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Vertical electrical resistivity soundings to locate ground water resources in Sabongida-Ora: A feasibility study

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Abstract

Subsurface geo-electrical survey using the electrical resistivity (VES) method was carried out in Sabongida-ora, Owan West Local Government Area of Edo State in order to investigate the aquifer characteristics and ground water potential of the subsurface formations. Three vertical electrical soundings were carried out using the Schlumberger array configuration. The data was interpreted using the conventional curve matching and computer iteration method. The results reveal seven geoelectric layers. The true resistivity of the top soil varies from 29 to 61Ωm while the thickness varies form 0.6 to 1.4m. The second layer has resistivity ranging from 14 to 109Ωm and thickness ranging from 2 to 3m. This layer is the laterite zone. The third, forth and the fifth geoelectric layers have resistivity ranging from 2 to 66Ωm and thickness ranging from 5 to 17m. The resistivity is diagnostic of clay layer. These layers act as a confining layer to the aquifer. The sixth geoelectric layer has resistivity ranging from 6 to 48Ωm and thickness varies from 15 to 33m this layer composes of fine to medium grain sand with little clay. This layer constitutes the first aquifer. The seventh geoelectric layer has a resistivity ranging from 24 to 177Ωm the thickness of this layer is not defined since it is the last layer. This layer consists of medium to coarser grained sand which constitutes an aquifer of very good quality groundwater. The average depth of this aquifer is between 48 to 53m. The result was correlated with lithological logs from boreholes drilled in the study area and was found to be consistent.

1.0 Introduction

In geophysical investigation for water exploration, depth to bedrock determinations, sand gravel exploration and so on, the Electrical Resistivity Method (ERM) can be used to obtain quickly and economically details about the locations, depth and resistivity of subsurface formations. [1] ERM uses an artificial source of energy rather than the natural fields of force such as used in gravity survey and so the source separation which effectively controls the depth of measurement. The water exploration survey with the help of ERM is low cost, easy for operation speedy and accurate. ERM is generally speedy and accurate ERM is generally employed for groundwater studies such as quality, quantity, mapping fresh water lenses, investigation of salt water intrusion and determination of the contamination

The ERM solves the problems of groundwater in the alluvium formation aquifer as an inexpensive and useful method. Some use of this method in groundwater are: determination of depth, thickness and boundary of an aquifer determination of interface, saline water and fresh water porosity of aquifer, hydraulic conductivity of aquifer, transmissivity of aquifer of groundwater, specific yield of aquifer contamination of groundwater.

The use of geophysics for groundwater resources and for water quality evaluation has increase dramatically over the last 10years. This in large part due to rapid advance in microprocessors and associated numerical modeling solutions. Ujuanbi and Asokhia (2001 [4]) successfully used the ERM to determine the extend of clay deposits in Sabongida-ora. Emenike (2001 [1]) used this method to also explore for groundwater in a sedimentary environment and Alile (2007 [5]) equally used the ERM method to locate groundwater in Edo South and Centre Senatorial District of Edo State (Table 5.4).

In the past, resident of Sabongida-ora depended on the slow-running Stream and the shallow hand dug wells for their domestic water needs, but today increase activities from human and companies which include infiltration of municipal water waste into the stream and the well from septic tank and agricultural activities, principally irrigation and fertilizer application have drastically polluted the stream and the well and rendered them unfit for use. Unfortunately, these are the only available sources despite increased demand for potable water in Sabongida-ora due to increase in pollution within the last few years. However, with recent technological development groundwater is the choice for domestic and industrial use. The purpose of this paper is to use the resistivity data and interpreted geoelectrical sounding to study the aquifer conditions such as depth and nature of the alluvium boundaries and location of the aquifer in Sabongida-ora, so as to protect groundwater supplies as a unique source of water.

1.1 Geology of the study area

The study area, Sabongia-ora is located in Owan West L.G.A of Edo State. It is underlain by the continental sands of the Benin formation. The geology of the Niger Delta has been extensively described by several authors including, Akpokodje and Etu-Efeotoeur (1987 [6]), Short and Stauble (1965 [7]), Asseez (1976). The subsurface sedimentary sequence has been subdivided into three stratigraphic units – the Benin, Agbada and Akata formations (Kogbe and Asseez (1979)). The Benin formation consists of sand, gravel sand, sandy clay and clay intercalations. The formation is known for its high aquifer potential. The lithological units of this area are generally composed of sands and clay.

