On optimal design of low-pass electronic filter using Chebyshev polynomial response

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Abstract

Application of optimization theory to engineering provides a design tool of quite extra ordinary power: it offers the prospect of solutions to problems for which no normal design methods exists. In dealing essentially with circuit specification in frequency domain, the design problem includes deciding the size or order of filter required and computing optimum values for the components. In this presentation, Chebyshev response which is known to give fast cut-off frequency for a given template and involves no finite frequency transfer function is formulated to determine optimum components by applying a conjugate direction method proposed by the author in minimizing the objective function. This is a didactic presentation of what to do in the presence of this problem. No. claim of originality in the theory of electronics and electrical engineering is made. Sensitivity analysis is undertaken.

Keywords: Template specification, Chebyshev response Conjugate direction quasi-Newton algorithm, Optimization, Computer-aided design, Design variables, Topology of the solution systems, Design objective function.

1.0 Introduction

A filter means an electronic circuit that allows certain frequencies to pass while stopping others. On the basis of magnitude response, filters can be classified as Low-Pass, High-Pass and Band Reject (or notch) filters. All use operational amplifiers as active elements and resistors and capacitors as passive elements.

The following definitions are pertinent. A filter that transmits all frequencies below a specified value substantially attenuating frequencies above this value in high-Pass filter. When a filter transmits only those current having a frequency lying within specified limit it is called a Band-Pass filter. Our emphasis shall be on low-Pass filters, shunted by the inductors while the high frequencies are transmitted through the capacitors. It was not until 1900 that a successful Low-Pass filter was constructed by Pupin. A high-Pass filter was built by Campbell in 1906. A Low-Pass filter can be constructed by interchanging the positions of the inductors and capacitors [12], [17]. Our most concern is the application of optimization techniques to the design of filters. The range of disciplines to which optimization has been applied goes way beyond the confines of electronics, even beyond those of engineering. Lawrence Dixon (1976) provides an interesting insight into this breadth, with applications from a wide cross – section of engineering and scientific fields. Specification for linear frequency – selective networks are often very demanding and the mathematical complexities of designing a suitable network are frequently very great. There are, indeed, many situations in which no analytic design methods are available, and

numerical techniques then provide the only course open to the designer. Hence the earliest published instance of the use of optimization in electronics involved the design of filters.

For greatest design generality and practicality, we would like each capacitor inductor to have its own particular loss factor, that is, the ratio of resistance (conductance) to inductance (capacitance), thus inductor loss factor, $d_1 = R_1 / L$ (1.1)

and similarly capacitor loss factor. $D_c = G_c / c$ (1.2)

From these definitions, we observe that ideal or loss less components have R_L and G_c equal to zero and hence the loss factors are zero. It is interesting to note that the design of filters with completely arbitrary loss factors is one of the situations for which no exact design method exists [3], [7], [10], [13, 14]. The published results showed that computer optimization allowed design solutions to be produced, although runtime were rather long. Some authors used steepest descent algorithm which (performed poorly) to obtain optimum component values. Lasdon and Warren [14] considered the same problem, but applied the more powerful DFP quasi-Newton algorithm with predictably better results. In this work the authors employed a conjugate direction quasi-Newton method proposed by them. This development of interest in applying optimization during the 1960's correspond to the growing availability of computing power and the subsequent years have witnessed a steady increase in the performance over cost ratio of digital computers. The position now is that comparatively low cost desktop machinery is available offering entirely adequate computational power and memory capacity to allow the method described in this paper to be used routinely in design, [4, 6, 7, 10].

The classical problem of filter design consists in:

(a) Obtaining a realizable network function H(s) whose corresponding amplitude and phase function $|H_1(w)|$ and arg $H_1(w)$, satisfy the given templates. This is referred to as approximation.

(b) Synthesizing a network by performing a sequence of mathematical operations on $H_1(S)$, leading to a network, which thus satisfies the original specifications.

This is referred to as synthesis.

Interested readers will find useful introduction to filter theory in [8]. Much of the difficulty of the approximation/synthesis cycle has been eased by the production of design tables and graphs relating gain and phase specifications to standard filter functions (Butterworth, Chebyshev, Bessel, Elliptic, etc), and giving component values for standard active and / or passive filter structure which realize these various filter functions.

