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Heat and mass transfer in the unsteady hydromagnetic free-convection flow in a rotating binary fluid II

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Abstract

The unsteady hydromagnetic free-convection flow near a moving infinite flat plate with heat and mass transfer is studied when chemical reaction is present. By adopting a further approximation on the steady temperature and concentration and in the absence of the soret term, the steady state equations are reduced to a set of coupled second order linear differential equations and solved. This together with the transient solution show that in the presence of chemical reaction D_f and k_r does not affect the temperature while R decreases it. Also increase in D_f , R and k_r causes a depletion in concentration with the depletion in concentration due to R in a narrow region near the flat plate boundary.

1.0 Introduction

In the previous paper (Alabraba et al [1]), the heat and mass transfer in the unsteady hydromagnetic flow of a thermally radiating binary mixture of hydrogen-air as a chemically inert gas pair was considered.

In this paper we extend the analysis to a chemically reacting dilute mixture of natural gas and oxygen as is obtained in gas flares. The following elementary reaction shows the combustion of one of the natural gas say methane

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O \qquad (1.1)$$

We can approximate this process to be a dilute mixture of CH_4 and O_2 and so use a single mass diffusion equation to represent the combustion of CH_4.

Nomenclature

(u, v)	dimensional velocity components
(x,y,z)	dimensional Cartesian coordinates
k	thermal conductivity
g	gravitational acceleration
c_p	specific heat at constant pressure
D_m	mass diffusivity
T	dimensional temperature
C	dimensional concentration
T_∞	reservoir temperature
C_∞	reservoir concentration
T_w	constant plate temperature
k_B	Boltzman constant
$H_0'^2$	constant transverse magnetic field
k_r'	constant associated with chemical reaction in the Arrhenius term
Pr	Prandtl number
R	radiation parameter
D_f	Dufour parameter
S_f	Soret parameter
S_c	Schmidt's number
K_T	thermo-diffusion constant
ε'	dimensional activation energy

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C_w constant plate concentration
T_m mean temperature
$q_{z'}$ radiative heat flux
q complex velocity
G_c free convection parameter due to concentration

Greek symbols

σ_c electrical conductivity
μ magnetic permeability
υ kinematic viscosity
β coeff. of volume expansion for temperature
ζ coeff. of volume expansion for concentration

$i = \sqrt{-1}$
E rotation parameter
M^2 magnetic parameter
G_r Free convection parameter due to temperature
ε small parameter
η constant exponent in the Arrhenius term
χ concentration susceptibility
$\bar{\varepsilon}$ dimensionless activation energy
Ω plate angular velocity
ρ_∞ reservoir density
α absorption coefficient
σ Stefan Boltzmann constant

2.0 Governing equations

The mathematical formulation and non-dimensionalization have been given in Alabraba et al [1]. Hence the steady flow and first order transient components after applying asymptotic expansion on the flow velocity, temperature and concentration are:

$$2iEq^{(0)} = \frac{d^2q^{(0)}}{dz^2} - M^2q^{(0)} + Gr\left(\theta^{(0)} - 1\right) + Gc\left(C^{(0)} - 1\right) \quad (2.1)$$

$$0 = \frac{d^2\theta^{(0)}}{dz^2} - R.\Pr\left(\theta^{(0)4} - 1\right) + D_f\frac{d^2C^{(0)}}{dz^2} \quad (2.2)$$

$$0 = \frac{d^2C^{(0)}}{dz^2} - k_r^{\,2}\exp\left(-\frac{\bar{\varepsilon}}{\theta^{(0)}}\right)\theta^{(0)\eta}C^{(0)} + S_f\frac{d^2\theta^{(0)}}{dz^2} \quad (2.3)$$

subject to

$$z = 0: q^{(0)} = 1,\ \theta^{(0)} = \theta_w,\ C^{(0)} = C_w \quad (2.4a)$$
$$z\to\infty: q^{(0)} = 0,\ \theta^{(0)} = 1,\ C^{(0)} = 1 \quad (2.4b)$$

$$\frac{\partial q^{(1)}}{\partial t} + 2iEq^{(1)} = \frac{\partial^2 q^{(10}}{\partial z^2} - M^2q^{(1)} + Gr\theta^{(1)} + GcC^{(1)} \quad (2.5)$$

$$\Pr\frac{\partial\theta^{(1)}}{\partial t} = \frac{\partial^2\theta^{(1)}}{\partial z^2} - 4.R.\Pr\theta^{(0)3}\theta^{(1)} + D_f\frac{\partial^2C^{(1)}}{\partial z^2} \quad (2.6)$$

$$Sc\frac{\partial C^{(1)}}{\partial t} = \frac{\partial^2C^{(1)}}{\partial z^2} - k_r^{\,2}\exp\left(-\frac{\bar{\varepsilon}}{\theta^{(0)}}\right)\left\{\theta^{(0)\eta-2}\theta^{(1)}C^{(0)}\left(\bar{\varepsilon} + \eta\theta^{(0)}\right) + \theta^{(0)\eta}C^{(1)}\right\} + S_f\frac{\partial^2\theta^{(1)}}{\partial z^2} \quad (2.7)$$

subject to

$$\left.\begin{aligned} &z = 0: q^{(1)} = 0, \theta^{(1)} = \theta_w, C^{(1)} = C_w \\ &z\to\infty: q^{(1)} = \theta^{(1)} = C^{(1)} = 0 \end{aligned}\right\} t \succ 0 \quad (2.8a,b)$$

$$t = 0: q^{(1)} = \theta^{(1)} = C^{(1)} = 0, \quad z \succ 0 \quad (2.8c)$$

3.0 Method of solution

To solve Equations. (2.2) and (2.3) we use the following approximations:

$\theta^{(0)} = \theta_w + \varphi$ (3.1a)
$C^{(0)} = C_w + \psi$ (3.1b)

where $\varphi < 1$ and order o(φ) ~ order o(ψ). From Alabraba et al [1] we find that S_f does not affect the flow and also from Bestman [2] η=1. Substituting $S_f = 0$ and η=1 in Equation.(2.3) we have

$$\frac{d^2\psi}{dz^2} - \kappa^2\psi - \kappa^2 C_w\left(1 + \frac{\varphi}{\theta_w}\right) = 0 \tag{3.2}$$

Now substituting Equations (3.1a,b) into Equation (2.3), ignoring order O(φ^2), simplifying and substituting $D_f \frac{d^2\psi}{dz^2}$ from equation.(3.2) and finally imposing $\psi < 1$ condition we arrive at $\frac{d^2\varphi}{dz^2} - \alpha_1{}^2\varphi - \alpha_2 = 0$ with solution as

$$\varphi = A_1^{(+)}e^{\alpha_1 z} + A_1^{(-)}e^{-\alpha_1 z} - \frac{\alpha_2}{\alpha_1{}^2} \tag{3.3}$$

By imposing the boundary conditions equations (2.4a,b) and noting that as z →∞ we have

(i) $\theta^{(0)} = 1 \Rightarrow \varphi = 1-\theta_w$
(ii) $A_1{}^{(+)} = 0$
(iii) $e^{-\alpha_1 z} = 0$

which gives the result $1-\theta_w = -\frac{\alpha_2}{\alpha_1{}^2}$. Combining this with z=0 boundary condition gives $A_1^{(-)} = \frac{\alpha_2}{\alpha_1{}^2}$.

The solution of equation (3.31) can therefore be written as $\varphi = (\theta_w - 1)(e^{-\alpha_1 z} - 1)$ which translates the expression for $\theta^{(0)}$ as

$$\theta^{(0)} = (\theta_w - 1)e^{-\alpha_1 z} + 1 \tag{3.4}$$

Substituting φ into equation (3.2) and simplifying we get

$$\frac{d^2\psi}{dz^2} - \kappa^2\psi - \left\{\kappa^2 C_w\left(1 - \frac{\alpha_2}{\alpha_1{}^2\theta_w}\right) + \frac{\kappa^2 C_w\alpha_2}{\alpha_1{}^2\theta_w}e^{-\alpha_1 z}\right\} = 0$$

with solution as $\psi = B_1^{(+)}e^{\kappa z} + B_1^{(-)}e^{-\kappa z} + \frac{\alpha_3}{\alpha_1{}^2 - \kappa^2}e^{-\alpha_1 z} - C_w\left(1 - \frac{\alpha_2}{\alpha_1{}^2\theta_w}\right)$. This together with equation (3.1b) gives $C^{(0)} = B_1^{(+)}e^{\kappa z} + B_1^{(-)}e^{-\kappa z} + \frac{\alpha_3}{\alpha_1{}^2 - \kappa^2}e^{-\alpha_1 z} + \frac{C_w\alpha_2}{\alpha_1{}^2\theta_w}$. By imposing the boundary condition equations (2.4a,b) and noting that as z → ∞

(i) $C^{(0)} = 1$
(ii) $B_1{}^{(+)} = 0$
(iii) $e^{-\alpha_1 z} = 0$
(iv) $e^{-\kappa z} = 0$

we get $1 = \frac{C_w\alpha_2}{\alpha_1{}^2\theta_w}$ and when this is combined with the z=0 boundary condition gives the expression for $B_1^{(-)}$ as $B_1^{(-)} = \left(C_w - 1 - \frac{\alpha_3}{\alpha_1{}^2 - \kappa^2}\right)$. The expression for $C^{(0)}$ therefore comes out as

$$C^{(0)} = \frac{\alpha_3}{\alpha_1^{\,2} - \kappa^2}\left(e^{-\alpha_1 z} - e^{-\kappa z}\right) + e^{-\kappa z}\left(C_w - 1\right) + 1 \tag{3.5}$$

To solve the first order equations (2.6) and (2.7) we adopt similar procedure of Laplace transform technique as in Alabraba et al [1]. The coupled transformed concentration and temperature equations are:

$$\left(D^2 - a_1\right)\overline{C}^{(1)} = a_2\overline{\theta}^{(1)} \tag{3.6}$$

$$\left(D^2 - b_1\right)\overline{\theta}^{(1)} + D_f D^2\overline{C}^{(1)} = 0 \tag{3.7}$$