2.0 Theory

It is conventional to designate the current electrodes A and B and the potential electrodes M and N. For a half – space solution we consider a single current electrode, a point source of current, on the surface of a homogeneous, isotropic half space, injecting a current I into the Earth [3]. The flow of electric current will be radially symmetric in the half space. This should be intuitively obvious, but is also a consequence of the two boundary conditions which must be satisfied at a contact between materials of differing conductivity, namely that the tangential potential must be continuous and the normal electric current density must be continuous. Since there is no current flowing in the atmosphere, there must be no normal component of J at the Earth's surface, i.e. *J* is tangential to surface. We balance the current flowing into the earth at the electrode with the total current flow out of a hemispherical surface. The total current across the hemisphere must be equal to 1 because there are no sources or sinks of current other than the electrode (i.e. charge is conserved). Because of the radial symmetry, the current density will be constant at a distance *r*, from the current electrode, so the total current flow across the hemispherical surface will be

$$I = \int_{hemisphere} J \cdot ds = 2\pi r^2 J \tag{2.1}$$

(*ds* is the surface element). Since current is always normal to the hemisphere, the integration is simply the surface area of the sphere times the (constant) current density. So, we have that at any distance r, the current density is

$$J = \frac{1}{2\pi r^2} r \tag{2.2a}$$

Where *r*, is the outward normal to the hemisphere. Substituting the above expression into Ohm's law gives $\sigma E = \frac{1}{2\pi r^2} r$ or using the definition of resistivity and rearranging

$$E = \frac{\rho I}{2\pi r^2}\mathrm{r}$$

obtained the potential at a distance R we integrate the electric field from infinity to R:

$$V_R = -\int_{\infty}^{R} E \cdot dr = -\int_{\infty}^{R} \frac{\rho I}{2\pi r^2} dr = \left.\frac{\rho I}{2\pi r}\right|_{\infty}^{R} \tag{2.2b}$$

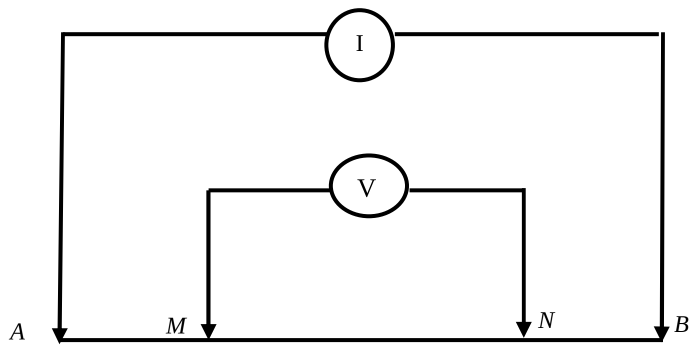

Figure 2.1: General 4–electrode array *A* and *B* are current electrode, M and *N* are potential electrodes.

Now for the general four – electrodes array (figure 2.1): we have that the potential at electrode M is simply the sum of the effects of the two current electrodes:

$$V_M = \frac{I\rho}{2\pi}\left(\frac{1}{AM} - \frac{1}{BM}\right)$$

and similarly, the potential at *N* is

$$V_N = \frac{I\rho}{2\pi}\left(\frac{1}{AN} - \frac{1}{BN}\right)$$

so the potential difference measured across *MN* is

$$\Delta V = V_M - V_N = \frac{I\rho}{2\pi}\left(\frac{1}{AM} - \frac{1}{BN} - \frac{1}{AN} + \frac{1}{BN}\right) = \frac{I\rho}{2\pi}\frac{1}{k}$$

where *k* is called the geometric factor:

$$k = \frac{I}{\left(\frac{1}{AM} - \frac{1}{BN} - \frac{1}{AN} + \frac{1}{BN}\right)}$$

We have derived the forward problem; in the above case the inverse problem is very easy and unique:

$$\rho = \frac{2\pi\Delta V}{I}k \tag{2.3}$$

That is, given a measurement of ΔV the above expression would correctly yield the resistivity of a homogeneous, isotropic half space. To aid interpretation, an apparent resistivity is defined for any measurement over any earth structure as

$$\rho_a = \frac{2\pi\Delta V}{I}k \tag{2.4}$$

If the Earth is not a homogeneous, isotropic half space the above expression would not yield the true resistivity of the earth. Although in principle the actual potential measurements could be interpreted in terms of more complex structure, in practice the apparent resistivity is the first step in the analysis of electrical sounding data. It is basically a way of normalizing away the geometry and current magnitude for the electrical measurement.

