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Effect of the variation of hollow core diameter in rectangular plain concrete beams

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Abstract

The paper reports the investigation of the effect of varying the diameter of hollow core in Rectangular Hollow Concrete Beams (RHCC).It applied the moment of inertia theory in its investigation, combining a theoretical and practical approach. Timber was used to prepare the moulds to* BS1881-109 *specification. A standard mix ratio of* 1:2:4:0.55 *was used which concrete gave a compressive strength of 45kN/mm^2. The longitudinal holes were achieved by cutting holes of the various diameters in the end faces of the moulds, through which pipes of the corresponding diameter were passed and withdrawn after a 1 hour setting time. The dimensions of the beams were kept constant and the following hollow diameters were investigated; 0, 30,50, 60,75and 105mm. The beams were subjected to flexural testing with the Universal Testing Machine (UTM) after curing for* 28 *days. Observation and recordings of deflections were carried out at 1kN intervals. At the ultimate loads, the beams failed by crack initiation at the bottom fibre which propagated to the top fibre. The investigation revealed that the ultimate failure loads of the beams increased up to an optimum 75mm diameter after which a considerable decrease in ultimate failure load was observed. The results show that hollow core enhances the flexural strength of rectangular concrete beams and by it, a saving of up to 19.64% by volume of concrete can be achieved. RHCC are usefully applied in voided slab decks in buildings, bridges and other structures.

Keywords: Hollowcore, Flexure, Moment of inertia, failure load and Economy.

1.0 Introduction and background of study

Hollowcore beams are horizontal structural elements having voids through their longitudinal axis. They carry loads primarily in flexure during service. Apart from saving construction materials and reducing the self weight of a structure, the void also provide channels for service installations. Literature reveals that the moment of inertia of a rectangular concrete beam is enhanced by the introduction of a central hollow core in their longitudinal axis .The hollow core makes the analysis of these beams complex (Ossai, 1988) [5]. Orie and Alutu (2005) [6] investigated thirty-nine reinforced rectangular hollowcore beams when the volume of concrete is kept constant and the depth and breadth dimensions are kept constant. They developed a model for the prediction of the ultimate failure load of the beams as

$$P_u = (0.1D)^{1.5} - 0.2D + P^{/} \tag{1.1}$$

$$0 \le D \le 0.72\,b$$

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where P_U = Ultimate failure load of hollow cored beam (kN)

D = Diameter of hollow core (mm)

b = Width of blind beam of equivalent concrete volume (mm)

$P^/$ = Failure load of blind beam of equivalent concrete volume (kN)

Orie and Alutu [6] concluded that the moment of inertia of the beams were enhanced by the hollow cores. Mansur and Hasnat (1979) [4] investigated concrete beams with small transverse circular openings under torsion and noted that, their test samples, upon loading to failure, exhibited failure by bending skewly. Joris (1999) [3] examined the shear and anchorage behaviour of fire exposed Hollow Cored slabs and observed that the load bearing capacity of Hollow Cored slabs decreases at elevated temperatures and the thermal elongation caused additional stress. Joris developed numerical models for a proper description and understanding of the fire behaviour of Hollow Cored slabs on flexible supports. Ugbaja (2005) [8] investigated the effect of varying the position of a constant diameter hollowcore, in the compression zone of RHCC theoretically, by using computer aided analysis. He concluded that there was no significant variation in the ultimate failure loads of the beams. In the construction industry, hollow cores (HC) are introduced into slabs and beams using ducts to create voids. In some cases the effect of the size of these ducts may not have been predetermined and in others, they were absent in the design of the structural elements. It is therefore necessary to investigate the effect of a void as it increases in size within the axis of a beam. Creation of void helps to decrease dead weight of concrete and this result in savings in cost of the supporting structures and foundations.

2.0 Background theory

2.1 Moment of inertia of a rectangular section

Let a rectangular lamina be '*b*' units wide and '*d*' units deep as shown in figure 1. Moment of inertia about centroidal axis *XX* parallel to the width: Consider an elemental component of lamina at a distance y from the *XX* axis and having the depth dy. Area of the elemental component = $dA = b \times dy$

Ryder (1982) [7] gives the moment of inertia of the elemental component about the axis x - x, $I_{xx} = dA \times y^2 = b \times dy \times y^2 = b\,y^2 dy$ Therefore, total moment of inertia of the lamina about the axis x-x,

$$I_{XX} = \int_0^{d/2} by^2 dy = 2b \times \frac{1}{3} \times \frac{d^3}{8} = \frac{bd^3}{12} \qquad (2.1)$$

Similarly, moment of inertia about the centroidal axis y - y parallel to the depth,

$$I_{XX} = \frac{bd^3}{12} \qquad (2.2)$$

2.2 Moment of inertia of a circular lamina

The figure 2.1 below shows a circular lamina of radius *R*. The lamina may be considered as consisting of elemental concentric rings. Consider one such elemental ring at a radius *r* and having a thickness *dr*.