By eliminating $\overline{C}^{(1)}$ we get a quartic equation in $\overline{\theta}^{(1)}$ as $aD^4\overline{\theta}^{(1)} - bD^2\overline{\theta}^{(1)} + c\overline{\theta}^{(1)} = 0$ with

solution as $$\overline{\theta}^{(1)} = A_1^{(+)}e^{\omega_1 z} + A_1^{(-)}e^{-\omega_1 z} + A_2^{(+)}e^{\omega_2 z} + A_2^{(-)}e^{-\omega_2 z} \tag{3.8}$$

where $\omega^2_{1,2} = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$ which translates to

$$\omega_1^{\,2} = \frac{\gamma s}{2} + \sqrt{\left(\gamma_2 s^2 + \gamma_3 s + \gamma_4\right)} + \frac{\gamma_1}{2} \tag{3.9a}$$

$$\omega_2^{\,2} = \frac{\gamma s}{2} - \sqrt{\left(\gamma_2 s^2 + \gamma_3 s + \gamma_4\right)} + \frac{\gamma_1}{2} \tag{3.9b}$$

We can write $\left(\gamma_2 s^2 + \gamma_3 s + \gamma_4\right)^{\frac{1}{2}} = \left[\left(\sqrt{\gamma_2}\,s + \sqrt{\gamma_4}\right)^2\right]^{\frac{1}{2}} = \sqrt{\gamma_2}\,s + \sqrt{\gamma_4}$ which is valid when $4\gamma_2\gamma_4$ $= \gamma_3^{\,2}$. Expressing this validity in terms of γ_1 gives $\gamma_1^{(1,2)} = \dfrac{-b_5 \pm \sqrt{b_5^{\,2} - 4a_5c_5}}{2a_5}$. The three conditions of the discriminant in this equation are:

(i) $b_5^{\,2} = 4a_5c_5$.

This translates to $\gamma^2 = 4\Pr Sc$ and holds when $Pr = Sc$ and $C_w = 0$.

(ii) $b_5^2 > 4a_5c_5$.

This translates to $\gamma^2 > 4PrSc$. By writing $b_5^2 - 4a_5c_5 > 0 = \zeta^2$, where $\zeta^2 = (\zeta_1\kappa^2 + \zeta_2)^2$ we find that ζ_1 and ζ_2 must take alternate signs thus giving the roots of γ_1 as:

(a) $\gamma_1^{(1)} = \dfrac{-b_5 + \zeta_1\kappa^2 - \zeta_2}{2a_5}$ and the corresponding expression for C_w as

$$C_w = \frac{\theta_w^{\,2}}{2\Pr Sc\overline{\varepsilon}D_f}\left\{\gamma\Pr - 2\Pr Sc + \sqrt{\gamma^2\Pr^2 - 4\Pr^3 Sc}\right\} \tag{3.10a}$$

(b) $\gamma_1^{(2)} = \dfrac{-b_5 - \zeta_1\kappa^2 + \zeta_2}{2a_5}$ with the expression for C_w as

$$C_w = \frac{\theta_w^{\,2}}{2\Pr Sc\overline{\varepsilon}D_f}\left\{\gamma\Pr - 2\Pr Sc - \sqrt{\gamma^2\Pr^2 - 4\Pr^3 Sc}\right\} \tag{3.10b}$$

(iii) $b_5^2 < 4a_5c_5$

This gives $\gamma^2 < 4PrSc$ with imaginary roots for γ_1. Equations (3.9a,b) can be reduced to the form $\omega_1^{\,2} = R_1 s + R_2$ and $\omega_2^{\,2} = R_3 s + R_4$. From equation (3.7) we have $D^2\overline{C}^{(1)} = \dfrac{1}{D_f}\left\{D^2 - b_1\right\}\overline{\theta}^{(1)}$

By following the same procedure as in Alabraba et al [1] we get

$$\overline{C}^{(1)} = \frac{\Omega_1}{\omega_1^{\,2}}\left(A_1^{(+)}e^{\omega_1 z} + A_1^{(-)}e^{-\omega_1 z}\right) + \frac{\Omega_2}{\omega_2^{\,2}}\left(A_2^{(+)}e^{\omega_2 z} + A_2^{(-)}e^{-\omega_2 z}\right) \tag{3.11}$$

Imposing the transformed boundary conditions on equations (3.8) and (3.11) gives

$$A_1^{(+)} = A_2^{(+)} = 0,\;\; A_2^{(-)} = \frac{\theta_w}{s} - A_1^{(-)}$$

$$A_1^{(-)} = \frac{\left(\dfrac{C_w}{s} - \dfrac{\Omega_2}{\omega_2^{\,2}}\dfrac{\theta_w}{s}\right)}{\left(\dfrac{\Omega_1}{\omega_1^{\,2}} - \dfrac{\Omega_2}{\omega_2^{\,2}}\right)}$$