The Schlumberger array is the most commonly used arrangement, and was developed by Conrad Schlumberger in the 1930's. like most of the standard arrays it is collinear and symmetrical. The particular feature of the Schlumberger array is that the spacing between potential electrodes is very much smaller than the current electrode spacing. To compute, the expression for apparent resistivity for this array we note first that for a symmetrical array $AM = BN$ and $BM = AN$ so that

$$\frac{1}{k} = \left(\frac{2}{AM} - \frac{2}{BM}\right)$$

Now for the Schlumberger array we write

$$BN = AN = \frac{AB}{2} - \frac{MN}{2} = \frac{(AB - MN)}{2}$$

and

$$AN = BM = \frac{(AB + MN)}{2}$$

so

$$\frac{1}{k} = \frac{4}{AB - MN} - \frac{4}{AB + MN} \tag{2.5}$$

$$= \frac{4(AB + MN) - 4(AB - MN)}{(AB - MN)(AB + MN)} = \frac{8MN}{(AB - MN)(AM + MN)}$$

$$K = \frac{(AB - MN)(AB + MN)}{8MN}$$

yielding an expression for apparent resistivity

$$P_a = \frac{\pi \Delta V}{4 \cdot I \cdot MN}(AB - MN)(AB + MN) \tag{2.6}$$

Because AB ϕϕ MN we may set $AB \pm MN \approx AB$ and also write

$$E \approx \Delta V / MN$$

$$\rho_a \approx \frac{\pi E}{4 \cdot I}(AB)^2 = \frac{\pi E}{I}\left(\frac{AB}{2}\right)^2 \tag{2.7}$$

3.0 Interpretation over a layered earth

If we allow conductivity to vary with depth by means of a sequence of layers with uniform conductivity in each layer, then it is clear that current injected at a point will have a cylindrically symmetric distribution. As a consequence, many solution s to EM problems over layered models involve a Hankel transform:

$$K(b)\int_0^\infty k(\lambda) J_n(b\lambda) d\lambda \tag{3.1}$$

For a real transform argument $b > 0$, and where Jn is a Bessel function of the first kind and order n, and $k(\lambda)$ is a kernel function, which may be complex. In electromagnetic problems, the transform argument is associated with distance from the electrode, n is 0 or 1, and the kernel function contains information about the layer thickness and conductivities.

For example, for a Schlumberger array over an earth composed of layers with resistivities $\rho_1, \rho_2 \ldots \rho_N$ and thickness $T_{n-1}, \ldots T_2, T_1$ (counting from the top down),

$$\rho_a\left(\tfrac{AB}{2}\right) = \left(\frac{AB}{2}\right)^2 \int_0^\infty T_1(\lambda) J_1\left(\tfrac{AB}{2}\right) \lambda d\lambda \qquad (3.2)$$

The apparent resistivity is given by a Hankel transform: where J_1 is the first-order Bessel function of the first kind and $T_1(\lambda)$ is the resistivity transform, given for the top of each layer by

$$T_1 = \frac{T_{i+1} + \rho_{i\,\tanh(\lambda t_i)}}{1 + T_{i+1} + \rho_i \tanh(\lambda t_i) / \rho_i} \qquad (3.3)$$

The transform at the top of each layer depends only on the resistivity and thickness of that layer and the transform of the layer below. By starting at the bottom with $T_N = \rho_N$, we can compute $T_{n-1}, \ldots T_2, T_1$ in succession. This is called a recurrence relation and is how one gets T_i at the surface of the earth [2].

The integration of the oscillation Bessel function is difficult to do numerically. This is almost always handed by a thing called a fast Hankel transform. If we make the change of variables $x = \ln(AB/2)$ and $y = \ln(1/\lambda)$. Then

$$\rho_{\alpha(x)} = \int_{-\infty}^{\infty} T_1(y) J_1(e^{x-y}) e^{2(x-y)} dy$$

By defining

$$F(x-y) = J_1(e^{x-y}) e^{2(x-y)}$$

We can see that we have made a convolution integral

$$\rho_a(x) = \int_{-\infty}^{\infty} T_1(\lambda) F(x-y) dy \qquad (3.4)$$

With $T_i(y)$ as the input function ρ_a as the output function, and $F(x-y)$ as the filter function. We can convert the continuous into a discrete convolution using the sampling theorem. If $T(y)$ is sampled at $y_0 + j\Delta y$ where Δy is a sampling interval, and assuming that T contains no power above the Nyquist frequency $1/(2\Delta y)$, then

$$T(y) = \sum_{J=-\infty}^{\infty} T(y_0 + j\Delta y) \sin c\left[\frac{\pi(y - y_0 - j\Delta y)}{y}\right] \qquad (3.5)$$