First, let us recall some basic knowledge about filters. Filters are networks whose gain or attenuation and associated phase vary with frequency. This frequency dependency can be exploited in order to separate wanted and unwanted components of a signal on the basis of frequency. The way in which a filter function varies with frequency is called frequency response. This can be represented mathematically by means of filter's transfer function (TF) or H.H. yields, the amount of amplification or attenuation, the phase shift experienced by the same signal. In general both magnitude and phase responses are frequency dependent.

Filters may be classified into analog or digital and passive or active. Analog filters are designed to process analog signals while the digital filters are designed to process digital signal depending on the type of elements used in their constructions. Passive filters use resistors, inductors and capacitors and cannot produce power gain while active filters use these in conjunction with some form of amplifying circuits and can therefore produce power gain. For example resistors – capacitors or RC filters are commonly used in audio or low frequency operations.

2.0 Mathematical requirements of low-pass filter design

While circuit structures may differ markedly from one style of filter realization to another, the underlying problem of filter design in all cases involves

- (*i*) Deciding what size of filter is required, that is, how many components
- (*ii*) Computing optimum values for the components.

The first point (i) relate to the approximation problem for we must decide how many transfer function poles there should be (whether zeros are required) and where they should be positioned. This shall be taken in isolation, that is, without any consideration of the proposed final filter format. We shall therefore choose a realizable network function $H_1(w)$ or $H_1(S)$, on which to base our computation of

component values. A study of literature on approximation theory convinces us that a Chebyshev function should be taken because no finite frequency transfer function (T.F) zeros is involved, only a constant appears in the T.F. Numerator (all practical transfer functions turn out to be rational functions as:

$$H(S) = \frac{N(S)}{D(S)}$$
(2.1)

where N(S) and D(S) are suitable polynomial in S with real coefficients and the order of N(S) never exceeds that of D(S). the order is of special significance and it is called order of filter (first order, second order, etc). The zeros of N(S) and D(S) are called, respectively, the zeros and the poles of H(S). To find H(S) we employ Ohm's Law, the voltage and current divider formulae and the super position principle. The topology of the circuit is fixed once the filter order is determine. We need to know the allowed pass band ripple level, and the gains and frequencies of the critical stop band points. The amplitude response function associated with a Low-Pass nth-order Chebyshev filter is given, for normalized frequency such

that the edge of the pass band occurs at:

$$H_{1}(w) = \frac{1}{\left[1 + E^{2}C_{N}^{2}(w)\right]^{\frac{1}{2}}}$$
(2.2)

where
$$E =$$
 ripple factor

where
$$E = \text{ripple factor}$$

$$C_{n} = \begin{cases} \cos(n\cos^{-1}w), & w \le 1 \\ \cos h(n\cos h^{-1}w), & w > 1 \end{cases}$$

$$C_{n}(1) = 1 \forall_{n}. \text{ For } w = 1, |H(jw)| = \frac{1}{\sqrt{1 + E^{2}}}$$
(2.3)

Having determined a suitable realizable network function N(S), the design process now proceeds to stage (ii), the computation of as set of nominal component values such that the filter will realize the specific transfer function coefficients. Optimization scheme for electronic circuit design is depicted below (Figure 2.1).

Figure 2.1: Optimization Scheme for electronic Circuit Design



To observe the next step we choose a suitable design objective, circuit analysis procedure and optimization method. These are heavily inter-related and dependent on what resources, particularly circuit analysis, are applicable to a given design problem. To choose a circuit analysis method, a simple recursive analysis scheme [3, 6, 14, 16] can be considered. Given an initial set of component values stored in vector X, the analysis procedure returns a vector whose elements are the S-plane voltage transfer function coefficients. Given this fast and very efficient analysis capability, we formulate a suitable design objective as a sum of squared residuals. We employ the bilinear property of network function to write H(S) in the form:

$$H(S) = \frac{N_1(S) + XN_2(S)}{D_1(S) + XD_2(S)}$$

where N_1, N_2D_1, D_2 are functions of the other network element but not X. Bode (1945) (cited in [17, 18]) was the first to point out that numerator and denominator polynomials of any network function H(S)are at most linear functions of a chosen element X. Let us denote a general T.F. coefficient as C_i (which can be either a numerator or denominator coefficient), we may thus write:

$$C_i = \alpha_{ij} X_j + \beta_{ij} \tag{2.5}$$

where X_j is the jth variable, and α_{ij} , β_{ij} are functions of the other variable, but not X_j . The partial

derivative of C_i with respect to X_i is given as:

$$\frac{\partial c_i}{\partial x_j} = \frac{\partial}{\partial x_j} (\alpha_{ij} X_j + \beta_{ij}) - \alpha_{ij}$$
(2.6)