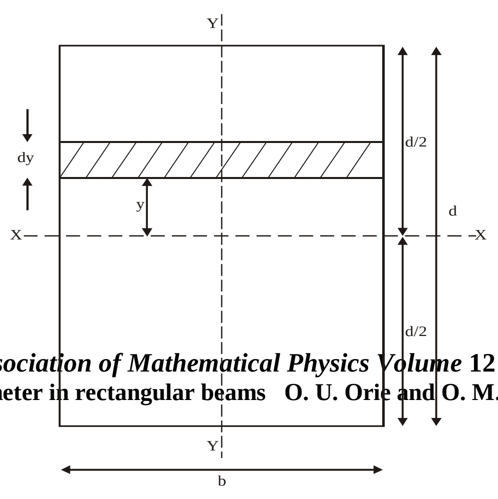

Figure 2.1: A rectangular section

The moment of inertia of elemental ring about the polar axis $=$ Area of ring $\times$ (radius)$^2 = 2\Pi rdr \times r^2$

$$= 2\Pi r^3 \times dr$$

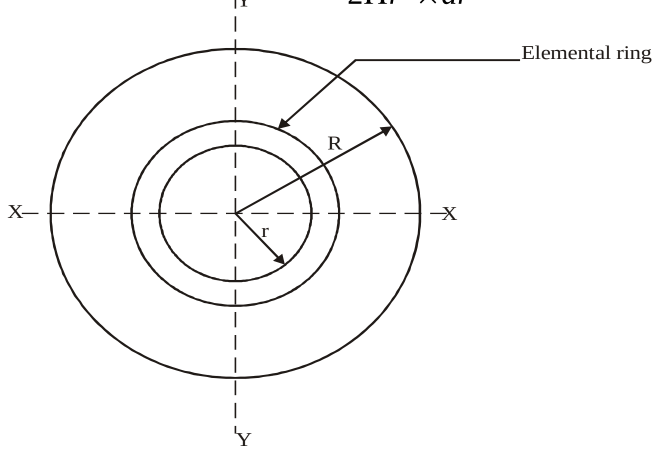

Figure 2.2: A circular section

Polar moment of inertia of whole lamina,

$$I = \int_0^R 2\Pi r^3 dr \tag{2.3}$$

$$I_P = \frac{2\Pi R^4}{4} = \frac{\Pi R^4}{2} \tag{2.4}$$

Let d be the diameter of the lamina, that is, $d = 2R$

$$I_P = \frac{\Pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\Pi}{32} d^4 \tag{2.5}$$

But $I_{XX} = I_{YY}$ and $I_{XX} + I_{YY} = I_P = \dfrac{\Pi d^4}{32}$ (2.6)

$$I_{XX} = I_{YY} = \frac{\Pi d^4}{64} \tag{2.7}$$

Hence, from equations 2.1 and 2.7 moment of inertia of a rectangle with a central hollow is given by

$$I = \frac{bd^3}{12} - \frac{\Pi d^4}{64} \tag{2.8}$$

(Ryder, 1982), [7] the maximum moment, $M_{\max} = \dfrac{\sigma I}{y}$ (2.9)

where σ = stress in concrete = modulus of rupture = f_t

y = depth of neutral axis from topmost fibre in the compression zone

I = moment of inertia of line concrete section

For beams point-loaded at mid-span,

$$M_{\max} = \frac{PL}{4} \tag{2.10}$$

where L = span of the beam.
From equations (2.9) and (2.10), we have

$$\frac{PL}{4} = \frac{\sigma I}{y} = f_f \frac{I}{y} \tag{2.11}$$

Therefore

$$P_u = \frac{4 f_t I}{yL} \tag{2.12}$$

where P_u = load on beam at failure and is the theoretically predicted failure load. The modulus of rupture, f_t, has been calculated for the bending test using the ultimate load equation,

$$f_t = \frac{P_u L y}{4I} \tag{2.13}$$

where I, y and L are constants.

Theoretically, the deflections of the beams have been calculated using the empirical formula:

$$\delta = \frac{PL^3}{48EI} \tag{2.14}$$

where E is the modulus of elasticity of concrete.