We can substitute this expression for T into the convolution integral and Fiddle with the order of the integration and sum to get

$$\rho_a(x) = \int_{-\infty}^{\infty} \sum_{J=-\infty}^{\infty} T(y_0 + j\Delta y) \sin c\left[\frac{\pi(y - y_0 - j\Delta y)}{y}\right] F(x-y) dy \qquad (3.6)$$

$$= \sum_{J=-\infty}^{\infty} T(y_0 + j\Delta y) \int_{-\infty}^{\infty} F(x-y) \sin c\left[\frac{\pi(y - y_0 - j\Delta y)}{y}\right] dy \qquad (3.7)$$

If for a given j we call the infinite integral f_j, we have a very simple expression for ρ as a convolution over filter coefficients f_j:

$$\rho_a(x) = \sum_{-\infty}^{\infty} T(y_0 + j\Delta y) f_j \tag{3.8}$$

We cannot do an infinite sum so we will have to truncate the summation at j_{min} and j_{max}, hopefully in a way that preserves enough accuracy. The tails of the sinc function are oscillatory with zer crossings intervals of Δy so the f_j will decay more rapidly if we shift things by S to line up with these by setting $Y_0 = x + S$, giving us more accuracy for a given filter length, or a shorter filter for a given accuracy. Finally, it is conventional (for some reason) to reverse the order of the coefficients with respect to the sign of S, so we have at last

$$\rho_a(x) = \sum_{j=j_{\min}}^{j_{\max}} T_1(x + s - j\Delta) f_j \tag{3.9}$$

a simple digital filter over some small set of real coefficients f_j.

All we need to do is to find a set of f_j. They may be found by numerical integration, but most commonly they are discovered using know Hilbert transform pairs. Then a set of f_j may be found by iterative nonlinear parameter estimation, or by Fourier transformation:

$$\rho(f) = T(f)F(f)$$

so that

$$F(f) = \frac{\rho(f)}{T(f)}$$

4.0 Material and methodology

The ABEMT Terrameter SAS 300B was used in data gathering in the study area using Schlumberger electrode array configuration, vertical electric sounding. The Vertical Electrical Sounding (VES) consist of four electrodes – two outer electrodes for current and two inner electrodes for the potential measurement in depth probing the potential electrodes are fixed while the current electrode spacing is extended symmetrically about the center of the spread. This process yields a rapidly decrease potential difference across the potential electrode spacing which exceeds the measuring capabilities of the instrument. At this point a new value of potential electrode spacing used, typically 2 to 4 times longer than the preceding value and the survey is continued. Field measurement was taken at half current electrode spacing equals 1.00, 1.47, 2.15, 3.16, 4.64, 6.81, 10.00, 14.70, 21.50, 31.60, 46.40, 68.10, 100.00, 147.00… the initial value of the spacing between potential electrodes is 0.15m and gradually to 0.5 and 5m.

Three sounding were taken in the study area to locate the water saturated zone Schlumberger sounding data processing and interpretation program software were used in the data interpretation.

5.0 Results and discussions

The results and field theoretical curves are presented.

