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Initial and final estimates of the Bilinear seasonal time series model

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Abstract

A particular class of non-linear models which has been found to be useful in many fields is the bilinear models. A special class of it is discussed in this paper. In getting the estimates of the parameters of this model special attention was paid to the problem of having good initial estimates as it is proposed that with good initial values of the parameters the estimates obtaining by the Newton-Raphson iterative technique usually not only converge but also are good estimates. In this paper we examined the initial and final estimates of the bilinear seasonal time series model. The Box-Jenkins linear convergence process, the Newton-Raphson iterative procedure, the Fortran Progran and the MINITAB software package were all employed in achieving both the initial estimates and the final estimates of the bilinear seasonal time series model studied. The results showed considerable closeness between the initial estimates and the final estimates for both simulations* (n - 100 *and n* - 500). *This confirmed that the initial estimates are good enough. The implication of this is that in estimations of this nature efforts should be made using the right procedures to achieve good initial estimates so that the final estimates could be achieved quickly after few iterations.

Keywords: Bilinear seasonal models, Box and Jenkins linear convergence process, Newton-Raphson iterative procedure, Initial estimates, Final estimates

1.0 Introduction

Linear time series models are widely used in many fields because these models can be analysed with considerable ease and they provide fairly good approximation for the true underlying generating random process. However, the underlying structure of the series- may not be linear and what is more, the series may not be Gaussian. In these situations, second-order properties, such as covariances and spectra, can no longer adequately characterize the properties of the series and one is lead then to consider non-linear models which can provide a better fit. A particular class of non-linear models which has been found to be useful in many fields is the bilinear models. Bilinear models have been extensively discussed in the control theory literature. One could check Rubert, Isidori and d'Alessandro (1972) [14] and Bruni, Dupillo and Koch (1974) [4] for further details. Until recently, the theory of bilinear models dealt with the structural theory of deterministic bilinear differential equations. The study ofbilinear models as stochastic models was initiated by Granger and Andersen (1978) [6] and Subba Rao (1981) [12].

Let e_t, $t \in Z$ be a sequence of independent and identically distributed random variables with $E(e_t) = 0$ *and* $E(e_t^2) = \sigma^2 < \infty$. Let $a_1, a_2 \cdots, a_p$,; c_1, $c_2 \cdots c_q$ and bij, $1 \le i \le m$, $1 \le j \le k$ be real constants. The general form of the bilinear models, as defined in Granger and Andersen (1978) is:

$$X_t \sum_{j=1}^{p} a_j X_{t-j} + \sum_{j=1}^{q} c_j e_{1t-j} + \sum_{i=1}^{m} \sum_{j=1}^{k} b_{ij} X_{t-j} e_{t-1} + e_t \qquad (1.1)$$

for every $t \in Z$. If X_t, $t \in Z$ satisfies (2.1), Subba Rao (1981) uses the notation that X_t, $t \in Z$, is BL (*p*,*q*,*m*,*k*) where BL is the abbreviation for bilinear model. Various simples forms of (1.1) are discussed in the literature by the following authors: Granger and Andersen (1978) [[6]; Subba Rao (1981) [12]; Pham, T.D [9] and Tran, L. T. (1981) ; Subba Rao and Gabr (1981) [13]; Tong (1981) [14], Quinn (1982) [10]; Bhaskara Rao M and Subramanyam (1986) [2].

2.0 Theoretical framework and methodology

According to Nwogu and Iwueze (2003) [[8] and Iwueze and Chikezie (2005) [7] the convariance analysis of the bilinear seasonal time series model;

$$X_t = \alpha X_{t-s} + \beta e_{t-s} + \gamma X_{t-s} + e_t,\ s \ge 1 \qquad (2.1)$$

which is a subset of (1.1) has been identified as behaving like the non-seasonal ARMA(1,1) except that for $s \ge 2$, the coefficients which appear at lags 1, 2, 3, …in the non-seasonal ARMA(1,1) now occur at multiples of lag $s(s,\ 2s,\ 3s,\ldots)$.

