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Temperature field in a flow over a stretching sheet with internal heat generation and uniform heat flux

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Abstract

Mathematical formulations of the temperature distribution in the flow of a viscous incompressible fluid past a stretching sheet with internal heat generation and subsequent analytical solutions are the subject of this article. The velocity of the sheet is proportional to the distance from the slit and the sheet is subject to a uniform heat flux. A closed form solution of temperature is obtained in terms of incomplete Gamma function. During the course of discussion, the effects of Prandtl number and heat generation parameter on temperature field is extensively discussed. It is hoped that the solution reported herein will serve as a stimulus for experimental work and as a vehicle for understanding the problem of a polymer strip extruded continuosly from a die, or a long thread travelling between a feed roll and a wind-up roll.

Keywords: heat generation, stretching sheet, uniform heat flux.

1.0 Introduction

The flow formation in the boundary layer of an incompressible viscous fluid due to moving solid surfaces was studied by Sakiadis [1,2]. Since the ambient fluid is at rest, the flow formation in the boundary layer is quite different from that in Blasius fow past a flat plate. These investigations have a bearing on the problem of a polymer strip extruded continuously from a die. Erikson *et al.* [3] extended the work of Sakiadis [1] by taking into account the suction /injection at the moving plate with heat and mass transfer. In the above studies the strip is assumed to be inextensible, but in polymer industry, situations arise to deal with extensible strip as pointed out by McCormack and Crane [4]. Such situations for non-Newtonian fluid were studied by Siddappa and Abel [5]. Again Dutta *et al*. [6] studied the temperature field in the flow over a stretching sheet with uniform heat flux.

However, the problem of determining the temperature field over a stretching sheet subject to uniform heat flux and temperature dependent heat generation, which is more realistic in many practical situations, does not seem to have received any attention. The present study is addressed to this situation.

2.0 Mathematical analysis

We consider the two dimensional flow of viscous, incompressible heat generating fluid past a horizontal stretching plate that issues from a thin slit at $x = 0$, $y = 0$, as in a polymer processing application (Figure 2.1). The

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derivation of equations governing the steady temperature distribution in the flow of a viscous incompressible fluid caused by the stretching of a sheet which issues from a slit into the fluid is given by Dutta *et al*. [6]. Following this treatment, on taking into account the temperature dependent heat generating fluid, the basic equations relevant to problem under boundary layer approximation are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}\quad , \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\nu}{pr}\frac{\partial^2 T}{\partial y^2}+\delta^*\frac{\partial T}{\partial y} \tag{2.3}$$

The relevant boundary conditions are $u = \alpha x,\ v = 0,\quad -\lambda\frac{\partial T}{\partial y}=A'$ at $y = 0$

$$u\to 0\ ,\ T\to T_\infty\quad as\quad y\to\infty \tag{2.4}$$

where A' is the uniform surface heat flux, δ^* is the heat source parameter and other quantities have their usual meanings as defined by Dutta *et al.* [6]. Since the fluid is incompressible, the momentum equation (2.2) and energy equation (2.3) can be solved consecutively. A stream function ψ defined by

$$u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x} \tag{2.5}$$

is introduced such that the continuity equation is identically satisfied. Defining a stream function

$$\psi=(\alpha\nu)^{\frac{1}{2}}xF(\eta),\ \eta=(\alpha/\nu)^{\frac{1}{2}}y$$

we have

$$u=\alpha xF'(\eta),\ v=(\alpha/\nu)^{\frac{1}{2}}F(\eta). \tag{2.6}$$

Using equation (2.6) in equation (2.2) we get

$$[F'(\eta)]^2-F(\eta)F''(\eta)=F'''(\eta), \tag{2.7}$$

with boundary conditions obtained from equation (2.4)

$$F(0)=0,\quad F'(0)=1,\ F'(\infty)=0 \tag{2.8}$$

where $F'(\eta)$ represent derivative with respect to η .

Crane [7] has given the solution of equation (2.7) which satisfies the boundary condition as

$$F(\eta)=1-\exp(-\eta) \tag{2.9}$$

To solve the energy equation (2.3), the temperature distribution can be taken in the form of a similar solution as

$$T-T_\infty=\frac{A'}{\lambda}(\nu/\alpha)^{\frac{1}{2}}G(\eta) \tag{2.10}$$

Using equation (2.10) in equation (2.3) we get

$$G''(\eta)+\Pr[F(\eta)+\delta]G'(\eta)=0, \tag{2.11}$$

with the boundary conditions

$$G'(0) = -1\,, \quad G(\infty) = 0 \quad , \tag{2.12}$$

where $\delta = \delta^* / (\alpha v)^{\frac{1}{2}}$, is the dimensionless heat source parameter.

