Viscous dissipative heat effect on radiative magnetic flow of an electrolyte in a vertical channel

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Abstract

This paper presents the effect of radiation on the magnentohydrodynamics flow of an electrically conducting fluid inside an open-ended vertical channel, permeated by a uniform transverse magnetic field. Of particular interest is the investigation of the behavior of velocity, temperature, induced magnetic field, heat transfer across the walls and flow rate. The optically thick limit case is considered. The fluid is assumed incompressible and viscous, while the flow is steady. The viscous dissipation term is therefore considered, in the "Governing equation". The difference in temperature between the channel walls and that of the undistributed fluid is large enough for and is therefore physically important. The standard equations of continuity, momentum and energy and assumed and non-dismensionalised. The non-dimensional equations feature some non-dimensional parameters i.e. Grashof-number (Gr.), Birkman-number (Br.), Raynold-number (Rr.), Prandtl-number (Pm), Hartmann-number (M), etc. The resulting non-linear partial differential equations were solved analytically using successive approximation technique. It is found that increase in the magnetic field, the flow rate and velocity, but a decrease in the heat transfer at the walls and the temperature.

1.0   Introduction

In the past few years, the interests of many researchers have been aroused on the motion of an electrically conduction liquid in a magnetic field. The interest stems from terrestrial problems concerning the motion of ionized gases in nuclear reactions from astronomical problems concerning the motion of molten magma in the atmosphere. Hartmann was the first to carry out laboratory experiments on the steady flow of a conducting liquid he used was mercury (Hg), he called the subject "Hg-dynamics". However, the term currently used is magnetohydrodynamics (MHD). When a conducting fluid moves at right angles to the magnetic field, a difference of electric potential is set up in a direction perpendicular to both of these; thus given rise to an electric current across the fluid and the internal (thermal) energy is directly converted into electrical energy. This is the basis of the (MHD) generators. A comprehensive review of work done in this area was given by Rowing in [1], Siegal [2], Permultter and Siegal [3] and Alpher [4], Gersgum and Zhukhavitsky [5], YU [6] all the flows subjected to transverse magnetic field. Andi, E. A. 1995 [7] focused on the influence of viscous dissipation on the radiative hydromagnetic combined effect of radiation and viscous dissipation on hydromagnetic flow of a fluid in a vertical channel.

Gupta, P. S. and Gupta, A. S. [9] studied the effect of radiation on the combined free and forced convection of an electrically conducting fluid flowing inside an open ended vertical channel in the presence of a uniform transverse magnetic field, conforming the analysis to the optically thin limits case, but neglected viscous and Ohmic dissipation.

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Thus, this work is concerned with the effects of viscous dissipation of energy which plays an important role in natural convective flow field of extremes size or in high gravity.

Bestman [10] studied the pulsatile flow in a heated porous channel and concluded that viscous dissipation heat and free convection parameter causes an increase in heat flux at the wall of the channel.

Viscous dissipative heat effect on radioactive (MHD) channel flow has not received much attention. However, the objective of this research is to investigate the viscous dissipative heat effect on radiative magnetic flow of an electrically conducting fluid flowing inside a vertical channel, when an optically thick limit case is considered.

2.0   Formulation of the boundary value problem

Considered here is the motion of an electrolyte (electrically conducting fluid) inside a vertical channel of two parallel walls with distance L apart. Taking the origin at the centre point of the channel, a magnetic field (B_0) is applied across the end of the channel (acting normal to the channel walls).
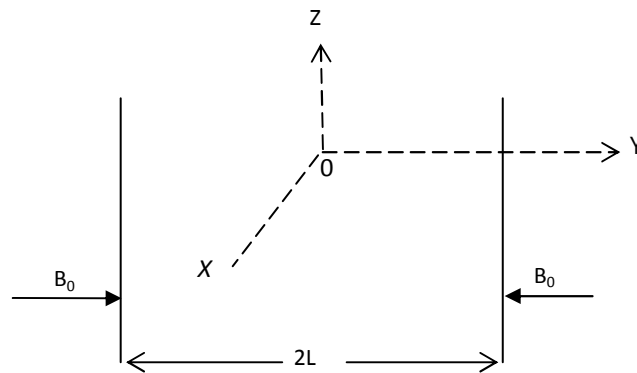
These there coupled non-linear differential equation (governing equation) are solved using successive approximation procedure to get the velocity, temperature and magnetic induction fields subject to the boundary conditions.

$$v = t = b = 0, \quad \Psi = \pm 1$$

Thus these equations to be solved are:

$$V = \frac{t''}{R} + m_2 t^3 t' + \frac{B_r}{P_r}(v^t)^2, \quad D = V'' + \frac{m^2}{R_m} b' - \frac{G_r}{N_r} t \qquad (2.1)$$

$$b'' = R_m V' \text{ subject to } T(\Psi) = 0, \; B(\Psi) = 0, \; V(\Psi) = 0 \qquad (2.2)$$

For $-1 \leq \Psi \leq 1$ where $M_2 = M_1(T_1 - T_0)/V_0 I$

$$B_r = \frac{V V_0^2}{P^C{}_p(T_1 - T_2)}$$ is the modified Eckert number $N_r = \frac{P T V_0}{\mu}, \quad R_m = \gamma \mu V_0 I$

$$G_r = \frac{gB(T_1 - T_0)L^3}{\upsilon I V_0}, \quad P_r = \frac{\upsilon}{\alpha}, \quad M_1 = \frac{-16\gamma^3}{3\alpha\rho C_p},$$

