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A heat model for temperature distribution in a laptop computer

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Abstract

In this study, the heat diffusion equation that captures the maximum variation of temperature with time in an electronic note book (hp Compag nX6310 laptop computer) from its hot underside to its cool top is derived. Using appropriate boundary conditions (BCs) and transformations, the second order differential heat equation was solved and the solution which gives the temperature distribution of the system obtained. The model was tested with time ranging from 0.0s to 2100.0s at a prevailing room temperature of 28.0℃. The maximum temperature distribution within this time was 12.8℃ at the bottom and 8.8℃ at the top (keyboard) above the prevailing room temperature. These results were in excellent agreement with the measured (experimental) results when comparison was made between the two results.

1.0 Introduction

Many of the processors in personal computers are narrow, thin rectangular blocks which are mounted on circuit boards, in which a poor design could trap heat generated by the power supply below **[1]**. The efficiency in the functioning of these systems is enhanced when they are subjected to external cooling devices like air conditioners, electric fans and so on which are all powered by electricity. Laptop computers (electronic notebooks) are designed with inbuilt storage devices that store electrical energy for them to function for sometime even without external source of power supply.

In Nigeria, there is high frequency of interrupted power supply which incapacitates cooling of computer systems by external devices like air conditioners and fans. This implies that in the event of per outage where there is no external power supply to facilitate the cooling of the system by the external cooling devices, the system continues to work and the heat distribution goes on. The resulting effect is that the efficiency and durability of the system will be affected if it is allowed to work in such a condition longer than necessary. In order to know how much the temperature increases with time when the computer is switched on, it is highly desirable in this paper to predict, through a heat model, the overall heat distribution with time in an electronic note book (laptop). This heat model is expected to capture the variation in temperature of the system from its hot underside to its cool top. Thus, the problem of how much heat is generated at any time when the system is switched on is hoped to be addressed by the heat model. The pain for this work is as follow. In section2, we derive the mathematical formulation of the heat model. In section 3, we solve the second order differential equation of our model and we present our results in section 4, then we conclude.

2.0 Mathematical formulation of the heat model.

This model assumes that the electronic note book starts at an initial temperature zero, which is

actually the prevailing room temperature, and that its top remains at the temperature while its bottom at temperature T_0. Since the height of the system is relatively small compare with its width, we simplify the problem by neglecting the variation of temperature in these directions. That is, we assume that the heat flows only in the vertical direction from the lower surface $y = 0$ at temperature T_0 to the upper surface $y = h$ at temperature zero (prevailing room temperature).

An appropriate control volume is a box of width Δx, height Δy and depth Δz. If the control volume is Δz deep, then the area of its top and bottom is $\Delta z \Delta x$. Conservation of energy requires that [1, 2, 3]

$$\textit{Changing in heat over } \Delta t = Q(t+\Delta t) - Q(t)$$

$$\textit{Heat energy in at } y = -K(\Delta z \Delta x)\frac{\partial T(y,t)}{\partial y}\Delta t$$

$$\textit{Heat energy out at } y+\Delta y = -K(\Delta z \Delta x)\frac{\partial T(y+\Delta y,t)}{\partial y}\Delta t\ ,$$

where K is the thermal conductivity of the system. Equating the terms as required by conservation of energy gives

$$\Delta t = Q(t+\Delta t) - Q(t)$$

$$= -K(\Delta z \Delta x)\frac{\partial T(y,t)}{\partial y}\Delta t + K(\Delta z \Delta x)\frac{\partial T(y+\Delta y,t)}{\partial y}\Delta t \qquad (2.1)$$

The value of the quantity of heat Q is expressed in terms of the density ρ and specific heat capacity c and volume v of the system as

$$Q = \rho v c T = \rho \Delta x \Delta y \Delta z c T \qquad (2.2)$$

$$\Delta x \Delta y \Delta z c \rho (T(t+\Delta t) - T(t))$$

$$= -K(\Delta z \Delta x)\left[\frac{\partial T(y,t)}{\partial y}\Delta t - \frac{\partial T(y+\Delta y,t)}{\partial y}\Delta t\right] \qquad (2.3)$$

Divide through by the area $\Delta z \Delta x$ to get

$$\Delta y c \rho (T(t+\Delta t) - T(t)) = -K\left[\frac{\partial T(y,t)}{\partial y}\Delta t - \frac{\partial T(y+\Delta y,t)}{\partial y}\Delta t\right] \qquad (2.4)$$