Substituting these results in equations (3.8) and (3.11) gives

$$\overline{\theta}^{(1)} = \frac{\omega_1^{\,2}\left(\omega_2^{\,2}C_w - \Omega_2\theta_w\right)}{s\left(\Omega_1\omega_2^{\,2} - \Omega_2\omega_1^{\,2}\right)}e^{-\omega_1 z} + \frac{\omega_2^{\,2}\left(\Omega_1\theta_w - \omega_1^{\,2}C_w\right)}{s\left(\Omega_1\omega_2^{\,2} - \Omega_2\omega_1^{\,2}\right)}e^{-\omega_2 z}$$

$$\overline{C}^{(1)} = \frac{\Omega_1\left(\omega_2^{\,2}C_w - \Omega_2\theta_w\right)}{s\left(\Omega_1\omega_2^{\,2} - \Omega_2\omega_1^{\,2}\right)}e^{-\omega_1 z} + \frac{\Omega_2\left(\Omega_1\theta_w - \omega_1^{\,2}C_w\right)}{s\left(\Omega_1\omega_2^{\,2} - \Omega_2\omega_1^{\,2}\right)}e^{-\omega_2 z}$$

Further substituting ω_1^2, ω_2^2, Ω_1 and Ω_2 in the last two equations and applying partial fractions we get

$$\overline{\theta}^{(1)} = \left(\frac{\beta_1}{s} + \frac{\beta_2 s}{(N_4{}^2 + N_5 s + N_6)} + \frac{\beta_3}{(N_4{}^2 + N_5 s + N_6)}\right)e^{-k_x\sqrt{(s+a_3)}} +$$

$$\left(\frac{\beta_4}{s} + \frac{\beta_5 s}{\left(N_4 s^2 + N_5 s + N_6\right)} + \frac{\beta_6}{(N_4 s^2 + N_5 s + N_6)}\right)e^{-k_y\sqrt{(s+a_4)}}$$

$$\overline{C}^{(1)} = \left(\frac{\chi_1}{s} + \frac{\chi_2 s}{(M_4 s^2 + M_5 s + M_6)} + \frac{\chi_3}{(M_4 s^2 + M_5 s + M_6)}\right)e^{-k_x\sqrt{s+a_3}} +$$

$$\left(\frac{\chi_4}{s} + \frac{\chi_5 s}{(M_4 s^2 + M_5 s + M_6)} + \frac{\chi_6}{(M_4 s^2 + M_5 s + M_6)}\right)e^{-k_y\sqrt{s+a_4}}$$

Employing the first shifting theorem and inverse Laplace transform we deduce $\theta^{(1)}$ and $C^{(1)}$ as

$$\theta^{(1)} = e^{-a_3 t}\left\{\begin{array}{l}\beta_1 L^{-1}\left(\dfrac{e^{-k_x\sqrt{s}}}{s-a_3}\right) + \beta_2 L^{-1}\left(\dfrac{(s-a_3)e^{-k_x\sqrt{s}}}{(N_4(s-a_3)^2 + N_5(s-a_3) + N_6)}\right) \\ + \beta_3 L^{-1}\left(\dfrac{e^{-k_x\sqrt{s}}}{(N_4(s-a_3)^2 + N_5(s-a_3) + N_6)}\right)\end{array}\right\}$$

$$+ e^{-a_4 t}\left\{\begin{array}{l}\beta_4 L^{-1}\left(\dfrac{e^{-k_y\sqrt{s}}}{s-a_4}\right) + \beta_5 L^{-1}\left(\dfrac{(s-a_4)e^{-k_y\sqrt{s}}}{(N_4(s-a_4)^2 + N_5(s-a_4) + N_6)}\right) \\ + \beta_6 L^{-1}\left(\dfrac{e^{-k_y\sqrt{s}}}{(N_4(s-a_4)^2 + N_5(s-a_4) + N_6)}\right)\end{array}\right\} \tag{3.12}$$

$$C^{(1)} = e^{-a_3 t}\left\{\begin{array}{l}\chi_1 L^{-1}\left(\dfrac{e^{-k_x\sqrt{s}}}{s-a_3}\right) + \chi_2 L^{-1}\left(\dfrac{(s-a_3)e^{-k_x\sqrt{s}}}{(M_4(s-a_3)^2 + M_5(s-a_3 + M_6)}\right) \\ + \chi_3 L^{-1}\left(\dfrac{e^{-k_x\sqrt{s}}}{(M_4(s-a_3)^2 + M_5(s-a_3) + M_6)}\right)\end{array}\right\}$$

$$+ e^{-a_4 t}\left\{\begin{array}{l}\chi_4 L^{-1}\left(\dfrac{e^{-k_y\sqrt{s}}}{s-a_4}\right) + \chi_5 L^{-1}\left(\dfrac{(s-a_4)e^{-k_y\sqrt{s}}}{M_4(s-a_4)^2 + M_5(s-a_4) + M_6)}\right) \\ + \chi_6 L^{-1}\left(\dfrac{e^{-k_y\sqrt{s}}}{M_4(s-a_4)^2 + M_5(s-a_4) + M_6)}\right)\end{array}\right\} \tag{3.13}$$

such that L^{-1} denotes the inverse Laplace transform.