3.0   Solution Procedure

Successive Approximation Procedure shall be used to solve the non-linear coupled differential equations, i.e. by neglecting non-linear terms, approximating some and neglecting others to get

approximate value for $v_1, v_2, t_1, t_2, b_1, b_2$. The iteration shall be carried out for successive value of v, t and b. Then v_{n+1}, t_{n+1} and b_{n+1} lead to a better approximate value for V_n, t_n and b_n respectively. Rewritten equation (2.2) in terms of V and substituting into equation (2.1) to have

$$V = \frac{t''}{R} + m_1 t^3 t^1 + \frac{B_r}{\rho_r R_m^2}(b'')^2 \tag{3.1}$$

differentiating equation (2.2) with respect to (Ψ) and substitute into equation (2.1) we have

$$b''' = m^2 b^1 + \frac{R_m G_r}{N_r} t = -DR_m \tag{3.2}$$

Also, integrating equation (2.2) with respect to (Ψ) and substitute into equation (3.2) to yield

$$b'' + m^2 R_m V - \frac{R_m G_r}{N_r} = (m^2 k_1 - DR_m) \tag{3.3}$$

However, substituting equation (3.1) into equation (3.3) results into

$$b''' t \lambda_1 (b'')^2 + \lambda_3 = -\lambda_4 t'' - \lambda_5 t^3 t' + \lambda_6 t \tag{3.4}$$

Neglecting non-linear terms and b''' in equation (3.4) gives

$$-h_4 t'' + \lambda_6 t - \lambda_3 = 0 \tag{3.5}$$

Also neglecting b''' in equation (3.1) gives

$$V_1 = \frac{t''}{R} + m_1 t^3 t' \tag{3.6}$$

but interpreting of equation (3.2) with respect to (Ψ) in views of (3.6) gives

$$b_1' = -R_m(v_1) + k_1 \tag{3.7}$$

From the equation (3.5), (3.6) and (3.7) we can get v_1, t_1 and b_1 i.e. first approximation, but not a better approximation. The neglected term (b'') and the non-linear terms in equation (2.1) are now introduced into equation

$$b''' = -\lambda_4 t'' - \lambda_5 t^3, t_1' + \lambda_6 t_1 - \lambda_3 \tag{3.8}$$

to get a better approximate value for b_2. b_2 in equation (3.8) is substituted into equation ( ) to get an approximate solution for V i.e. V_2 and

$$V_2 = t'' + m_1 t^3 \; t_1' + \lambda_1 + \lambda_1 (b'')^2 \tag{3.9}$$

b'' is introduced to get better approximation for t i.e. t_2 we shall stop at the 3rd iteration for convenience. Hence,

$$b_2''' + \lambda_1 (b_2'') + \lambda_3 = -\lambda_4 t + \lambda_6 t \tag{3.10}$$

The solution of equation (3.10) gives t_3. Substituting t_3 and (b_2'') into equation (3.1) as

$$V = t_3'' + \lambda_1 (b_2)^2 \tag{3.11}$$

Then the solution V_3 form equation (3.11) is substituted into equation (3.11) is substituted into equation

$$(V^1)^2 = \left(\frac{1}{R_m} b''\right)^2 \tag{3.12}$$

as

$$b^1 - R_m(V_3 + k_1) \tag{3.13}$$

Hence V_3 and b_3 are obtained.

4.0   Successive approximate solution

The solution to equations (3.2), (3.6), (3.7) gives v_1, t_1 and b_1 respectively, therefore

$$\lambda_4 t'' - \lambda_6 t = \lambda_3 \tag{4.1}$$

Solving (4.1) completely and applying the boundary condition gives

$$t_1 - \left[A_1 (e^{R_1 \Psi} + e^{-R_1 \Psi}) + \lambda_7 \right] \tag{4.2}$$

where

$$A_1 = -\lambda_7 / e^{R_1} + e^{2R_1} \tag{4.3}$$

Substitute (4.2) into (3.6) after manipulation we have

$$V_1 = M_1 A_1 R_1^2 \left(e^{4R_1 \Psi} - e^{4R_1 \Psi} \right) + 3 A_1^3 R_1 M_1 \lambda_7 \left(e^{3R_1 \Psi} - e^{3R_1 \Psi} \right)$$

$$+ M_1 R_1 (2A_1^4 + 3A_1^2 \lambda_7) \left(e^{2R_2 \Psi} - e^{-2R_2 \Psi} \right) \frac{A_1 R_1^2}{R} \left(e^{R_1 \Psi} - e^{-2R_2 \Psi} \right)$$
$$+ \left(A_1^3 \lambda_4 M_1 R_1 + \lambda_7^3 A_1 M_1 R_2 \right) \left(e^{R_1 \Psi} + e^{-R_1 \Psi} \right) \tag{4.4}$$

In the same vain,

$$b_2 = \lambda_8 \left(e^{4R_1 \Psi} + e^{-4R_1 \Psi} \right) - \lambda_9 \left(e^{3R_1 \Psi} + e^{-3R_1 \Psi} \right) - \lambda_{10} \left(e^{2R_1 \Psi} + e^{-2R_1 \Psi} \right) - \lambda_{11} \left(e^{R_1 \Psi} + e^{-R_1 \Psi} \right)$$

$$- \lambda_{12} \left(e^{R_2 \Psi} - e^{-2R_2 \Psi} \right) - \lambda_{13} \Psi^4 + k_3 \frac{\Psi^2}{2} + k_4 \Psi + k_5 \tag{4.5}$$

where $k_3 = 2 \left[\lambda_8 \left(e^{4R_1} + e^{-4R_1} \right) + \lambda_9 \left(e^{3R_1} + e^{-3R_1} \right) + \lambda_{10} \left(e^{2R_1} + e^{-2R_1} \right) + \lambda_{11} \left(e^{R_1} + e^{R_1} \right) \right.$ (4.6)

$$k_4 = \lambda_{12} \left(e^{-R_2} - e^{R_2} \right) + \lambda_{13} \tag{4.7}$$

However, for a better approximate solutions

Let $b_1 + b_2 = B_2$, therefore

$$B_2 = -\left[\frac{R_m A_1^4 M_1 + \lambda_9}{4} \right] \left(e^{4R_1 \Psi} + e^{-4R_1 \Psi} \right) - \left[\frac{R_m A_1^3 M_1 + \lambda_9}{3} \right] \left(e^{3R_1 \Psi} + e^{-3R_1 \Psi} \right)$$