Separation of variables into Δy and Δt leads to

$$\Delta y c \rho (T(t+\Delta t) - T(t)) = K\frac{\partial}{\partial y}\{T(y+\Delta y,t) - T(y,t)]\Delta t$$

Therefore,

$$\frac{c\rho(T(t+\Delta t - T(t))}{\Delta t} = \frac{K\dfrac{\partial}{\partial y}[T(y+\Delta y,t) - T(y,t)]}{\Delta y}$$

Taking limits as Δy and Δt tends to zero, we have

$$c\rho\frac{\partial}{\partial t}T(y,t) = K\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}T(y,t)\right) = K\frac{\partial^2}{\partial y^2}T(y,t)$$

If the quotient $K/c\delta = \kappa$, then the heat equation becomes

$$\frac{\partial}{\partial y}T(y,t) = \kappa \frac{\partial^2}{\partial y^2}T(y,t) \tag{2.5}$$

Equation (2.5) is the required heat diffusion equation for our system and is a second order differential equation.

3.0 Solution to the head diffusion equation

To solve the heat diffusion equation derived in section 2.0 for our problem, we initialize the prevailing room temperature as 0°C. In doing so we assume that the system of height *h* is initially at a temperature of 0^0C. One end of the system (top) is held at 0°C while the other (bottom) is supplied with heat at constant rate per unit area of *H*. The boundary conditions (BCs) can be written as [4]

$$T(y,0) = 0,\ T(0,t),$$
$$\frac{\partial T(h,t)}{\partial y} = \frac{H}{K}, \tag{3.1}$$

the last of which is inhomogeneous, and can cause difficulties. We therefore transform the BCs into equivalent homogeneous ones [4]

$$v(y,0) = -\frac{Hy}{K},$$
$$v(0,t) = 0,\ \frac{\partial v}{\partial y}(h,t) = 0,$$

which are homogeneous in *y*. We now assume the solution to our problem to take the form

$$T(y,t) = v(y,t) + \omega(y), \tag{3.2}$$

with

$$\omega(y) = \frac{Hy}{K}.$$

From the method of separation of variables, the separated solution for the one dimensional diffusion equation is

$$v(y,t) = (A\ \cos\ \lambda y + B\ \sin\ \lambda y)\exp(-\lambda^2 Kt),$$

corresponding to separation constant $-\lambda^2$. To satisfy

$$v(0,t) = 0,$$

we require $A = 0$. Further more, since

$$\frac{\partial v}{\partial y} = B\ \exp(-\lambda^2 Kt)\,\lambda \cos \lambda y,$$

in order to satisfy

$$\partial v(h,t)/\partial y = 0,$$

we required $\cos \lambda h = 0$, and so λ is restricted to take the values

$$\lambda = \frac{n\pi}{2h}, \tag{3.3}$$

where *n* is an odd integer. Thus, to satisfy the boundary condition

$$v(y,0) = -Hy/K,$$

we must have

$$\sum_{n\ odd} B_n \sin\left(\frac{n\pi y}{2h}\right) = \frac{Hy}{K}$$

in the range $y = 0$ to $y = h$. This leads to Fourier sine series coefficients containing only odd-numbered terms. These corresponding Fourier series coefficients are found to be

$$B_n = -\frac{8Hh}{K\pi^2}\frac{(-1)^{(n-1)/2}}{n^2}$$

for n odd and thus the final formula for $T(y,t)$ is

$$T(y,t) = \frac{Hy}{K} - \frac{8Hh}{K\pi^2}\sum_{n\ odd}\frac{(-1)^{(n-1)/2}}{n^2}\sin\left(\frac{n\pi y}{2h}\right)\exp-\left(\frac{Kn^2\pi^2 t}{4h^2 c\rho}\right), \qquad (3.4)$$

giving the temperature for all positions $0 \le y \le h$ and for all positions $t \ge 0$.