2.3 Load deflection characteristics

For a simply supported beam under a point load at mid-span, the load deflection characteristic is described below.

OA = Elastic Region
AB = Plastic Region

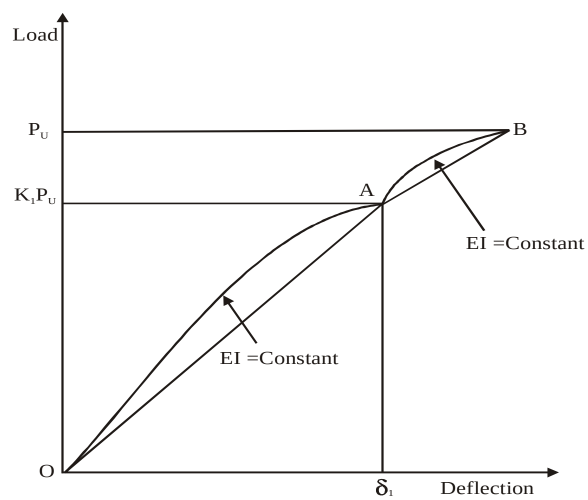

Figure 2.3: Load deflection Curve

The elastic equation for the deflection of a beam is applicable within elastic region (see figure 2.3). Thus, the elastic portions *OA* have a deflection δ_1 and load $K_1 P_U$. Where K_1 is the factor of the ultimate load at roll over point. From analysis, we have

$$\frac{P}{\delta} = \frac{48EI}{L^3} = Slope \text{ of the line OA} \tag{2.15}$$

from which we have: $$\delta = \frac{PL^3}{48EI} \tag{2.16}$$

The modulus of elasticity may be assumed to be, $E = 3.8\sqrt{\ }$ Concrete strength in N/mm^2 (Jackson and Dhir, 1988) [2]. Hence, for concrete strength of 45*kN*, $E = 25.49N/mm^2$. therefore, with a *K* factor of 0.9, the deflections have been estimated.

3.0 Experimental programme

The hollow cored beam variants were prepared using standard rectangular timber moulds and round pipes of diameters; 0, 30,50, 60,75and 105mm. The moulds measured 150*mm* × 150*mm* × 750*mm* (BS 1881- 109 [1983, 1]). The pipes were withdrawn from the specimens about three hours after casting of samples and the samples were demoulded after 24 hours. They were then transferred to the curing tank until maturity at 28 days. The cured samples were transferred to a Universal Testing Machine, (UTM)

made in England by Avery-Denison Limited with identity, EN770467113DCJ of maximum capacity 600kN, fitted with a dial gauge. The beams were simply supported and point-loaded at mid-span. The deflections of the beams were read-off the dial gauge at gradual load increments of 1kN intervals, until the sample failed at ultimate load. At the ultimate loads, the beams failed by crack initiation at the bottom fibre. These cracks propagated to the top fibre where the beams finally failed.