$$- \left[\frac{R_m M_1}{2} (2A_1^4 + 3A_1^2 \lambda_7^2) - \lambda_{10} \right] \left(e^{2R_1 \Psi} + e^{-2R_1 \Psi} \right) - \left[R_m (A_1^3 \lambda_4 M_1 + \lambda_7^3 A_1 M_1) - \lambda_{11} \right] \left(e^{R_1 \Psi} + e^{-R_1 \Psi} \right)$$

$$- \left(\frac{R_m A_1 + \lambda_{12}}{R_1} \right) \left(e^{R_1 \Psi} + e^{-R_1 \Psi} \right) - \lambda_{13} \Psi^3 + \frac{k_3 \Psi^2}{2} + (k_4 + k_1) \Psi + (k_5 + k_2) \tag{4.8}$$

and lastly $V_2(\Psi) = \dfrac{1}{R_m} \left[\lambda_{18} \left(e^{4R_1 \Psi} - e^{-4R_1 \Psi} \right) + \lambda_{22} \left(e^{3R_1 \Psi} - e^{-3R_1 \Psi} \right) + \lambda_{20} \left(e^{2R_1 \Psi} - e^{-2R_1 \Psi} \right) \right.$

$$\left. + \lambda_{21} \left(e^{R_1 \Psi} - e^{R_1 \Psi} \right) + \lambda_{22} \left(e^{R_1 \Psi} + e^{-R_1 \Psi} \right) + 3\lambda_{13} \Psi^2 - k_3 \Psi - \lambda_{23} \right] \tag{4.9}$$

$$T_2(\Psi) = \lambda_{50} \left(e^{8R_1 \Psi} + e^{-8R_1 \Psi} \right) + \lambda_{51} \left(e^{7R_1 \Psi} - e^{-7R_1 \Psi} \right) + \lambda_{52} \left(e^{6R_1 \Psi} + e^{-6R_1 \Psi} \right) + \lambda_{53} \left(e^{5R_1 \Psi} \right)$$

$$+ \lambda_{54} e^{-5R_1 \Psi} + \lambda_{55} e^{-4R_1 \Psi} + \lambda_{56} e^{-4R_1 \Psi} + \lambda_{57} e^{3R_1 \Psi} + \lambda_{58} e^{-3R_1 \Psi} + \lambda_{59} e^{2R_1 \Psi} + \lambda_{60} e^{-2R_1 \Psi}$$

$$+ \lambda_{61} e^{R_1 \Psi} + \lambda_{62} e^{-R_1 \Psi} + \lambda_{63} \Psi_4 - \frac{k_3 \Psi^3}{6} - \lambda_{67} \Psi^2 + k_6 \Psi + k_7 \tag{4.9}$$

where

$$k_7 = \lambda_{50} \left(e^{8R_1} + e^{-8R_1} \right) + \lambda_{51} \left(e^{7R_1} + e^{-7R_1} \right) + \lambda_{52} \left(e^{6R_1} + e^{-6R_1} \right) \tag{4.10}$$

$$+ \lambda_{53} e^{5R_1} + \lambda_{54} e^{-5R_1} + \lambda_{55} e^{4R_1} + \lambda_{56} e^{-4R_1} + \lambda_{57} e^{3R_1} + \lambda_{58} e^{-3R_1} + \lambda_{59} e^{2R_1}$$

$$+ \lambda_{60}e^{-2R_1} + \lambda_{61}e^{R_1} + \lambda_{62}e^{-R_1} + \lambda_{53} + \lambda_{63} - \lambda_{67} \qquad (4.11)$$

$$k_6 = \frac{k_3}{6} \qquad (4.12)$$

Group by the same process, we have $T_3(\Psi), V_3(\Psi)$ and $B_3(\Psi)$ which are better and preferable approximations for ultimate results. Thus:

$$T_3(\Psi) = \left(B_1e^{R_1\Psi} + B_1e^{-R_1\Psi}\right) + N_1\left(e^{8R_1\Psi} + e^{-8R_1\Psi}\right) + N_2\left(e^{7R_1\Psi} + e^{-7R_1\Psi}\right) + N_3\left(e^{6R_1\Psi} + e^{-6R_1\Psi}\right)$$

$$+ N_4e^{5R_1\Psi} + N_5e^{-5R_1\Psi} + N_6e^{4R_1\Psi} + N_7e^{-4R_1\Psi} + N_8e^{3R_1\Psi} + N_9e^{-3R_1\Psi}$$

$$+ N_{10}e^{2R_1\Psi} + N_{11}e^{-2R_1\Psi} + N_{13}e^{-R_1\Psi} + N_{14}e^{\Psi^4} + N_{15}\Psi - \frac{k_3}{6}\Psi^3$$

$$+ \rho_{18}\Psi\left(e^{R_1\Psi}\right) + \rho_{19}\Psi e^{-R_1\Psi} + N_{16} \qquad (4.13)$$

$$V_3(\Psi) = N_{16}\left(e^{8R_1\Psi} + B_1e^{-8R_1\Psi}\right) + N_{17}\left(e^{7R_1\Psi} + e^{-7R_1\Psi}\right) + N_{18}\left(e^{6R_1\Psi} + e^{-6R_1\Psi}\right)$$

$$+ N_{19}e^{5R_1\Psi} + N_{20}e^{-5R_1\Psi} + N_{21}\Psi e^{4R_1\Psi} + N_{22}e^{-4R_1\Psi} + N_{23}e^{4R_1\Psi} + N_{24}\Psi e^{-4R_1\Psi}$$