4.0 Result and discussion

The derived heat model as well as its solution as given by equations (2.5) and (3.4) respectively is applicable to any lab top computer system. However, since there are variations in the values of the parameters such as height of the computer h, density ρ, and H as we move from one computer system to another, one needs to get the correct values of these parameters for a given brand of computer. For this reason, the model was tested for hp Compaq $nX6310$ in this paper. The height h and density ρ of the system were measured and calculated to be $0.022m$ and $650.277kgm^3$ respectively. The values of thermal conductivity K and specific heat conductivity c for the system are $1.06J/K$ and $1002J/Kg/K$ respectively, while the rate of heat generated by the system, H, is $400.0J/m^3s$.

Table (4.1) shows the calculated values of temperature distribution with time when the system was switched on at the time the prevailing room temperature was 28.0°C. The middle column gives the temperature distribution from the bottom (hot underside) while the last column gives the temperature distribution from the top (key board). Table 4.2 gives the values of temperature distribution with time as measured using a thermometer and a stopwatch. Both tables are plotted on a graph of temperature against time in figure 4.1 Series T1 (bottom) and T2 (top in figure 4.1 are the theoretical temperature distribution in the system from its hot underside to its cool top when the system was switched on for a period of 2100 seconds respectively, while series T3 (bottom) and T4 (top) are the corresponding temperature distribution in the system from its hot underside to its cool top respectively as measured on a thermometer within the same time limit.

The results of our theoretical work are in excellent agreement with the experimental (measured) values when comparison was made between them. The maximum temperature distributions when they system was switched on for a period of 2100 seconds from the bottom and top are 12.8°C and 8.8 °C respectively above the prevailing room temperature of 28.0°C for the theoretical values while that of the measured values are 13.0°C and 7.0°C from bottom to top respectively above the same prevailing room temperature of 28.0°C and within the same range of time.

The implication of the above result is that, if the lap top is used in an environment where the ambient temperature is generally high (33.0°C – 36.0°C) as it is the case in some part of the world, the efficiency of the system will be affected and its durability may be reduced since these systems are sensitive to high temperatures.

Table 4.1: Calculated values of temperature distribution with time in an hp (Campaq *n*X3610) lap top computer when it was switched on.

Time(s)	Temperature (°C) (bottom)	Temperature (°C) (top)
0.0	27.8	28.1
300.0	31.0	30.6
600.0	33.2	31.0
900.0	36.0	33.5
1200.0	37.8	34.8
1500.0	39.5	35.6
1800.0	40.4	36.2
2100.0	40.8	36.8

Table 4.2: Measured values of temperature distribution with time with time in an hp (Campaq nX3610) lap top computer when it was switched on.

Time(s)	Temperature (°C) (bottom)	Temperature (°C) (top)
0.0	28.0	28.0
300.0	32.0	30.0
600.0	34.5	31.5
900.0	37.5	32.0
1200.0	39.0	33.0
1500.0	40.8	34.5
1800.0	41.0	35.0
2100.0	41.0	35.0

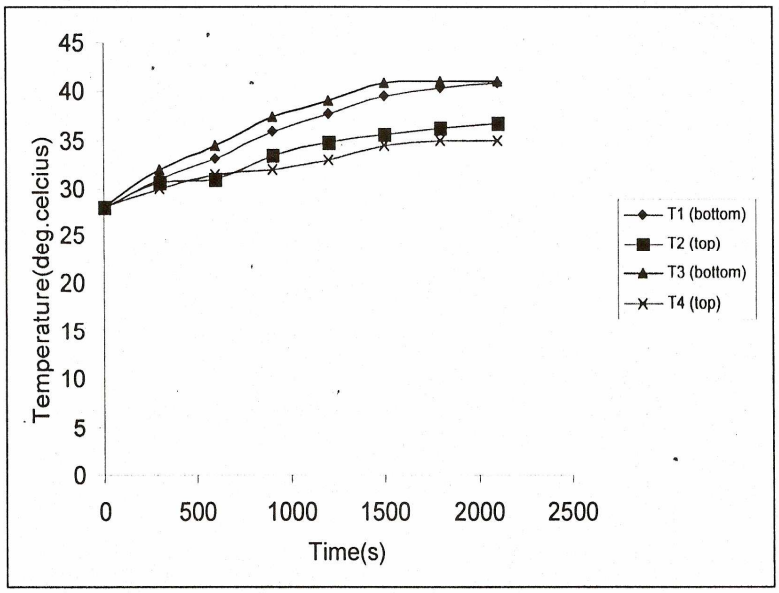

Figure 4.1: Variation of temperature distribution with time in a laptop when is switched on.

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