Figure 5.1

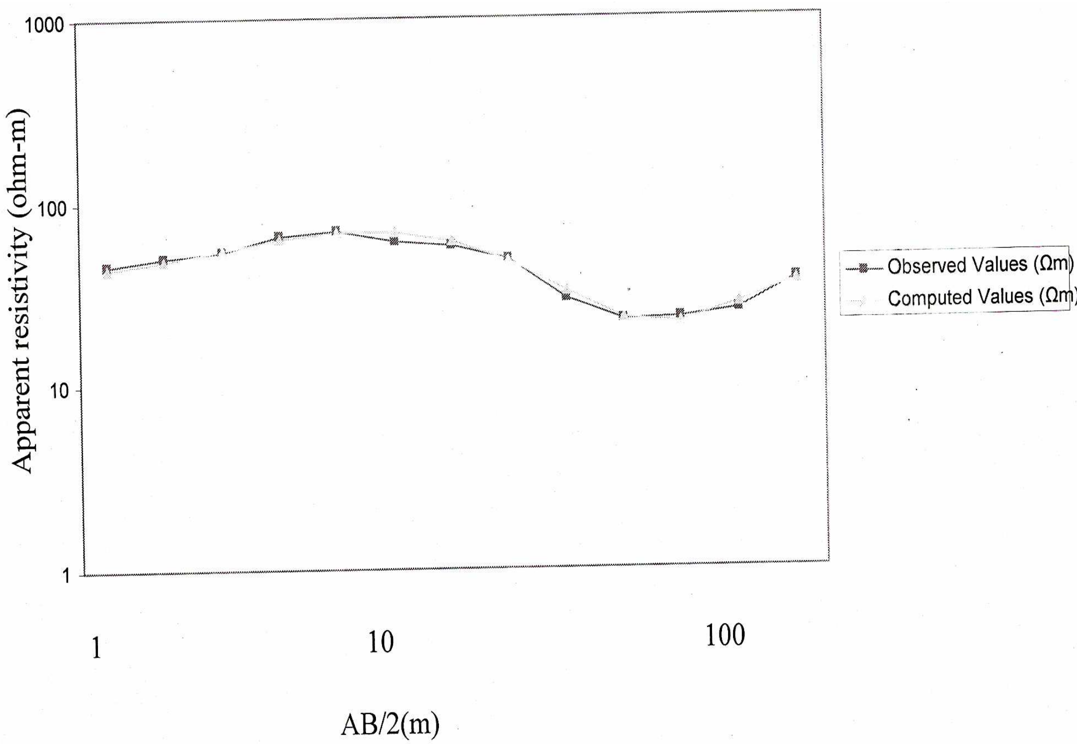

Table 5.1: Observed (Field) Computer (Theoretical) Data Model Parameters

AB/2 Value(m)	Oberved Values (Ωm)	Computed Values (Ωm)
1	45.32	43.85
1.47	49.91	47.83
2.15	53.47	54.43
3.16	65.54	62.5
4.64	68.92	67.32
6.81	60.68	67.42
10	56.41	60.54
14.7	47.47	46.93
21.7	29.02	31.92
31.6	21.95	22.37
46.4	22.52	21.32
68.1	25	26.64
100	36.66	35.9
147	54.92	48.34

Geoelectric layer	Resistivity (Ωm)	Thickness (m)	Cumulative thickness(m)
1	41.3	1.08	1.08
2	109	1.4	2.48
3	66.2	5.62	8.1
4	11.1	8.97	17.07
5	14.8	16.63	33.7
6	48	15.2	48.9
7	177	Infinity	Infinity