$$+ N_{25}e^{-3R_1\Psi} + N_{26}e^{3R_1\Psi} + N_{20}e^{3R_1\Psi} + N_{28}\Psi e^{-3R_1\Psi} + N_{29}e^{2R_1\Psi} + N_{30}e^{-2R_1\Psi}$$

$$+ N_{31}\Psi e^{R_1\Psi} + N_{32}\Psi e^{-2R_1\Psi} + N_{33}e^{2R_1\Psi} + N_{34}e^{-R_1\Psi} + N_{35}\Psi e^{2R_{14}} + N_{36}e^{R_1\Psi}$$

$$+ N_{37}e^{\Psi^2} + K_3\psi + N_{38} \qquad (4.14)$$

$$B_3(\Psi) = N_{39}\left(e^{8R_1\Psi} - e^{-8R_1\Psi}\right) - N_{40}\left(e^{7R_1\Psi} - e^{-7R_1\Psi}\right) - N_{41}\left(e^{6R_1\Psi} + e^{-6R_1\Psi}\right)$$

$$- N_{42}e^{5R_1\Psi} + N_{43}e^{-5R_1\Psi} - N_{44}\Psi e^{4R_1\Psi} + N_{45}e^{-4R_1\Psi} - N_{46}\Psi\left(e^{4R_1\Psi} - e^{-4R_1\Psi}\right)$$

$$+ N_{47}\left(e^{4R_1\Psi} + e^{-4R_1\Psi}\right) - N_{48}e^{-3R_1\Psi} - N_{49}e^{3R_1\Psi} - N_{50}\left(e^{3R_1\Psi} + e^{-3R_1\Psi}\right)$$

$$+ N_{51}\left(e^{3R_1\Psi} + e^{-3R_1\Psi}\right) + N_{52}e^{2R_1\Psi} + N_{53}e^{-2R_1\Psi} - N_{54}\left(e^{2R_1\Psi} - e^{-2R_1\Psi}\right)$$

$$+ N_{55}\left(e^{2R_1\Psi} + e^{-2R_1\Psi}\right) + N_{56}e^{-R_1\Psi} + N_{57}e^{3R_1\Psi} - N_{58}\left(e^{R_1\Psi} - e^{-R_1\Psi}\right)$$

$$+ N_{59}\left(e^{R_1\Psi} + e^{-R_1\Psi}\right) - N_{60}\Psi^3 - \frac{k_3\Psi^2}{2} + k_7 \qquad (4.15)$$

Hence, the flow rate is obtained thus:

$$U_3(\Psi) = S_1'\left(e^{8R_1} - e^{-8R_1}\right) + S_3'\left(e^{6R_1} - e^{-6R_1}\right)$$

$$S_4'\left(e^{5R_1} - e^{-5R_1}\right) + S_5'e^{4R_1} + S_6'e^{-4R_1} + S_7'e^{3R_1}$$

$$+ S_8'e^{-3R_1} + S_9'e^{2R_1} + S_{10}'e^{-2R_1} + S_{11}'e^{R_1} + S_{12}'e^{-R_1} + S_{13}' \qquad (4.16)$$

Finally, let H be the heat transfer at the wall due to thermal conduction. Thus:

$$H = \frac{dt_3}{d\Psi} \text{ for } \Psi = 1 \text{ and } H = S_{14}'\left(e^{8R_1} - e^{-8R_1}\right) + S_{15}'\left(e^{7R_1} - e^{-7R_1}\right)$$

$$+ S_{16}'\left(e^{6R_1} - e^{-6R_1}\right) + S_{17}'e^{5R_1} - S_{18}'e^{-5R_1} + S_{19}'e^{4R_1}$$

$$- S_{20}'e^{-4R_1} + S_{21}'e^{3R_1} - S_{22}'e^{-3R_1} + S_{23}'e^{2R_1}$$

$$- S'_{24}e^{-2R_1} + S'_{25}e^{R_1} + S'_{26}e^{-R_1} - S'_{27} \tag{4.17}$$

where $\lambda_1 = m^2 \, B_r / P_r R_m,\ \lambda_3 = DR_m - m^2 k_1,\ \lambda_4 = m^2 R_m / R$

$$\lambda_5 = m^2 R_m m_1,\ \lambda_6 = R_m G_r / N_r,\ \lambda_7 = (Q - m^2 k_1) N_r / G_r$$

$$\lambda_8 = \lambda_5 A_1^{\Psi} / 64 R_1^2,\ \lambda_9 = A_1^3 \lambda_7 \lambda_5 / 9R_1,\ \lambda_{10} = \lambda_5(2A_1^4 + 3A_1^2 \lambda_7^2) / R_1^2$$

$$\lambda_{11} = \lambda_5(3A_1^3 \lambda_7 + \lambda_7^3 A_1) / R_1^2,\ \lambda_{12} = (\lambda_6 - R_1^2 A_1 A_1 \lambda_4) / R_1^2,\ \lambda_{13} = \lambda_3 / 6$$

$$\lambda_{14} = \left[(R_m A_1^4 m_1) + \lambda_8 \right] 16 R_1^2 / 4,\ \lambda_{15} = 9R_1^2 \left(\frac{R_m A_1^3 m_1 \lambda_7 + \lambda_9}{3} \right)$$

$$\lambda_{16} = 4R_1 \left[\frac{R_m m_1}{2} (2A_1^4 + 3A_1^2 \lambda_7^2 + \lambda_{10}) \right]$$

$$\lambda_{17} = R_1^2 [R_m(A_1^3 \lambda_4 m_1 + \lambda_1^3 A_1 m_1) + \lambda_{11}],\ \lambda_{18} = 4R_1 \left(\frac{R_m A_1^4 m_1}{4} \right)$$

$$\lambda_{18} = \left(\frac{R_m A_1}{R_1} + \lambda_{12} \right) R_1^2,\ \lambda_{19} = 3R_1 \left(\frac{R_m A_1^3 m_1 \lambda_7}{3} + \lambda_9 \right)$$