4.0 Results

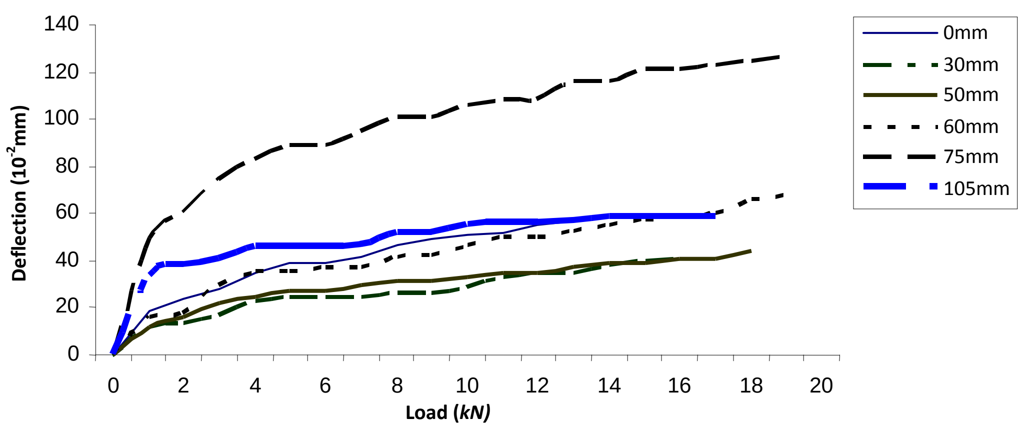

Figure 4.1: Load-Deflection curves for beams

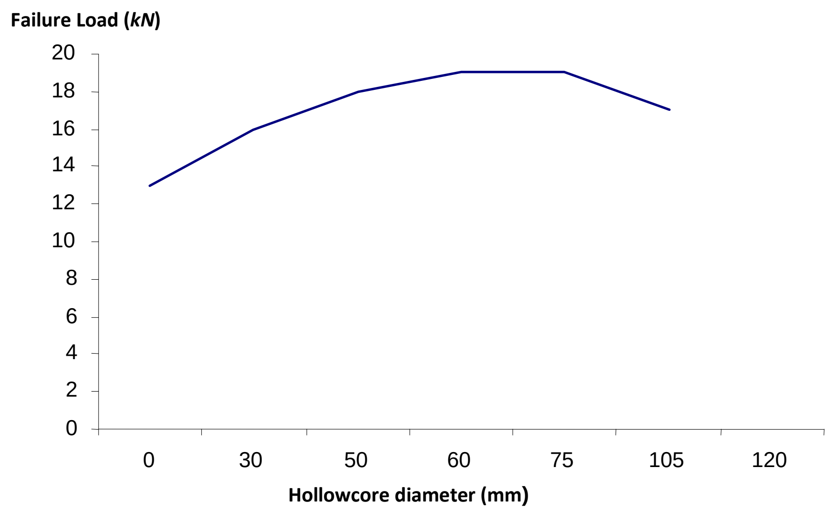

Figure 4.2: Variation of failure load with hollowcore diameter

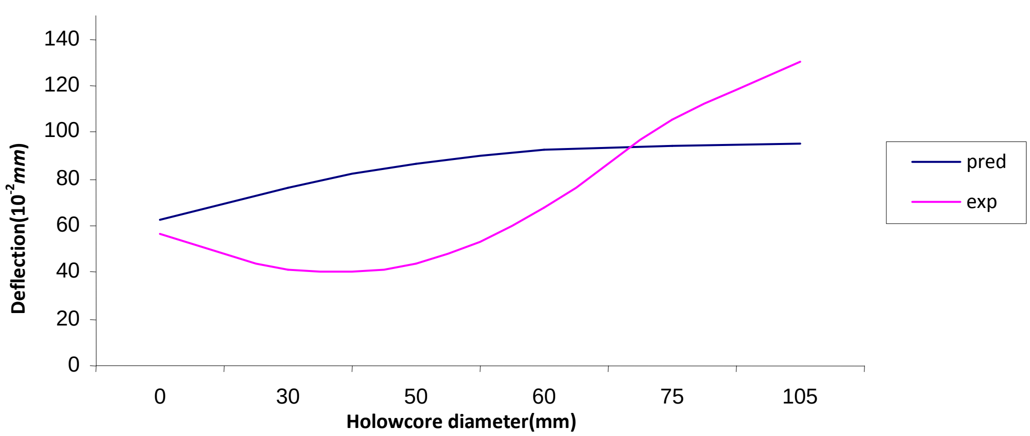

Figure 4.3: Predicted and Experimental defletions for Core diameters

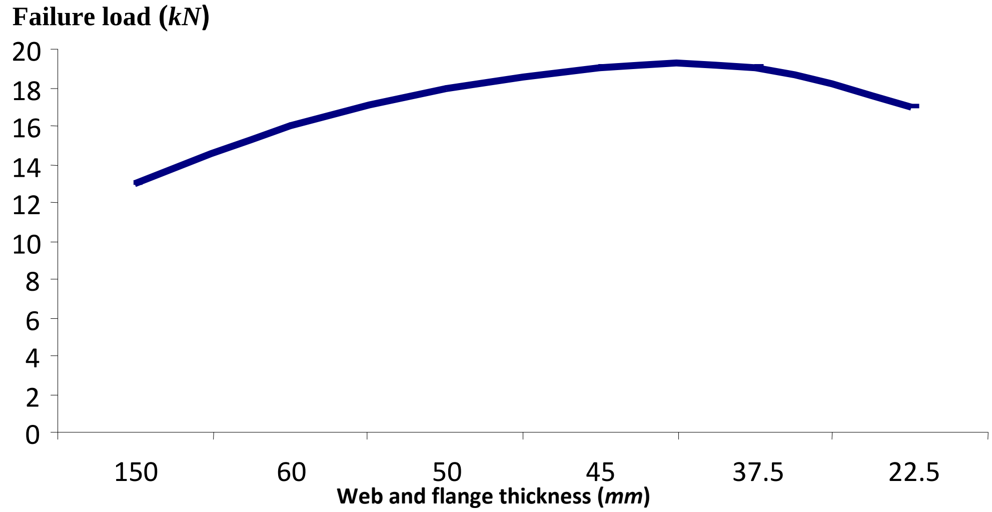

Figure 4.4: Failure load Vs web and flange thickness

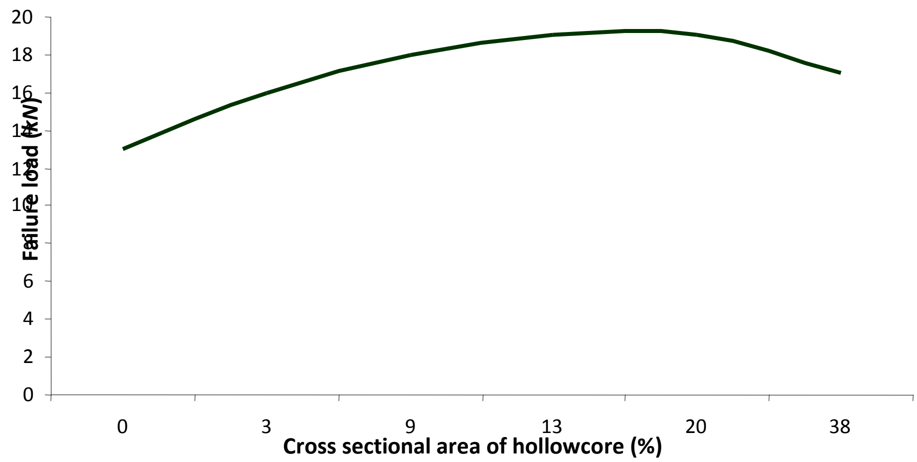

Figure 4.5: Variation of failure load with cross sectional area of hollowcore

4.0 Discussion of results

Figure 4.1 shows that for all the specimens tested, the deflection of the beams between loads of 1kN and 5kN were proportional to the applied load. This is expected as the material of the beams obeys Hooke's law. The figure reveals a curvilinear increase in deflection of the beams as the load is increased. A plot of the failure load against hollowcore diameter shown in figure 4.2, reveal that the ultimate load increased as the diameter of the hollowcore increased up to 75*mm*, beyond which the ultimate load dropped. This hollowcore diameter corresponds to a percentage cross sectional area of 19.64% (see figure 4.5). The specimens, showed optimal deflection performance at a core diameter of 60mm (see figure 4.2) with a percentage cross sectional area of hollowcore of 12.58%, a flexural strength of 19kN and a deflection of 0.68. HCC beams are more economical than their solid counterparts as the concrete which would otherwise have occupied the voids constitute a saving.

5.0 Conclusion