 α_{ii} can be determined by a simple perturbation approach here because of the linear form of the expression for the network function coefficient. Incrementing X_i to a new perturbed value X_i^* by a convenient $X_{i}^{*} = X_{i} + 1$ amount usually unit, we obtain: (2.7)

And this gives rise to new coefficient values C_i^* where

$$C_j^* - C_i = \alpha_{ij} X_j + \beta_{ij}$$
(2.8)

$$C_{j}^{*} - C_{i} = \alpha_{ij}(X_{j} + 1) + \beta_{ij}$$
(2.9)

The required derivative can be found simply by subtracting equation (2.5) from equation (2.9): or

$$C_{j}^{*} - C_{i} = \alpha_{ij}(X_{j} + 1) + \beta_{ij} - (\alpha_{ij}N_{1} + \beta_{ij})$$
(2.10)

or

$$C_j^* - C_i = \alpha_{ij_{ij}} \tag{2.11}$$

(2.12)

The above procedure implies that we can make use of property of the coefficient of a network function to develop a special – purpose derivative evaluation procedure. Therefore, in order to find the required complete set of coefficient partial derivative with respect to given variable, X_j , we re-analyze the network with X_i , set equal to $X_i + 1$ and then subtract the original (unperturbed) coefficient from the corresponding new (perturbed) set, as in equation (2.11). this provides a s fast efficient and above all accurate strategy for computing derivatives. The design function $\phi(x)$ can be formulated to be sum of

squared residuals of the form:
$$\phi(x) = \sum_{i=0}^{n} (a_{si} - a(x))^2$$

 $a_{si}(x)$ are the n+1 specified denominator coefficients, $a_{ri}(x)$ are the n+1 realized corresponding coefficients by the current set of component values and n is the filter order. Since the aim is to minimize $\phi(x)$ using any suitable minimization algorithm we present the following conjugate direction quasi-Newton algorithm.

3.0 A quasi-Newton method algorithm

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The algorithm minimizes the function $\{\phi(x) \mid x \in IR^n\}$ where ϕ is assumed to be twice continuously differentiable and can be summarized by the following steps [2].

3.1 Algorithm

Step 0:

Select an initial guess $X_0 \in \Re^n$, compute $\nabla \phi(x)$ initial gradient vector,

$$abla^2 \phi(X_0) = H_0 = (h_{ij}^{(0)}) = \text{Hessian matrix at } X_0, i, j = 1, 2, \text{K}, n$$

Set

$$B_0 = \alpha H_0^T = \alpha H_0$$
 since H_0 is symmetric

where
$$\alpha = \frac{1}{\left\| H_0^T \right\| \infty \left\| H_0 \right\| 1}$$
 and $\left\| H_0^T \right\| \infty = \max_i \sum_j \left| h_{ij}^{(0)} \right| = \max$ column sum $\left\| H_0^T \right\|_1 \infty = \max_i \sum_j \left| h_{ij}^{(0)} \right| = \max$ row sum [1]

Step 1:

If convergence criterion is achieved then stop, else

Step 2:

Compute a quasi-Newton direction (Vector).

$$\underbrace{S_k}_{B_k} = -B_k \nabla \phi(x_k)$$

$$B_k = B_{k-1} + B_{k-1}R_{k-1}$$

$$R_{k-1} = 1 - H_{k-1}B_{k-1}, \ k = 1, 2, \text{K} \ n-1$$

Step 3:

Set
$$\underline{x}_{k-1} = \underline{x}_k + \beta_k \underline{s}_k = \underline{x}_k - \beta_k B_k \underline{\nabla} \phi(\underline{x}_k)$$

where β_k is chosen to be $\phi(\underline{x}_k + \beta_0 \underline{s}_k)$ [1, 2] such that $\phi(\underline{x}_{k-1}) < \phi(\underline{x}_k)$ and by algorithm (2.11) in [2].