$$\lambda_{20} = 2R_1 \left[\frac{R_m M_1}{2} (2A_1^4 + 3A_1^2 \lambda_7^2) - \lambda_{10} \right],\ \lambda_{12} = R_1 \left[R_m(A_1^3 \lambda_4 m_1 + \lambda_7^3 A_1 M_1) \right],$$

$$\lambda_{22} = R_2 \lambda_{12},\ \lambda_{23} = (k_4 - \lambda_2),\ \lambda_{28} = 2\lambda_{14}\lambda_{15}$$

$$\lambda_{29} = 2\lambda_{14}\lambda_{16} + \lambda_{15}^2,\ \lambda_{30} = 2[\lambda_{14}\lambda_{17} + \lambda_{12}\lambda_{14}\lambda_{15}\lambda_{16}],$$

$$\lambda_{31} = 2[\lambda_{14}\lambda_{17} - \lambda_{12}\lambda_{14} + \lambda_{15}\lambda_{16}],\ \lambda_{32} = 12\lambda_{13}\lambda_{14},\ \lambda_{33} 12\lambda_{13}\lambda_{14},$$

$$\lambda_{34} = [-2k_3\lambda_{14} + \lambda_{15}\lambda_{17} + 2\lambda_{12}\lambda_{15} + \lambda_{16}^2$$

$$\lambda_{35} = [-2k_3\lambda_{14} - 2\lambda_{15}\lambda_{12} + 2\lambda_{15}\lambda_{17} + \lambda_{16}^2],\ \lambda_{36} = 2[\lambda_{14}\lambda_{17} - \lambda_{12} + \lambda_{14} - \lambda_{13}k_3 + \lambda_{16}\lambda_{17} + \lambda_{12}\lambda_{16}]$$

$$\lambda_{37} = 2[\lambda_{16}\lambda_{17} + \lambda_{14} + \lambda_{17} - \lambda_{12}\lambda_{16} + \lambda_{14}\lambda_{12} + k_3\lambda_{15}],\ \lambda_{38} = 12[\lambda_{13}\lambda_{15}],\ \lambda_{39} = 12\lambda_{13}\lambda_{15}$$

$$\lambda_{40} = 2[\lambda_{14}\lambda_{16} + \lambda_{15}\lambda_{17} - \lambda_{12}\lambda_{15} + \lambda_{17}\lambda_{12} - k_3\lambda_{16}] + \lambda_{17}^2 + \lambda_{12}^2$$

$$\lambda_{41} = 2[\lambda_{14}\lambda_{16} + \lambda_{15}\lambda_{17} + \lambda_{12}\lambda_{15} - \lambda_{17}\lambda_{12} - k_3\lambda_{16}] + \lambda_{17}^2 + \lambda_{12}^2$$

$$\lambda_{42} = \lambda_{43} = 12\lambda_{13}\lambda_{16},\ \lambda_{44} = 2[\lambda_{14}\lambda_{15} + \lambda_{15}\lambda_{16} + \lambda_{16}\lambda_{17} - \lambda_{13}\lambda_{16} - k_3\lambda_{17}] - k_3\lambda_{12}$$

$$\lambda_{45} = 2[\lambda_{14}\lambda_{15} + \lambda_{16}\lambda_{17} - k_3\lambda_{17}] + 4\lambda_{15}\lambda_{16} + k_3\lambda_{12},\ \lambda_{46} = 12[\lambda_{12}\lambda_{13}\lambda_{17} + \lambda_{13}\lambda_{12}],$$

$$\lambda_{47} = 12(\lambda_{13}\lambda_{17} - \lambda_{13}\lambda_{12}),\ \lambda_{48} = 36\lambda_{13}^2,\ \lambda_{49} = 2[\lambda_{14}^2 + \lambda_{15}^2 + \lambda_{16}^2 + \lambda_{17}^2 - \lambda_{12}^2] - k_3^2,$$

$$\lambda_{50} = \lambda_1 \lambda_{14}^2 / 64 R_1^2,\ \lambda_{51} = \lambda_1 \lambda_2 / 49 R_1^2,\ \lambda_{52} = \lambda_1 \lambda_{29} / 36 R_1^2,\ \lambda_{53} = \lambda_1 \lambda_{30} / 25 R_1^2,$$

$$\lambda_{54} = \lambda_1 \lambda_{32} / 25 R_1^2,\ \lambda_{54} = \lambda_{55} = \left(M_1 R_1 A_1^4 + \lambda_{54} = \lambda_1 \lambda_{34} - \lambda_{54} = \lambda_{18} / R_m \right) / 16 R_1^2$$

$$\lambda_{56} = \frac{\left(\lambda_1 \lambda_{35} + \dfrac{\lambda_{18}}{R_m} - M_1 R_1 A_1^4 \right)}{16 R_1^2},\ \lambda_{57} = \frac{\left(\lambda_1 \lambda_{36} + \dfrac{\lambda_{19}}{R_m} + M_1 A_1^3 \lambda_1 R_1 \right)}{9 R_1^2},$$

$$\lambda_{58} = \frac{\left(\lambda_1\lambda_{37} - \dfrac{\lambda_{19}}{R_m} + M_1 R_1 A_1^4\right)}{9R_1^2}, \quad \lambda_{59} = \frac{\left(\lambda_1\lambda_{40} - \dfrac{\lambda_{20}}{R_m} + R_1 M_1(2A_1^4 + 3A_1^2\lambda_7^2)\right)}{4R_1^2},$$

$$\lambda_{60} = \frac{\left(\lambda_1\lambda_{41} + \dfrac{\lambda_{20}}{R_m} - R_1 M_1(2A_1^4 + 3A_1^2\lambda_7^2)\right)}{4R_1^2}$$