Proceeding from the foregoing and having obtained the following useful results for model 1.2:

(1) $$\mu = E(X_t) = \sigma^2\gamma/(1-\alpha),\ |\alpha| < 1 \qquad (2.2)$$

(2) $$\mu_2 = E(X_1^2) = \frac{\sigma^2\{1+\beta^2+2\alpha\beta+2\gamma\mu(1+\alpha+\beta)\}}{1-\alpha^2-\sigma^2\gamma^2} \qquad (2.3)$$

provided $\alpha^2 + \sigma^2\gamma^2 < 1$

(3) $$\begin{aligned} R(s) &= \alpha R(0) + \sigma^2(\beta + \gamma\mu) \\ &= \alpha R(0) + \sigma^2 + (1-\alpha)\mu^2 \qquad (2.4) \end{aligned}$$

(4) $$R(2s) = \alpha R(s) \qquad (2.5)$$

(5) $$\rho 2s = \alpha\rho s \qquad (2.6)$$

we solve to obtain

(6) $$\alpha = \rho 2s / \rho s \qquad (2.7)$$

(7) $$\beta = \frac{R9s) = \alpha(0) - 1(1-\alpha)\mu^2}{\sigma^2} \qquad (2.8)$$

(8) $$\gamma = \frac{(1-\alpha)\mu}{\sigma^2} \qquad (2.9)$$

(9) $$\sigma^2 = \frac{\mu_2(1-\alpha^2)}{1+\beta^2+\mu_2\gamma^2+2\alpha\beta+2\gamma\mu(1+\alpha+\beta)} \qquad (2.10)$$

The estimates of the parameters of (1.1) can be achieved by replacing theoretical moments with their sample equivalents in (2.7) through (1.10).

Thus:

$$\hat{\alpha} = r_{2s} / r_s \tag{2.11}$$

$$\hat{\sigma} = \frac{M_2(1-\alpha^2)}{1+\hat{\beta}^2 + M_2\hat{\gamma}^2 + 2\hat{\alpha}\hat{\beta} + 2\hat{\gamma}\overline{X}(1+\hat{\alpha}+\hat{\beta})} \tag{2.12}$$

$$\hat{\beta} = \frac{C_s - \hat{\alpha}C_0 - (1-\hat{\alpha})\overline{X}^2}{\hat{\sigma}^2} \tag{2.13}$$

and

$$\hat{\gamma} = \frac{(1-\hat{\alpha})\overline{X}}{\hat{\sigma}} \tag{2.14}$$

where

$$\left.\begin{aligned} \overline{X} &= \sum X_i / n \\ M_2 &= \sum X_i / n \\ C_k &= \frac{\sum (X_t - \overline{X})(X_{t+k} - \overline{X})}{n} \qquad k = 0,\ 1,\ 2, \ldots \end{aligned}\right\} \tag{2.15}$$

3.0 Numerical illustrations

Having obtained $\alpha = r_{2s} / r_s$, we adopt an iterative procedure called "Linear convergence process" by Box and Jenkins (1976, p. 202) [3] to obtain initial estimates of β, Υ and σ^2. We can compute the estimates $\hat{\sigma}^2, \hat{\beta}, \hat{\gamma}$ in this precise order using the iteration (2.12), (2.13) and (2.14). The parameters β and Υ and σ^2 to be used in any subsequent calculation are the most up to date values available. See Table 1.1 for illustration.

Table 1. An illustration of Box and Jenkins iterative procedure involving σ^2, β and Υ for Region 4, (X(bar) = - 0.13, M_2 = 4.06, C_0 = 4.04, C_s = -3.15).

Iteration	σ^2	β	Υ
0	-	0.00	0.00
1	2.33	-0.24	-0.09
2	1.66	-0.33	-0.13
3	1.45	-0.38	-0.14
4	1.36	-0.40	-0.15
5	1.32	-0.42	-0.16
6	1.26	-0.43	-0.16
7	1.28	-0.44	-0.17
8	1.24	-0.44	-0.17
9	1.24	-0.44	-0.17

Table 1.2 and Table 1.3 give a summary of the sample estimates of model (1.2) in Region 1, s = 1, 2, 3, 4, 6, 12 for n =100 and 500 respectively.