Step 4:

Compute the next inverse Hessian approximation

$$B_{k+1} = B_k R_k + B_{kj}, \ k = 0, 1, 2, K \ n-1$$
$$R_k = 1 - H(x_0) B_k H_0 = H(x_0)$$

Step 5:

Set k = k+1 and go to step 2.

We shall next select a template for a lower-pass antialiasing filter intended for a particular communication system and determine a suitably realizable filter specification.

Computation Result 3.1

In this section let us suppose that we want to design an RC-active Low-Pass filter based on the extend Sallen and key structure to realize the 0.5dB the filter is to be designed for a cut-off frequency of 3.5KHz: that is the allowable pass band ripple level, upper should (0dB) and lower 05dB) and the gain (0.5dB-20dB, 60dB) and normalized frequency of the critical stop band point (1, 2, 3). The voltage simplifications is to be regarded as an ideal voltage controlled source gain (amplitude response), of 2. We shall need to determine the specific function coefficient for the numerically designed filter (see Figure 3.1).

Figure 3.1: RC-active low-pass filter



The first step in solving the design problem is to determine the filter order. Thus since

$$|jw| \downarrow w = 1 = \frac{1}{[1+\epsilon^2]^{\frac{1}{2}}}$$



Figure 3.1: Information flow chart for quasi-Newton algorithm

We must compute so as to obtain attenuation of better than -20dB at normalized frequency w = 2 and better than -60dB at w = 3. For w = 2 we have

$$\begin{bmatrix} 1 + \varepsilon^2 C_n^2(w) \end{bmatrix}^{\frac{1}{2}} = -20 dB$$

$$\Rightarrow -10 \log w [1 + 0.1220 C_N(2)] = -20$$

or
$$1 + 0.1220 \left[\cos h(n \cos h^{-1} 2) \right]^2 = 10^2$$

$$\Rightarrow \left[\cos h(n\cos h^{-1}2)^{2} = \frac{(10^{2}-1)}{0.1220}\right]$$
$$= \frac{99}{0.1220} = 811.4$$
$$\Rightarrow \left[n\cos(n\cos h^{-1}2)\right] = 28.484$$
$$\Rightarrow n\cos h^{-1}2 = n\cos h^{-1}28.484$$
$$\Rightarrow n(1.370) = 4.0422$$
$$\Rightarrow n(1.370) \approx 3.1$$

Since we must have integral order, n = 4 will be required to ensure that the specification is satisfied at this point. Repeating the above calculation for w = 3 with minimum attenuation = 60Db, we obtain n = 4.9 and we round this up to 5. Hence to satisfy both specification constraints simultaneously, we must have n = 5. This is the required size of our network. According to Weinberg's table [18], the s-plane T.F for a 5th – order 0.5dB ripple Chebyshev filter is given as

$$H_1S = \frac{1}{a_5S^5 + a_4S^4 + S^3 + a_2S^2 + a_1S + a_0}$$

where

$$a_5 = 5.889839, a_4 = 6.5530328$$

 $a_3 = 10.827916, a_2 = 7.3191919,$
 $a_1 = 5.5889839, a_0 = 1.0$

It is easy to compute the amplitude response by noting that the steady-state sinusoidal response H(jw) is found by setting s = jw in the above. It is |H(jw)|. The design objective can then be formulated as sum of square discrepancies between realized and specified amplitude responses. Suppose that we have a small level specific performance features a_{si} , i = 1, 2K, n and we require the corresponding realized values, a_{ri} to be matched to them. We formulate the sum of squares design objective as:

$$\phi(x) = \sum_{1=0}^{n} \left[a_{si} - a_{ri}(x) \right]^{2}$$

where *n* is the filter order, a_{si} = specific values found by evaluating |H(jw)| for each of the frequencies *w*, *a*. *ri* are the corresponding realized values, which are stored in vector \underline{x} . The results are tabulated and are shown in Table 1. The design was completed in just 4.3 seconds (CPU time) using Pentium 133). The stopping criterion is that $\|\nabla \phi\| \le 10^{-12}$.