$$\lambda_{61} = \frac{\left(\lambda_1\lambda_{44} - \dfrac{\lambda_{21}}{R_m} - \dfrac{\lambda_{22}}{R_m} + R_1 M_1(3A_1^3 + \lambda_7 + \lambda_7^3)\right)}{R_1^2}, \quad \lambda_{62} = \frac{\left(\lambda_1\lambda_{45} + \dfrac{\lambda_{21}}{R_m} - \dfrac{\lambda_{22}}{R_m} + R_1 M_1(3A_1^3 + \lambda_7 + \lambda_7^3)\right)}{R_1^2},$$

$$\lambda_{63} = \frac{(\lambda_1\lambda_{48} + 3\lambda_{13})}{12R_{13}}, \quad \lambda_{64} = \lambda_1\lambda_{49} + \lambda_{22} - M_1\lambda_7^3 \tag{4.18}$$

$$P_1 = \frac{\lambda_1\lambda_{14}^2}{(\lambda_6 - 64\lambda_4^2 R_1^2)}, \quad P_2 = \frac{\lambda_1\lambda_{28}}{(\lambda_6 - 49\lambda_4^2 R_1^2)}, \quad P_3 = \frac{\lambda_1\lambda_{29}}{(\lambda_6 - 36\lambda_4^2 R_2)}$$

$$P_4 = \frac{\lambda_1\lambda_{30}}{(\lambda_6 - 25R_1^2\lambda_4)}, \quad P_5 = \frac{\lambda_1\lambda_{31}}{(\lambda_6 - 25\lambda_4^2 R_1^2)}, \quad P_6 = \frac{(\lambda_1\lambda_{34} - 4R_1\lambda_{14})}{(\lambda_6 - 16R_1^2\lambda_4^2)}$$

$$P_7 = \frac{(\lambda_1\lambda_{36} + 3R_1\lambda_{15})}{(\lambda_6 - 9R_1^2\lambda_4^4)}, \quad P_8 = \frac{(\lambda_1\lambda_{36} - 3R_1\lambda_{15})}{(\lambda_6 - 9R_1^2\lambda_4^2)}, \quad P_9 = \frac{(\lambda_1\lambda_{37} - 3R_1\lambda_{15})}{(\lambda_6 - 9R_1^2\lambda_4^2)}$$

$$P_{10} = \frac{(\lambda_1\lambda_{40} + 2R_1\lambda_{15})}{(\lambda_6 - 9R_1^2\lambda_4^2)}, \quad = P_{11} = \frac{(\lambda_1\lambda_{41} + 2R_1\lambda_{16})}{(\lambda_6 - 4R_1^2\lambda_4^2)}, \quad P_{12} = \frac{(\lambda_1\lambda_{45} + R_1\lambda_{17} + R_1\lambda_{12})}{(\lambda_6 - 9R_1^2\lambda_4^2)},$$

$$P_{14} = zero.$$

$$P_{15} = \frac{(4\lambda_3\lambda_{14})}{(\lambda_6 - 16R_1^2\lambda_4^2)}, \quad P_{16} = \frac{4\lambda_3\lambda_{15}}{(\lambda_6 - 9R_1^2\lambda_4^2)}, \quad P_{17} = \frac{4\lambda_3\lambda_{16}}{(\lambda_6 - 4R_1^2\lambda_4^2)}$$

$$P_{18} = \frac{4\lambda_3(\lambda_{17} + \lambda_{18})}{(\lambda_6 - 16R_1^2\lambda_4^2)}, \quad P_{19} = \frac{4\lambda_3(\lambda_{17} - \lambda_{18})}{(\lambda_6 - R_1^2\lambda_4^2)}, \quad P_{20} = \frac{4\lambda_3 k_3}{\lambda_6}, \quad P_{21} = \frac{k_6}{\lambda_6} \tag{4.19}$$

$$N_1 = \lambda_{50} + P_1, \quad N_2 = \lambda_{51} + P_2, \quad N_3 = \lambda_{52} + P_3, \quad N_4 = \lambda_{53} + P_4, \quad N_5 = \lambda_{54} + P_5$$

$$N_6 = \lambda_{55} + P_6, \quad N_7 = \lambda_{56} + P_7, \quad N_8 = \lambda_{57} + P_8, \quad N_9 = \lambda_{58} + P_9, \quad N_{10} = \lambda_{59} + P_{10}$$

$$N_{11} = \lambda_{60} + P_{11}, \quad N_{12} = \lambda_{61} + P_{13}, \quad N_{13} = \lambda_{62} + P_{13}, \quad N_{14} = \lambda_{63} + P_{14}, \quad N_{15} = \lambda_{64} + P_{15}$$

$$N_{16} = (\lambda_1\lambda_{14}^2 + 64R_1^2 N_1), \quad N_{17} = (\lambda_1\lambda_{20} + 4R_1^2 N_2), \quad N_{18} = (\lambda_1\lambda_{29} + 36R_1^2 N_3)$$

$$N_{19} = (\lambda_1\lambda_{30} + 25R_1^2 N_4), \quad N_{20} = (\lambda_1\lambda_{31} - 25R_1^2 N_5), \quad N_{21} = (\lambda_1\lambda_{31})$$

$$N_{22} = (\lambda_1\lambda_{33}, \quad N_{23} = (\lambda_1\lambda_{34}, + 16R_1^2 N_9)), \quad N_{24} = (\lambda_1\lambda_{35} - 16R_1^2 N_7), \quad N_{25} = (\lambda_1\lambda_{37} - 9R_1^2 N_9)$$

$$N_{26} = (\lambda_1\lambda_{36} + 9R_1^2 N_8), \quad N_{27} = (\lambda_1\lambda_{38}, \quad N_{28} = (\lambda_1\lambda_{39}, \quad N_{29} = (\lambda_1\lambda_{40} + 4R_1^2 N_{10})$$