RMS Error(%) = 2.75

Figure 5.2

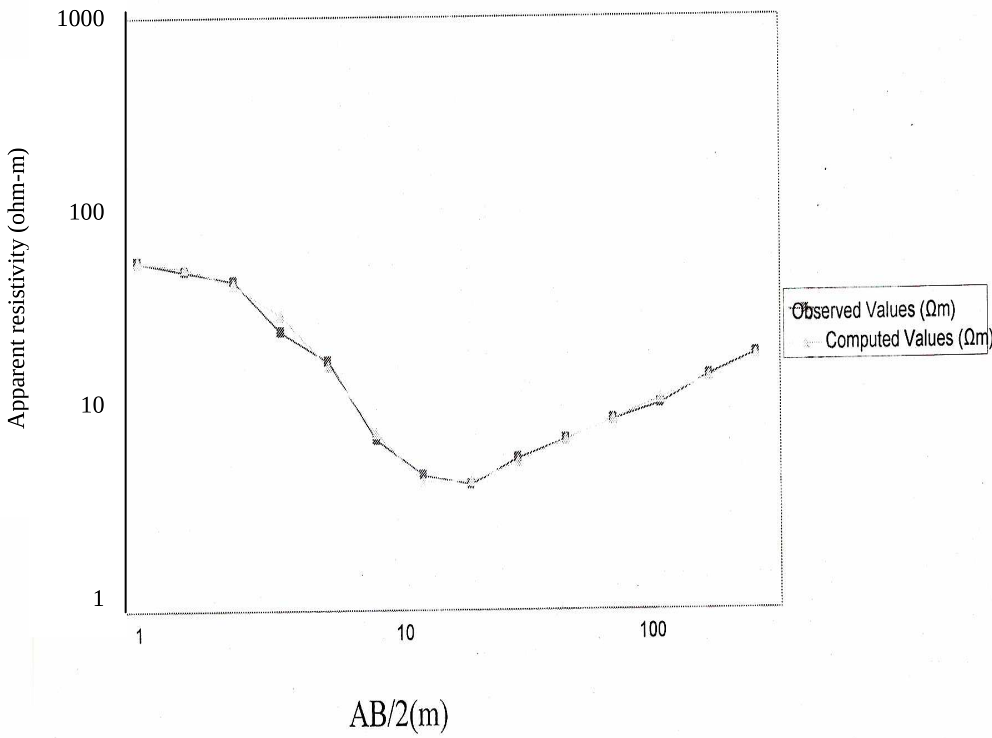

Table 5.2: Observed (Field) Computer (Theoretical) Data Model Parameters

AB/2 value(*m*)	Oberved values (Ω*m*)	Computed values (Ω*m*)
1	57.69	58.47
1.47	51.72	53.87
2.15	46.24	44.54
3.16	25.35	30.72
4.64	18.02	16.95
6.81	7.2	7.88
10	4.82	4.45
14.7	4.3	4.47
21.7	5.83	5.66
31.6	7.3	7.2
46.4	9.2	9.08

Geoelectric layer	Resistivity (Ωm)	Thickness (m)	Cumulative thickness(m)
1	61.4	1.39	1.39
2	14.2	1.68	3.07
3	2.1	5.26	8.33
4	13.2	15.98	24.31
5	11.5	12.11	36.42
6	27.2	16.1	52.52
7	44.7	Infinity	Infinity