By expressing the following fractions in partial fractions i.e.

$$\frac{1}{s-a_3} = \frac{1}{2}\left\{\frac{1}{\sqrt{s}\left(\sqrt{s}-\sqrt{a_3}\right)} + \frac{1}{\sqrt{s}\left(\sqrt{s}+\sqrt{a_3}\right)}\right\}$$

$$\frac{1}{s-a_4} = \frac{1}{2}\left\{\frac{1}{\sqrt{s}\left(\sqrt{s}-\sqrt{a_4}\right)} + \frac{1}{\sqrt{s}\left(\sqrt{s}+\sqrt{a_4}\right)}\right\}$$

with the following approximations

$$\left.\begin{aligned}
\frac{(s-a_3)}{N_4(s-a_3)^2+N_5(s-a_3)+N_6} &= \frac{1}{2N_4\sqrt{c_1}}\left\{\frac{1}{\sqrt{s}-\sqrt{c_1}}-\frac{1}{\sqrt{s}+\sqrt{c_1}}\right\}\\
\frac{(s-a_4)}{N_4(s-a_4)^2+N_5(s-a_4)+N_6} &= \frac{1}{2N_4\sqrt{c_2}}\left\{\frac{1}{\sqrt{s}-\sqrt{c_2}}-\frac{1}{\sqrt{s}+\sqrt{c_2}}\right\}\\
\frac{1}{N_4(s-a_3)^2+N_5(s-a_3)+N_6} &= \frac{1}{N_5}\left\{\frac{1}{s-a_3}-\frac{1}{s-c_1}\right\}\\
\frac{1}{N_4(s-a_4)^2+N_5(s-a_4)+N_6} &= \frac{1}{N_5}\left\{\frac{1}{s-a_4}-\frac{1}{s-c_2}\right\}
\end{aligned}\right\}, s\to\infty$$

and substituting into equations (3.12) and (3.13), finally employing Abramowitz and Stegun [3] we get the result

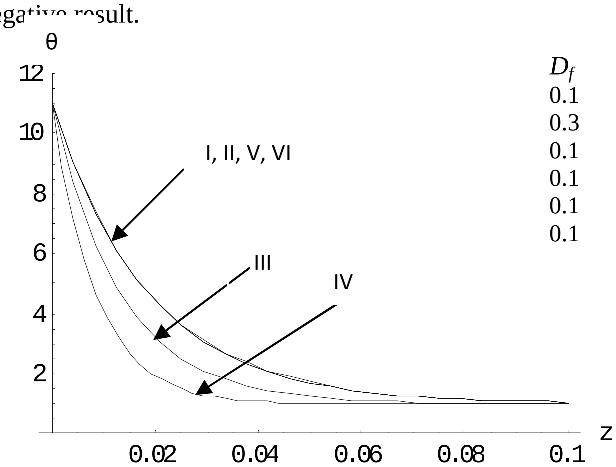
$$\begin{aligned}
\theta^{(1)} = e^{-a_3t}\Bigg\{&\frac{\beta_1}{2}\left[e^{\left(a_3t-k_x\sqrt{a_3}\right)}erfc\left(\frac{k_x}{2\sqrt{t}}-\sqrt{a_3t}\right)+e^{\left(a_3t+k_x\sqrt{a_3}\right)}erfc\left(\frac{k_x}{2\sqrt{t}}+\sqrt{a_3t}\right)\right]\\
&+\frac{\beta_2}{2N_4}\left[e^{\left(c_1t-k_x\sqrt{c_1}\right)}erfc\left(\frac{k_x}{2\sqrt{t}}-\sqrt{c_1t}\right)+e^{\left(c_1t+k_x\sqrt{c_1}\right)}erfc\left(\frac{k_x}{2\sqrt{t}}+\sqrt{c_1t}\right)\right]\\
&+\frac{\beta_3}{2N_5}\Bigg[e^{\left(a_3t-k_x\sqrt{a_3}\right)}erfc\left(\frac{k_x}{2\sqrt{t}}-\sqrt{a_3t}\right)+e^{\left(a_3t+k_x\sqrt{a_3}\right)}erfc\left(\frac{k_x}{2\sqrt{t}}+\sqrt{a_3t}\right)\\
&-e^{\left(c_1t-k_x\sqrt{c_1}\right)}erfc\left(\frac{k_x}{2\sqrt{t}}-\sqrt{c_1t}\right)-e^{\left(c_1t+k_x\sqrt{c_1}\right)}erfc\left(\frac{k_x}{2\sqrt{t}}+\sqrt{c_1t}\right)\Bigg]\Bigg\}\\
+e^{-a_4t}\Bigg\{&\frac{\beta_4}{2}\left[e^{\left(a_4t-k_y\sqrt{t}\right)}erfc\left(\frac{k_y}{2\sqrt{t}}-\sqrt{a_4t}\right)+e^{\left(a_4t+k_y\sqrt{t}\right)}erfc\left(\frac{k_y}{2\sqrt{t}}+\sqrt{a_4t}\right)\right]\\
&+\frac{\beta_5}{2N_4}\left[e^{\left(c_2t-k_y\sqrt{c_2}\right)}erfc\left(\frac{k_y}{2\sqrt{t}}-\sqrt{c_2t}\right)+e^{\left(c_2t+k_y\sqrt{c_2}\right)}erfc\left(\frac{k_y}{2\sqrt{t}}+\sqrt{c_2t}\right)\right]\\
&+\frac{\beta_6}{2N_5}\Bigg[e^{\left(a_4t-k_y\sqrt{a_4}\right)}erfc\left(\frac{k_y}{2\sqrt{t}}-\sqrt{a_4t}\right)+e^{\left(a_4t+k_y\sqrt{a_4}\right)}erfc\left(\frac{k_y}{2\sqrt{t}}+\sqrt{a_4t}\right)\\
&-e^{\left(c_2t-k_y\sqrt{c_2}\right)}erfc\left(\frac{k_y}{2\sqrt{t}}-\sqrt{c_2t}\right)-e^{\left(c_2t+k_y\sqrt{c_2}\right)}erfc\left(\frac{k_y}{2\sqrt{t}}+\sqrt{c_2t}\right)\Bigg]\Bigg\}
\end{aligned}$$