The solution of equation (2.11) satisfying equation (2.12) in terms of incomplete Gamma function [8] is

$$G(\eta) = \exp(\Pr)\Pr^{-C\Pr}\,\gamma(C\Pr, \Pr\exp(-\eta)) \;, \tag{2.13}$$

where $\gamma(\Pr, \xi)$ is the incomplete Gamma function defined as

$$\gamma(\Pr, \xi) = \int_0^{\xi} x^{\Pr-1}\exp(-x)dx \quad , \tag{2.14}$$

and C=1+ δ .

Rewriting equation (2.14) in terms of confluent hypergeometric function [8]

$$\gamma(\Pr, \xi) = \Pr^{-1}\,\xi^{\Pr}\phi[\Pr; 1+\Pr; -\xi] \tag{2.15}$$

where ϕ stands for the confluent hypergeometric function. Hence

$$G(\eta) = (\exp(\Pr(1-\eta C))/C\Pr)\varphi[C\Pr; 1+C\Pr; -\Pr\exp(-\eta)] \tag{2.16}$$

In the absence of heat source ($\delta = 0$) , all the above results reduce to that reported by Dutta *et al.* [6].

Finally the wall temperature T_∞ is obtained from (2.10) as

$$T_W - T_\infty = \frac{A'}{\lambda}\left(\frac{v}{\alpha}\right)^{\frac{1}{2}} G(0) \tag{2.17}$$

where $G(0) = \exp(\Pr)\ \Pr^{-C\Pr}\,\gamma(C\Pr; \Pr)$

3.0 Results and discussion

In order to point out the influence of the heat source parameter and Prandtl number into the problem on the temperature field, we have computed numerically $(T - T_\infty)/(T_W - T_\infty)$, using equations (2.10), (2.16) and (2.17).

For illustration purposes, the temperature variations have been shown in the Table for different values of the Prandtl number (Pr) and heat generation parameter (δ). The results pertain three values of Prandtl numbers 0.1, 0.7 and 1.0 .The fluids correspond to these Prandtl numbers exhibit decreasing heat conductivities in that order. From table it is clear that thermal boundary layer thickness increases with decreasing Prandtl number due to the increased conductivity of the fluid. Furthermore the impact of heat generation parameter in reducing the temperature is also evident from the table. The results in table yield quantitative estimate of the counterbalancing effects of Prandtl number and heat generation parameter on the temperature profile. We also observe from the table that the convergence of the temperature values to the corresponding free stream values are considerably influenced by the Prandtl number and heat generation parameter. We believe that these results will be useful in the problem of cooling of a polymer sheet extruded continuously from die.

Table 3.1: Variation of temperature field

		η					
Pr	δ	0.0	2.0	4.0	6.0	8.0	10.0
0.1	0.0	1.0000	0.8250	0.6762	0.5537	0.4533	0.3712
	0.5	1.0000	0.7490	0.5557	0.4118	0.3051	0.2260
	1.0	1.0000	0.6798	0.4566	0.3061	0.2052	0.1376
	0.0	1.0000	0.3098	0.0790	0.0196	0.0048	0.0012

0.7	0.5	1.0000	0.1637	0.0209	0.0026	0.0003	0.0000
	1.0	10000	0.0851	0.0054	0.0003	0.0000	0.0000
1.0	0.0	1.0000	0.2002	0.0287	0.0039	0.0005	0.0001
	0.5	1.0000	0.0808	0.0043	0.0002	0.0000	0.0000
	1.0	1.0000	0.0317	0.0006	0.0000	0.0000	0.0000

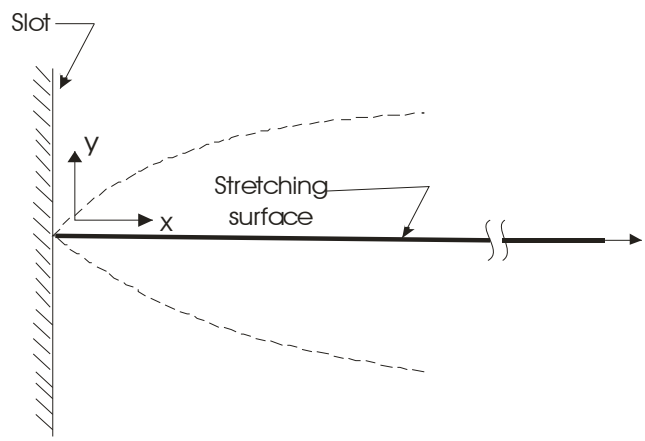

Figure 2.1: Boundary layer on stretching plate

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