RMS Error(%) = 2.75

68.1	10.8	11.54
100	15	14.78
147	19.4	18.86

Figure 5.3

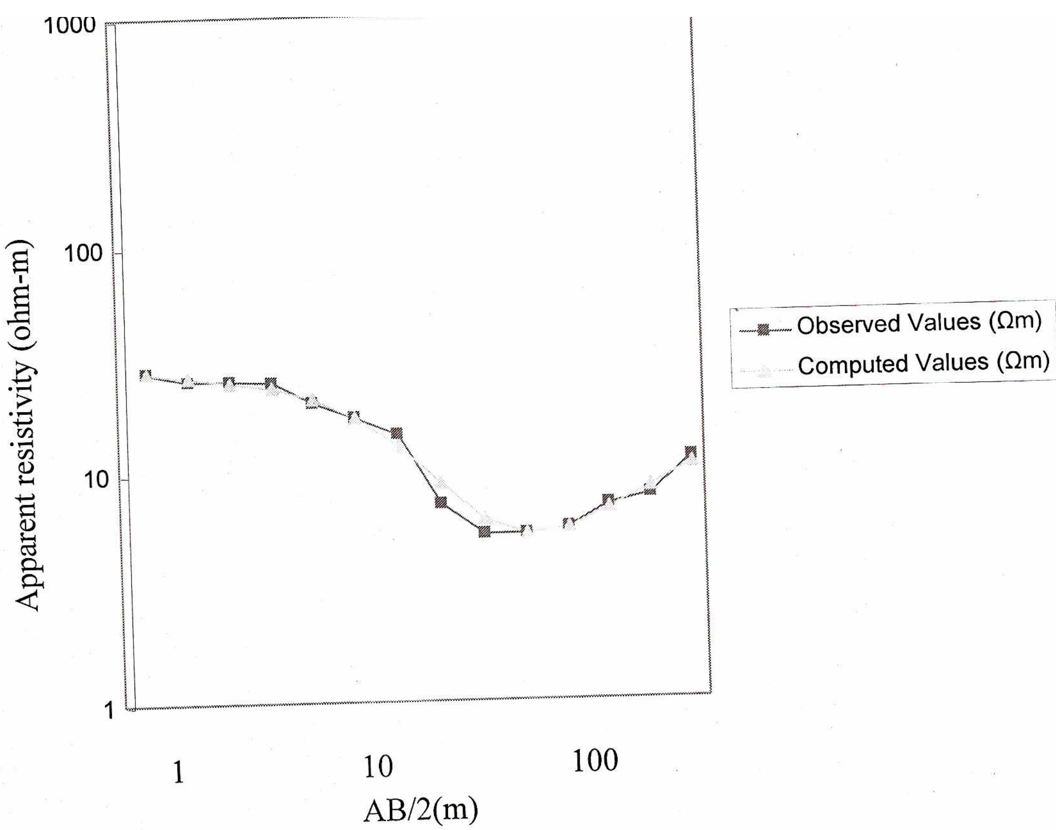

Table 5.3: Observed (Field) Computer (Theoretical) Data Model Parameters

AB/2 value(*m*)	Oberved values (Ω*m)*	Computed values (Ω*m*)
1	28.54	28.19
1.47	25.75	26.97
2.15	25.67	25.47
3.16	25.56	23.73
4.64	20.67	21.37
6.81	17.60	17.74
10	14.65	13.15
14.7	7.26	8.91
21.7	5.30	6.26
31.6	5.30	5.34
46.4	5.60	5.69

Geoelectric layer	Resistivity (Ω*m*)	Thickness (*m*)	Cumulative thickness(*m*)
1	29.55	0.68	0.68
2	23.96	1.53	2.21
3	22.49	2.14	4.35
4	7.22	6.35	10.70
5	2.90	8.96	19.66
6	6.70	32.80	52.46
7	24.15	Infinity	Infinity

RMS Error(%) = 3.64

68.1	7.16	6.81
100	7.90	8.52
147	11.20	10.76

Table 5.4: Depth and approximate thickness of aquifer in some locations in Edo State [5]

Locations	**Approx. depth of aquifer(m)**	**Approx. thickness of aquifer(m)**
Igieduma, Uhunmwonde LGA	197.49	75.00
Ubiaja, Esan North East LGA	240.76	116.00
Igueben, Igueben LGA	174.91	130.12
Edumebo, Ekpoma, Esan West LGA	266.32	90.05
Iruekpen, Esan West LGA	252.57	115.43
Ehor Uhumwonde LGA	262.57	103.02
Obedo Uromi	207.41	175.00
Obaretin, Ikpoba Okha LGA	59.80	26.00
Idumebo Irrua Esan Centre LGA	168.41	175.00

6.0 Discussion

The resistivities, thickness and curve types from the interpreted sounding curves are presented above. In VES 1 the curve has a belbowl (KH) $(\rho_1 < \rho_2 > \rho_3 < \rho_4)$ shape. Showing a seven layered earth and the curves types for VES 2 and 3 are seven layers $QH(\rho_1 < \rho_2 > \rho_3 < \rho_4)$ curve. The first geoelectric layer corresponds to the top soil with resistivity ranging from 29 to 61Ωm reflecting the various compositions and moisture content of the top soil. It is composed of clay, fine sand and decomposed organic mateicals the thickness varies from 0.6 to 1.4m. The second geoelectric layer has resistivity ranging from 14 to 109 Ωm and thickness varies from 2 to 3m constitute the latarite layer. The third, forth and the fifth geoelectirc layers have resistivity ranging from 2 to 66Ωm and thickness ranging from 5 to 17m. The resistivity is diagnostic of clay layer. These layers act as a confining layer to the aquifer. The sixth geoelectric layer has resistivity ranging from 6 to 48Ωm and thickness ranging from 15 to 33m. This layer composes of fine to medium grain sand with little clay. This layer constitutes the first aquifer. The seventh geoelectric layer has a resistivity ranging from 24 to 177Ωm the thickness of this layer is not defined since it is the last layer. This layer consists of medium to coarser grained sand which constitutes an aquifer of very good quality groundwater. The average depth of this aquifer is between 48 to 53m. The result was correlated with lithological logs from boreholes drilled in the study area and was found to be consistent.

7.0 Conclusion

Three sounding were used to evaluate the sub surface hydrogeological conditions to a depth of about 53.m. Based on the interpretation of geoelectric data, the following conclusion were drawn: the use of geoelectric sounding provides an inexpensive method to characterizing the groundwater condition in sabongida-ora. Interpretation of the VES tests indicates the presence of a confined aquifer that mainly contains medium to coarser grained sand. The average depth of the aquifer is between 48 to 53m. The VES test also revealed seven geoelectric layers consisting of surface layer (top soil), alluvium layers and saturated bottom layer, depth and thickness of all the layer were identified. The result was correlated with lithological logs from boreholes drilled in the study area and was found to be consistent.

8.0 Recommendation

From the study it is recommended that boreholes are drilled to 48-53m to harness potable water within the 2^{nd} aquifer in sabongida-ora.

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