$$
\begin{aligned}
C^{(1)} = e^{-a_3 t}\Bigg\{ & \frac{\chi_1}{2}\left[e^{\left(a_3 t - k_x\sqrt{a_3}\right)} erfc\left(\frac{k_x}{2\sqrt{t}} - \sqrt{a_3 t}\right) + e^{\left(a_3 t + k_x\sqrt{a_3}\right)} erfc\left(\frac{k_x}{2\sqrt{t}} + \sqrt{a_3 t}\right)\right] \\
& + \frac{\chi_2}{2M_4}\left[e^{\left(c_3 t - k_x\sqrt{c_3}\right)} erfc\left(\frac{k_x}{2\sqrt{t}} - \sqrt{c_3 t}\right) + e^{\left(c_3 t + k_x\sqrt{c_3}\right)} erfc\left(\frac{k_x}{2\sqrt{t}} + \sqrt{c_3 t}\right)\right] \\
& + \frac{\chi_3}{2M_5}\left[e^{\left(a_3 t - k_x\sqrt{a_3}\right)} erfc\left(\frac{k_x}{2\sqrt{t}} - \sqrt{a_3 t}\right) + e^{\left(a_3 t + k_x\sqrt{a_3}\right)} erfc\left(\frac{k_x}{2\sqrt{t}} + \sqrt{a_3 t}\right)\right. \\
& \left. - e^{\left(c_3 t - k_x\sqrt{c_3}\right)} erfc\left(\frac{k_x}{2\sqrt{t}} - \sqrt{c_3 t}\right) - e^{\left(c_3 t + k_x\sqrt{c_3}\right)} erfc\left(\frac{k_x}{2\sqrt{t}} + \sqrt{c_3 t}\right)\right]\Bigg\} \\
+ e^{-a_4 t}\Bigg\{ & \frac{\chi_4}{2}\left[e^{\left(a_4 t - k_y\sqrt{a_4}\right)} erfc\left(\frac{k_y}{2\sqrt{t}} - \sqrt{a_4 t}\right) + e^{\left(a_4 t + k_y\sqrt{a_4}\right)} erfc\left(\frac{k_y}{2\sqrt{t}} + \sqrt{a_4 t}\right)\right] \\
& + \frac{\chi_5}{2M_4}\left[e^{\left(c_4 t - k_y\sqrt{c_4}\right)} erfc\left(\frac{k_y}{2\sqrt{t}} - \sqrt{c_4 t}\right) + e^{\left(c_4 t + k_y\sqrt{c_4}\right)} erfc\left(\frac{k_y}{2\sqrt{t}} + \sqrt{c_4 t}\right)\right] \\
& + \frac{\chi_6}{2M_5}\left[e^{\left(a_4 t - k_y\sqrt{a_4}\right)} erfc\left(\frac{k_y}{2\sqrt{t}} - \sqrt{a_4 t}\right) + e^{\left(a_4 t + k_y\sqrt{a_4}\right)} erfc\left(\frac{k_y}{2\sqrt{t}} + \sqrt{a_4 t}\right)\right. \\
& \left. - e^{\left(c_4 t - k_y\sqrt{c_4}\right)} erfc\left(\frac{k_y}{2\sqrt{t}} - \sqrt{c_4 t}\right) - e^{\left(c_4 t + k_y\sqrt{c_4}\right)} erfc\left(\frac{k_y}{2\sqrt{t}} + \sqrt{c_4 t}\right)\right]\Bigg\}
\end{aligned}
$$

4.0 Results and discussion

The problem posed in Alabraba et al [1] with the extension of chemically reacting pair say methane and oxygen as is obtained in gas flares has been solved. The steady flow resulting from asymptotic approximation has been tackled by a further approximation of equation (3.1a,b). We have used similar parameters as in Alabraba et al [1] for the numerical computation i.e. Pr=0.71, Sc = 2.0, $\theta_w = 10$, $t = 0.01$, $\varepsilon = 0.1$.

Equation (3.10a) has been chosen to calculate the wall concentration because (3.10b) gives a negative result.