From the study, the following conclusions are made; the moment of inertia of rectangular concrete beam is reduced by the introduction of a longitudinal HC if the cross sectional dimensions of the beam is kept constant. This is unlike the introduction of HC in beams while allowing the dept to vary, a case which causes an increase in moment of inertia as revealed in the literature.

- The flexural performance of a rectangular beam is enhanced by the introduction of a longitudinal circular hollowcore with an optimal core diameter amounting to 19.64% of the total cross sectional area of the beam. This will amount to a significant saving in volume of concrete.

References

[1] BS 1881- 109 (1983), “Specifications for Concrete Moulds“, BSI, London.

[2] Jackson, N. and Dhir, R. K. (1998), “Civil Engineering Materials”, Macmillan Education Press Limited, London.

[3] Joris, H. H. F. (1999), “Shear and Anchorage Behaviour of Fire exposed Hollow Core Slabs “, DCT. 4406 Delft University of Technology, Delft.

[4] Mansur, M. A. and Hasnat, A (1979), “Concrete Beams with Small Opening under Torsion”, ASCE Journal, Structural Div. Vol. 105 pp 2433 – 22447.

[5] Ossai, G.I (1988), “Structural Characteristics of Unreinforced Hollow Concrete Beams”, Project Report, University of Nigeria, Nsukka.

[6] Orie, O.U. and Alutu, O. E. (2005), “Flexural Characteristics of Reinforced Hollow Cored Concrete Beams Point Loaded at Mid-span”, Nigerian Journal of Technical Development, Ilorin, (Accepted).

[7] Ryder, G. H. (1982), “Strength of materials “, Macmillan Education Press Limited, London.

[8] Ugbaja, J. C. (2005), “Effect of Varying the Position of Hollow Core in the Compression Zone of a [9] Concrete Beam”, B. Eng. Project, University of Benin, Nigeria.