4.0 Sensitivity analysis

It seems highly probable that any design problem will be subjected to substantial specification errors. Consequently sensitivity analysis of the effects of specification errors upon the efficiency of the optimization models would be highly desirable. Consider a system having some performance feature $\phi(x)$ which is a function of the set system parameters, \underline{x} . The performance function $\phi(x)$ could be an amplitude or phase response in the case of a frequency – selective circuit, a dc operating point in the case of transistor amplifier configuration, or indeed, any other performance measure of interest to the designer. A very important question is the extent to which the performance is likely to be affected by a change in the value of the system parameters. We therefore speak of the sensitivity of the function ϕ to the parameters, say, x. Here we shall denote this symbolically as S^{ϕ} and define it in differential form as

$$S_{si}^{\phi} = \frac{x_i \partial \phi(x_i)}{\phi(X_i) \partial x_i}$$
(4.1)

The reader would recall that use is made of a circuit system procedure which returns values of the performance measure of interest given the current set of design variables. This implies, necessarily, that

a numerical value of the design objective $\phi(\underline{x})$ can be set up for any current vector, but that corresponding exact values of the gradient vector $\nabla \phi(X)$ are not available. The practical significance of the measure of performance can be brought out by considering the small-change approximation.

$$S_{si}^{\phi} = \frac{x_i \Delta}{\phi \Delta x_i} = \frac{\Delta \phi}{\frac{\Delta \phi}{\Delta x_i}}$$
(4.2)

From which we can see that the sensitivity analysis is therefore concerned with analyzing the behavior of a local solution when the problem functions are perturbed slightly. This perturbation might be due to inexactness with which certain parameter values in the problems are calculated or because the optimization model was parameterized and one is interested in the solution for a variety of values of the parameters [9, 12]. What is expressed in equation (4.1) – (4.2) is termed first – order sensitivity. There has been some work in second-order sensitivity analysis which derived an expression for the second derivative with respect to the perturbation term. The essence of sensitivity analysis in nonlinear programming is the application of the implicit function theorem to the Karush-Kuhn-Tucker (KKT) necessary conditions for the parameterized problem [15]. The active Low-Pass filter under consideration is of order 5 and comprises 5 capacitors and 5 resistors. Let us suppose that it is desired to minimize amplitude response (gain). Sensitivity to the passive components at the normalized bandage $\omega = 1$ rad/s while maintaining all components nonnegative. The filter realized a 5th order all-pole voltage transfer function H(S), so that equation (3.1) holds with a_0 the realized value of the ith coefficient. The filter satisfies a 5th order transfer function specification H(S) where,

$$H_0(S) = \frac{1}{a_5 S^5 + a_4 S^4 + a_3 S^3 + a_2 S^2 + a_1 S + a_0}$$
(4.3)

With a_{xi} the specified value of the coefficient and, in practices, simply a user defined number. It is further required that the summed value of the capacitor should not exceed a specified value C. The aim of the problem is to formulate this specification as an equation/inequality – constrained minimization problem. The sensitivity of the gain function |H(jw)| to the ith component at frequency w is according to equation (4.1) and (4.2).

$$S_{xi}^{H(w)} = \frac{X_i \partial |H(jw)|}{|H(jw)| \partial X_i}$$

$$\tag{4.4}$$

We shall formulate this as sum of square objective

$$\phi(X) = \sum_{i=1}^{n} \left\{ S^{H(jw)} \right\}^2$$
(4.5)

We shall need to minimize $\phi(x)$. The constraints imposed by the specification are formulated as the six equality condition relating to the voltage transfer function coefficients.

$$C_{11} = a_{S_0} - a_{i01}, C_{E2} = a_{si}a_{ri}K, C_{E6} = a_{S5} - a_{r5}$$
(4.6)

and the 11 inequality conditions

$$C_{11} \le x_1, C_{12} \le x_2, C_{13} \le x_3, C_{14} \le x_4, C_{15} \le x_5, C_{16} \le x_6, C_{17} \le x_7, C_{18} \le x_8, C_{19} \le x_9,$$

$$C_{110} = x_{10} \text{ and } C_{111} = C_r = \sum_{i=1}^5 x_i.$$

where the capacitors value are stored as the first elements of x and the resistor as the remaining 5 elements. Recall that the filter order is 5. We have thus reformulated our very practical filter design problem as the optimization problem:

Maximize $\phi(x)$

Subject to:
$$C_{ij} = 1, K 6, C_{ij} \le 0, j = 1, 2, K = 11$$

with C_{ij} and C_{IJ} given as above.