D_f	R	k_r	Graphs
0.1	1.0	1.0	I
0.3	1.0	1.0	II
0.1	2.0	1.0	III
0.1	5.0	1.0	IV
0.1	1.0	2.0	V
0.1	1.0	5.0	VI

Figure 4.1: Temperature profile θ against boundary layer z for different D_f , R and k_r

Figure 4.1 shows the temperature profile as it is affected by D_f, R and k_r. The result shows that in the presence of chemical reaction, D_f and k_r do not affect the temperature even when k_r is increased to 5.0. However increase in R causes a decrease in temperature like the case without chemical reaction.

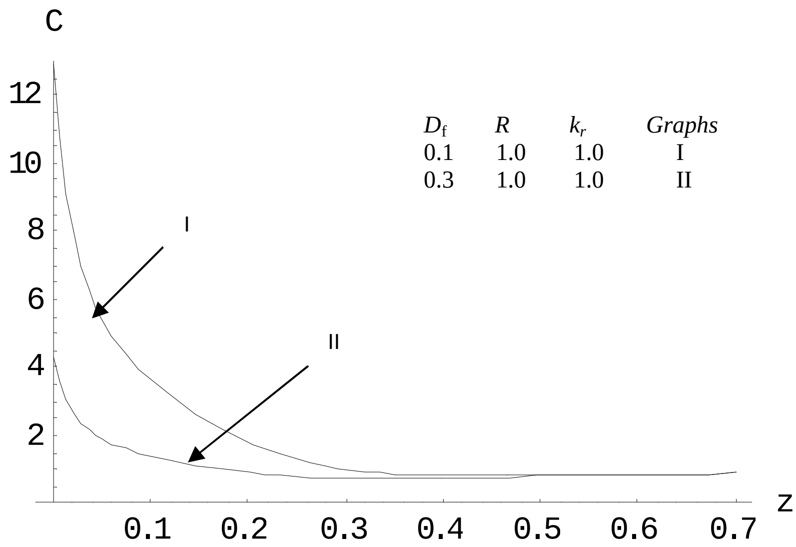


Figure 4.2: Concentration profile C against boundary layer z for different D_f

Figure 4.2 gives the concentration profile for increased value of D_f. The result shows that increase in D_f in the presence of chemical reaction and radiation causes a depletion in concentration.

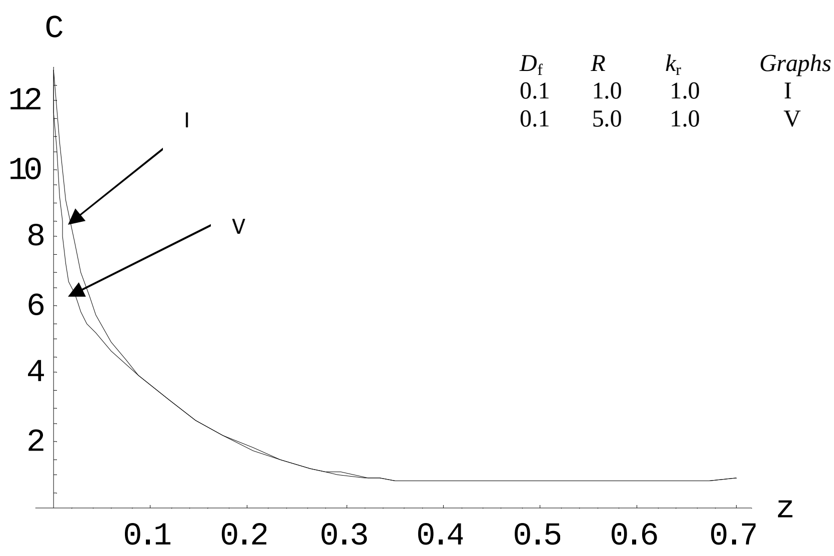
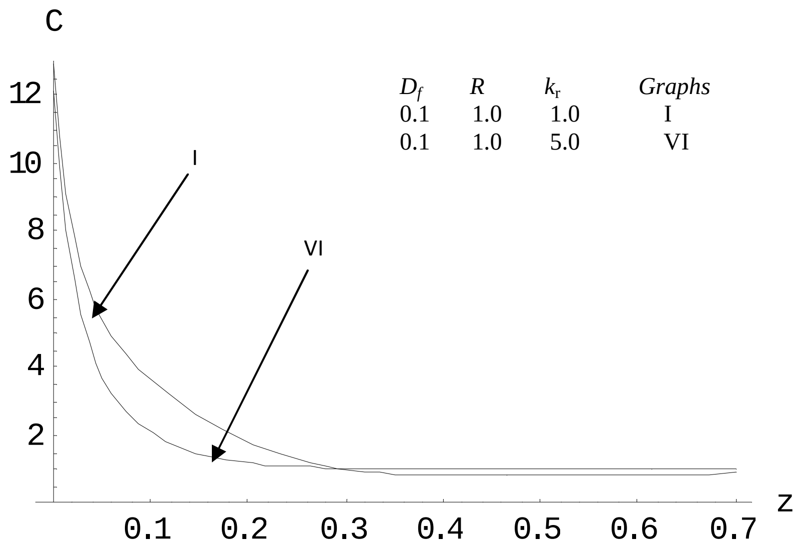


Figure 4.3: Concentration profile C against boundary layer z for different R

Figure 4.3 shows that increase in R in the presence of chemical reaction causes depletion in concentration only in a narrow region near the flat plate boundary.



