\ & -\frac{26711t^7}{24000} + \frac{571t^8}{3600} - \frac{11t^9}{800} \\ & +\frac{t^{10}}{1500} - \frac{t^{11}}{72000} \end{aligned}$	$\frac{17017}{655360}$	0	0	0	0
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	5	$ \begin{aligned} & -14 t + \frac{481 t^2}{20} - \frac{917 t^3}{288} \\ & -\frac{199661 t^4}{11520} + \frac{179731 t^5}{11520} - \frac{298927 t^6}{46080} \\ & +\frac{71969 t^7}{46080} - \frac{5303 t^8}{23040} + \frac{473 t^9}{23040} \\ & -\frac{47 t^{10}}{46080} + \frac{t^{11}}{46080} \end{aligned} $	$\frac{109395}{2097152}$	0	0	0	0
	6	$ \begin{aligned} & +\frac{8011 t^4}{486} - \frac{151673 t^5}{9720} \\ & +\frac{21799 t^6}{3240} - \frac{803 t^7}{480} \\ & +\frac{329 t^8}{1296} - \frac{113 t^9}{4860} \\ & +\frac{23 t^{10}}{19440} - \frac{t^{11}}{38880} \end{aligned} $	$\frac{17017}{196608}$	0	0	0	0
	7	$ \begin{aligned} & -\frac{32 t}{3} + \frac{6052 t^2}{315} \\ & -\frac{799 t^3}{210} - \frac{22231 t^4}{1680} + \frac{261733 t^5}{20160} \\ & -\frac{11041 t^6}{1920} + \frac{939 t^7}{640} - \frac{4603 t^8}{20160} \\ & +\frac{3 t^9}{140} - \frac{t^{10}}{896} + \frac{t^{11}}{40320} \end{aligned} $	$\frac{109395}{262144}$	0	0	0	0
	8	$ \begin{aligned} & \frac{64 t}{7} - \frac{584 t^2}{35} + \frac{163 t^3}{45} \\ & +\frac{28517 t^4}{2520} - \frac{114931 t^5}{10080} \\ & +\frac{7423 t^6}{1440} - \frac{54137 t^7}{40320} \\ & +\frac{269 t^8}{1260} - \frac{59 t^9}{2880} \\ & +\frac{11 t^{10}}{10080} - \frac{t^{11}}{40320} \end{aligned} $	$\frac{222757759081}{332943851520}$	0	0	0	0
	$\frac{17}{2}$	1	1				
	9	$ \begin{aligned} & -\frac{32645251 t}{7144200} + \frac{5586468847 t^2}{666792000} \\ & -\frac{270167325259 t^3}{144027072000} - \frac{650938097539 t^4}{115221657600} \\ & +\frac{331115488943 t^5}{57610828800} - \frac{287280747409 t^6}{109734912000} \\ & -\frac{3590718983 t^7}{5225472000} - \frac{8461926899 t^8}{76814438400} \\ & +\frac{2457243709 t^9}{230443315200} - \frac{1320881893 t^{10}}{230443315200} \\ & +\frac{30300391 t^{11}}{230443315200} \end{aligned} $	$\frac{222757759081}{332943851520}$	$ \begin{aligned} & \frac{7444 t}{2835} - \frac{1276693 t^2}{264600} + \frac{62647321 t^3}{57153600} \\ & +\frac{148337101 t^4}{45722880} - \frac{75811307 t^5}{22861440} \\ & +\frac{66018271 t^6}{4354600} - \frac{828377 t^7}{2073600} \\ & +\frac{1960541 t^8}{30481920} - \frac{572011 t^9}{91445760} \\ & +\frac{309067 t^{10}}{91445760} - \frac{7129 t^{11}}{91445760} \end{aligned} $	$\frac{29582839}{132120576}$	$ \begin{aligned} & -\frac{4 t}{9} + \frac{86 t^2}{105} - \frac{8599 t^3}{45360} \\ & -\frac{19909 t^4}{36288} + \frac{5119 t^5}{9072} - \frac{8959 t^6}{34560} \\ & +\frac{791 t^7}{11520} - \frac{269 t^8}{24192} + \frac{79 t^9}{72576} \\ & -\frac{43 t^{10}}{725760} + \frac{t^{11}}{725760} \end{aligned} $	$\frac{12155}{524288}$

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