An alternative of the objective is provided by p(x) defined below and derived from equation

(4.1) taking
$$\phi(x)$$
 as $\phi(x) = \sum_{i=1}^{2n} (a - x_i)^2$ (4.7)

That is

$$p(x) = \sum_{i=1}^{2n} \left[\frac{x_i \partial \phi(x)}{\phi(x) \partial x_i} \right]^2 = \sum_{i=1}^{2n} \left[S_{xi}^{\phi(x_i)} \right]^2$$
(4.8)

 $\underline{x} = (x_1, x_2, x_3 \mathbf{K} \ x_{2n})^T$ where the X_{si} , are the capacitors and resistors to be determined.

Table 4.1: Optimal Component Values

Iteration	C ₁	R ₁	C ₂	R ₂	C ₃	R ₃	C_4	R_4	C ₅	
1	5.0038	1.2071	0.5231	42.827	11.278	1.1074	1.5894	50.689	40.81	0.5
2	5.7811	1.2484	0.4992	47.927	13.8921	1.2649	1.91	63.486	40.919	0.6
3	5.9112	1.3471	0.5681	51.892	15.181	1.2972	1.8111	76.894	44.783	0.6
4	6.8271	1.3961	0.5892	50.984	18.178	1.3986	1.33121	77.898	45.891	0.8
5	7.869	1.3012	0.5891	53.639	17.698	1.4686	1.3689	80.789	47.791	0.7
6	7.874	1.2876	0.6173	57.5530	17.589	1.4861	1.3986	80.639	47.69	0.8
7	8.994	1.2494	0.8163	60.989	16.589	1.4791	1.4910	80.749	50.583	0.8
8	8.562	1.4049	0.8531	62.697	18.67	1.3791	1.6910	80.674	50.987	0.9
9	8.714	1.4276	0.8861	64.989	19.438	1.2678	1.7691	82.816	52.04	0.9
10	9.786	1.4620	0.8789	65.942	19.647	1.3021	1.7847	82.93	52.116	0.7
11	9.760	1.7980	0.8678	65.872	18.764	1.3421	1.8647	83.826	52.860	0.9
12	11.857	1.8698	0.9467	75.213	20.76	1.5834	1.5658	85.764	56.590	1.0

Key

 C_i = capacitors, R_i = Resistors, Passband = 0, Stopband = 0.5, Gain = 20, Frequency = 2, Gain = 60, Frequency = 3, Order = 5

Sensitivity to Specification Errors



Journal of the Nigerian Aon of Mathematical Physics Volume 12 (May, 2008), 297 - 306Low –pass electronic filter using Chebyshew PolynomialF. M. Aderibigbe and T. A. AdewaleJ of NAMP

Figure 4.1: Normal component values

5.0 Discussion of results and perspectives

Thus far, the example and design case studies we have considered may be viewed as involving deterministic circuit design, we have been concerned with the computation of set of circuit component values with an implied understanding that these values will be accurately realized when the circuit is built, so that optimum circuit performance will result. In the case of design of high-precision normal values, and this is done but the procedure is time consuming and costly and this will call for other circuit

design methods. Also, the sensitivity data obtained are essential as they allow the designer to make important practical decision on component tolerancing and provide a basis for comparing alternative circuit realization of a given specification. A circuit offering low sensitivity to component variation will, other things of being equal, be more reliable in long-term use and less susceptible to the temperature variation on component values. The example considered demonstrates the potential merits of using a coefficient matching phase design strategy namely the availability of fast analysis procedures and the very attractive facility of exact derivative computation using the multi-linearity property of the network function coefficients. The design objective function is also a comparatively simple function of the design variables. In order to deal with dangers of local minima the optimization runs were initialized by the component value sets obtained by Massara [13] on the basis that the modified solution should not be very far from this starting point. This proved a very effective strategy. Finally, since the optimizing model is based on zeroing the Taylor series, taking more terms by employing methods of higher orders than two would yield better component values. This will be combined with choosing elliptic function response in the next paper.

6.0 Conclusion

When the nominal circuit designed has been completed and a final full analysis carried out, the designer should have all possible information about the behavior of his circuit. However circuit is far from complete. A design may appear to be perfectly acceptable on the basis of nominal behavior and sensitivity yet be quite useless in practice. Indeed, many of the publish RC active circuit filter realizations (sallen and key filter and Bessel filter inclusive) belong to this category. Design will be completed only when tolerances have been assigned, and a full random simulation carried out to assess yield, production costs, and behaviours as components vary together. Most of the above have not been considered here.

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