$$N_{30} = (\lambda_1\lambda_{41} - 4R_1^2 N_{11}), \quad N_{31} = (\lambda_1\lambda_{42}, \quad N_{32} = (\lambda_1\lambda_{43}, \quad N_{33} = (\lambda_1\lambda_{44}$$

$$N_{39} = \frac{R_m N_{16}}{8R_1}, \ N_{40} = \frac{R_m N_{17}}{7R_1}, \ N_{41} = \frac{R_m N_8}{8R_1}, \ N_{42} = \frac{R_m N_{19}}{7R_1}, \ \ N_{43} = \frac{R_m N_{20}}{5R_1}$$

$$N_{44} = \frac{R_m N_{21}}{16R_1}, \ N_{45} = \frac{R_m N_{24}}{4R_1}, \ \ N_{49} = \frac{R_m N_{26}}{4R_1}, N_{47} = \frac{R_m N_{21}}{16R_1}, \ N_{40} = \frac{R_m N_{25}}{3R_1}, \ \ N_{49} = \frac{R_m N_{26}}{3R_1}$$

$$N_{50} = \frac{R_m N_{30}}{3R_1}, \ N_{51} = \frac{R_m N_{27}}{9R_1}, \ \ N_{52} = \frac{R_m N_{29}}{2R_1} \ N_{53} = \frac{R_m N_{30}}{2R_1}, \ N_{54} = \frac{R_m N_{33}}{2R_1}, \ \ N_{55} = \frac{R_m N_{32}}{2R_1}$$

$$N_{56} = \frac{R_m N_{34}}{R_1}, \ N_{57} = \frac{R_m N_{36}}{R_1}, \ \ N_{50} = \frac{R_m N_{35}}{R_1} \ N_{59} = \frac{R_m N_{33}}{R_1^2}, \ N_{60} = \frac{R_m N_{375}}{3} \qquad (4.20)$$

$$S_1' = \frac{N_{16}}{4R_1}, \ S_2' = \frac{N_{17}}{7R_1}, \ S_3' = \frac{N_{10}}{3R_1}$$

$$S_{41}' = \frac{(N_{19} - N_{20})}{5R_1}, \ S_5' = \left(\frac{N_{21}}{4R_1^2} - \frac{N_{21}}{4R_1^3} + \frac{N_{22}}{4R_1^3} - \frac{N_{22}}{4R_1^3} + \frac{N_{23}}{4R_1} - \frac{N_{24}}{4R_1} \right)$$

$$S_6' = \left(\frac{N_{21}}{4R_1^2} - \frac{N_{21}}{4R_1^2} - \frac{N_{22}}{4R_1^2} + \frac{N_{22}}{4R_1^3} + N_{23}4R_1 - \frac{N_{24}}{4R_1} \right)$$

$$S_7' = \left(\frac{N_{25}}{3R_1} + \frac{N_{26}}{3R_1} + \frac{N_{27}}{3R_1} - \frac{N_{27}}{3R_1^2} - \frac{N_{20}}{R_1} - \frac{N_{20}}{3R_1} \right)$$

$$S_8' = \left(\frac{N_{27}}{3R_1} - \frac{N_{26}}{3R_1} - \frac{N_{25}}{3R_1} - \frac{N_{20}}{3R_1^2} - \frac{N_{20}}{R_1^2} - \frac{N_{20}}{3R_1^2} \right)$$

$$S_9' = \left(\frac{N_{29}}{2R_1} + \frac{N_{30}}{2R_1} - \frac{N_{31}}{2R_1^2} - \frac{N_{32}}{2R_1^2} - \frac{N_{32}}{2R_1^2} + \frac{N_{31}}{2R_1} \right)$$

$$S_{10}' = \left(\frac{N_{31}}{2R_1} + \frac{N_{31}}{2R_1^2} - \frac{N_{29}}{2R_1} - \frac{N_{30}}{2R_1} - \frac{N_{32}}{R_1^2} - \frac{N_{32}}{2R_1} \right)$$

$$S_{12}' = \left(\frac{N_{33}}{R_1} + \frac{N_{33}}{R_1^2} - \frac{N_{34}}{R_1} - \frac{N_{35}}{R_1} - \frac{N_{36}}{R_1^2} - \frac{N_{36}}{R_1} \right)$$

$$S_{13}' = \left(\frac{2}{3} N_{27} + 2N_{30} \right)$$

$$S_{14}' = 8R_1 N_1, S_{15}' = 7R_1 N_2$$

$$S_{16}' = 6R_1 N_3, S_{17}' = 5R_1 N_4$$

$$S_{18}' = 5R_1 N_5, \ S_{19}' = 4R_1 N_6, \ S_{20}' = 4R_1 N_7$$

$$S_{21}' = 3R_1 N_8, \ S_{22}' = 3R_1 N_9$$

$$S_{23}' = 2R_1 N_{10}, S_{24}' = 2R_1 N_{11}$$

$$S_{25}' = (R_1 N_{12} + P_{10} R_1 + P_{10} + R_1 B_1)$$

$$S_{26}' = (R_1 N_{13} - P_{19} R_1 + R_1 + R_1 + R_1 + B_1)$$

$$S'^{27} = \left(N_{15} - \frac{1}{2} k_{31} \right)$$

(4.21)

5.0   Remark on remarks

In order to illustrate the foregoing analysis, $T_3(\Psi), V_3(\Psi)$ and $B_3(\Psi)$ are fluid variables, $\dfrac{B_r}{P_r}$ (viscous dissipative heat parameters), M_2 (radiation parameters) conclusions are drawn from the observation and suggestions are made for further research thus. The results from our observation of the parameter behavior, reads that as viscous dissipative heat (B_r) increases, the induced magnetic filed also increases, when $M_2 = 1$, and all other dimensionless parameters $G_r = 5, D = 1, M_2 = 10$ as shown in Figure 5.1. Also as shown in Figure 5.2, increase in viscous dissipative heat causes decrease in velocity distribution, for different values of Hartmann number when $M_2 = 1, G_r = 5, D = 1$ and $M^2 = 10$. While at point $\Psi = 0$, the three graphs over-lapped.