D_f	R	k_r	*Graphs*
0.1	1.0	1.0	I
0.1	1.0	5.0	VI

Figure 4.4: Concentration profile C against boundary layer z for different k_r

We observe in Figure 4.4 that increase in chemical reaction k_r causes a depletion in concentration up to about z=0.3 and then it reverses but only very slightly. This is in good agreement with the result of Bestman [2] where the ozone budget is modeled with the result that when the ozone concentration is high in the outer atmosphere, chemical reaction causes depletion while when it is high near the earth, chemical reaction causes an increase in ozone concentration.

5.0 Conclusion

In conclusion therefore the unsteady free-convection flow near a moving infinite flat plate in a rotating chemically reacting binary mixture as affected by D_f , R and k_r show that the temperature field is only affected with reduction by R similar to the case without chemical reaction. The concentration is affected by R with depletion only in a narrow region near the plate boundary while k_r causes a depletion in concentration from the plate up to about z=0.3, beyond which there is a very slight increase.

Appendix A

The following constants have been used

$$\kappa^2 = k_r^{\,2} e^{-\frac{\bar{\varepsilon}}{\theta_w}} \theta_w, \qquad a = 1$$

$$\alpha_1^{\,2} = 4R\Pr\theta_w^{\,3} - \frac{\kappa^2 D_f C_w}{\theta_w}, \qquad a_1 = \kappa^2 - Scs$$

$$\alpha_2 = R\Pr\theta_w^{\,4} - \kappa^2 D_f C_w, \qquad a_2 = \kappa^2 \frac{C_w}{\theta_w}\left(\frac{\bar{\varepsilon}}{\theta_w} + 1\right)$$

$$\alpha_3 = \frac{\kappa^2 C_w \alpha_2}{\alpha_1^{\ 2}\theta_w}, \qquad a_3 = \frac{R_2}{R_1}$$

$$c = a_1 b_1$$

$$c_1 = a_3 - \frac{N_5}{N_4}, \qquad a_4 = \frac{R_4}{R_3}$$

$$c_2 = a_4 - \frac{N_5}{N_4}, \qquad a_5 = \Pr Sc,\ b = a_1 + b_1 + a_2 D_f$$

$$c_3 = a_3 - \frac{M_5}{M_4}, \qquad b_1 = 4R\Pr\theta_w^{\ 3} + \Pr s$$

$$b_5 = -\left(4\gamma R\Pr Sc\theta_w^{\ 3} + \gamma\Pr\kappa^2\right)$$

$$c_4 = a_4 - \frac{M_5}{M_4}$$

$$c_5 = 4R\Pr\theta_w^{\ 3}\gamma^2\kappa^2 - 16R\Pr^2 Sc\theta_w^{\ 3}\kappa^2 + 16R^2\Pr^2 Sc^2\theta_w^{\ 6} + \Pr^2\kappa^4 + 8R\Pr^2 Sc\kappa^2\theta_w^{\ 3}$$

$$k_x = \sqrt{R_1}z,\ k_y = \sqrt{R_3}z$$

γ = Pr+Sc, $\gamma_4 = -4R\Pr\theta_w^{\ 3}\kappa^2 + \frac{\gamma_1^{\ 2}}{4}$

$$\gamma_1 = \kappa^2 + 4R\Pr\theta_w^{\ 3} + \kappa^2\frac{C_w}{\theta_w}\left(\frac{\bar{\varepsilon}}{\theta_w} + 1\right)D_f,\ \gamma_5 = \frac{\gamma}{2}$$

$$\gamma_2 = \frac{\gamma^2 - 4\Pr Sc}{4}$$

$$\gamma_3 = \frac{\gamma\gamma_1}{2} = 4R\Pr Sc\theta_w^{\ 3} - \Pr\kappa^2,\ \gamma_6 = \frac{\gamma_1}{2}$$

$$R_1 = \gamma_5 + \sqrt{\gamma_2}, \qquad R_5 = \frac{R_1 - \Pr}{D_f}$$

$$R_2 = \gamma_6 + \sqrt{\gamma_4}, \qquad R_6 = \frac{4R\Pr\theta_w^{\ 3} - R_2}{D_f}$$

$$R_3 = \gamma_5 - \sqrt{\gamma_2}$$

$$R_4 = \gamma_6 - \sqrt{\gamma_4}, \qquad R_7 = \frac{R_3 - \Pr}{D_f}$$

$$\Omega_1 = R_5 s - R_6$$

$$\Omega_2 = R_7 s - R_8,\ R_8 = \frac{4R\Pr\theta_w^{\ 3} - R_4}{D_f}$$

References

[1] M.A. Alabraba, Ojo Aghoghophia, K.D. Alagoa, Heat and mass transfer in the unsteady hydromagnetic free-convection flow in a rotating binary fluid I, J. of NAMP 11(2007) 533-544

[2] A. R. Bestman, Radiative heat transfer to chemically reacting flow between concentric rotating spheres, Il Nuovo cimento 11c (8) (1998) 257-273

[3] M. Abramowitz, I.A. Stegun : Handbook of Mathematical functions. Dover publications Inc., New York 1965