$B_3(\psi)$

$B_r = 3, + B_r = 6, \lozenge B_r = 9$
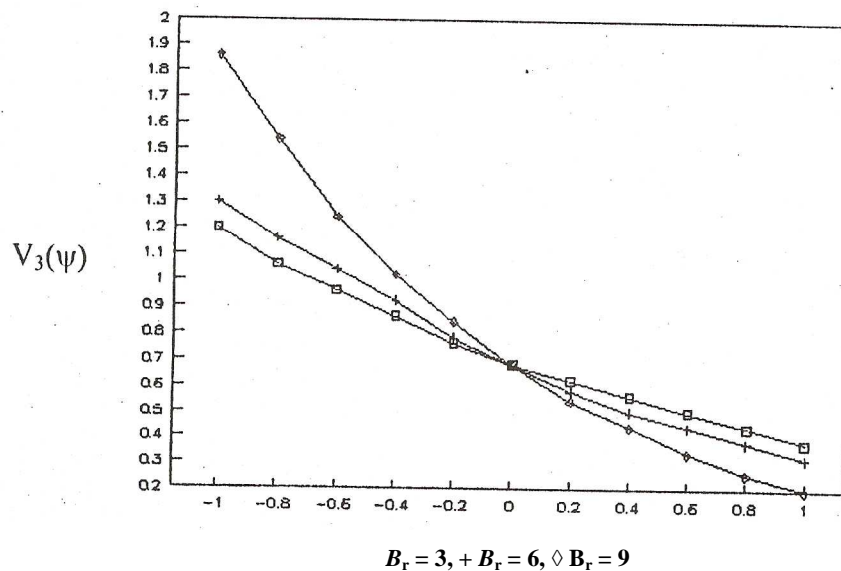
$B_r = 3, + B_r = 6, \Diamond B_r = 9$
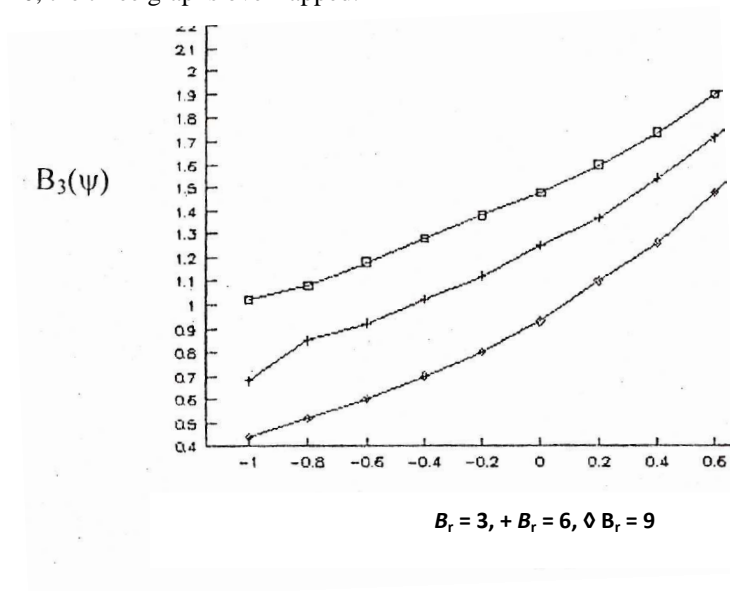
Table 5.2: Velocity distribution for different Hartmann Number $M_1 = 1$

In figure 5.3, increase in viscous dissipative heat causes decrease in temperature distribution for different Hartmann number for $M_2 = 1, D = 1, M^2 = 10, G_r = 5$. When $B_r = 3$ to the graph overlapped in the a-axis and at point $\psi = 0.8$ then B_r tends to increase with increase in temperature distributions. However, the flow rate increases with increase in viscous dissipative heat for constant values of $M_1^2 D_1$ and M_2.
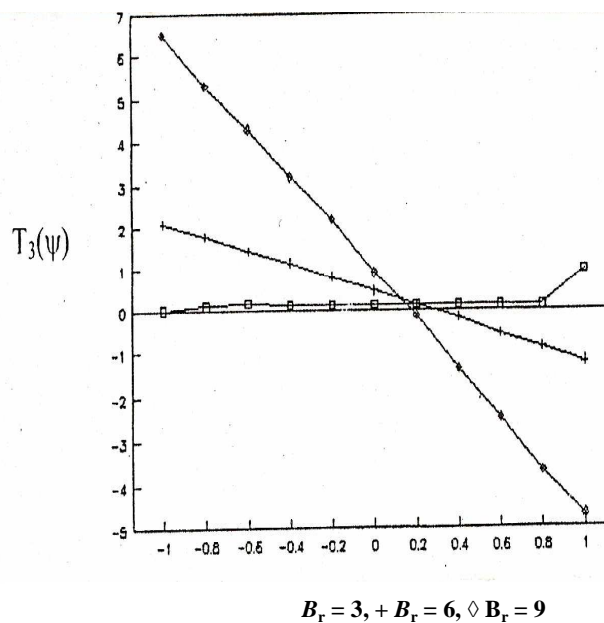

$B_r = 3, + B_r = 6, \Diamond B_r = 9$

Table 5.2: Temperature distribution for different Hartmann Number $M_1 = 1$

6.0   Conclusion

A close form solution is presented for the viscous dissipative heat influences the velocity, induced magnetic field, temperature and flow rate of the fluid in a channel in the presence of radiation causing a variation in the values of these variables and reduces the radiation parameter $(M_{2)}$ for fixed value of G_r, M_r^2 and B_r .

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