Table 1.4 and Table 1.5 give the initial and the final estimates. The convergence of the final estimates adopting the Newton-Raphson iterative technique {see Table 1.6} and the closeness to the "true' values make the initial estimates good enough and the entire procedure of achieving the final estimates adequate. The program for estimation adopting the Newton-Raphson procedure is written in Fortran 77 language. However, for want of space only as much are reported here. Other regions discussed by Nwogu and Iwueze (2003) [8] and Iwueze and Chikezie [7] can be obtained similarly.

Table 1.2: Sample estimates of model (1.2) in Region 1, s = 1, 2, 3, 4, 6, 12 for n =100

s	X(bar)	C_0	C_s	C_{2s}	M_2	α
1	0.8455	6.4006	5.3612	3.6873	7.1155	0.69
2	0.7778	5.956	5.0965	3.8824	6.5645	0.76
3	0.9340	8.0267	7.098	5.7925	8.8990	0.82
4	0.9168	6.9508	5.9164	4.3821	7.7913	0.74
6	0.9939	7.7649	6.7171	5.1642	8.7527	0.77
12	0.6547	5.8501	4.6278	3.0314	6.2788	0.66

Table 1.3: Sample estimates of model (1.2) in Region 1, s = 1, 2, 3, 4, 6, 12 for n =500

s	X(bar)	C_0	C_s	C_{2s}	M_2	α
1	1.1556	8.6551	7.7553	6.3715	9.9906	0.82
2	1.0206	6.2559	5.3795	3.9920	7.8968	0.74
3	1.0100	6.6295	5.7376	4.4287	7.6496	0.77
4	0.9812	6.2970	5.3811	3.9816	7.2597	0.74
6	0.9107	5.6130	4.7360	3.8412	6.4425	0.74
12	.9592	7.0123	60.857	4.7866	7.9324	0.79

Table 1.4: Initial and Final estimates for Region 1, s = 1, 2, 3, 4, 6, 12 for n =100

	Initial estimates				**Final estimates**			
S	α	β	γ	σ^2	α	β	γ	σ^2
1	0.70	0.50	0.20	1.30	0.78	0.27	0.25	0.97
2	0.76	0.36	0.16	1.17	0.76	0.36	0.16	0.96
3	0.82	0.26	1.12	1.40	0.80	0.31	0.07	1.31
4	0.74	0.50	1.21	1.00	0.80	0.33	0.23	0.97
6	0.77	0.41	1.18	1.24	0.83	0.35	0.05	1.40
12	0.66	0.36	1.13	1.72	0.79	0.43	0.19	0.99

Table 1.5: Initial and Final estimates for Region 1, s = 1, 2, 3, 4, 6, 12 for n =500

	Initial estimates				**Final estimates**			
S	α	β	γ	σ^2	α	β	γ	σ^2
1	0.82	0.40	0.20	1.01	0.81	0.40	0.20	1.00
2	0.74	0.41	0.23	1.04	0.80	0.40	0.20	1.00
3	0.77	0.38	0.13	1.36	0.79	0.37	0.05	1.34
4	0.74	0.39	0.21	1.14	0.76	0.42	0.11	1.25
6	0.76	0.18	0.14	1.41	0.78	0.35	0.02	1.37
12	0.80	0.16	0.10	1.64	0.81	0.36	0.04	1.37

4.0 Conclusion

In this paper we have examined the initial and final estimates of the bilinear seasonal time series model (1.2). Table 1.6 showed convergence after few iterations which lends credence to the proposition that with good initial values of the parameters the estimates obtained by the Newton-Raphson iterative technique usually not only converge but also are good estimates. The closeness of the initial estimates and the final estimates for both simulations (n = 100 and n = 500) also confirms that the initial estimates are good enough.

Table 1.6: An Illustration of the Newton-Raphson iterative procedure for arriving at the Final estimates for Region 1, s = 3 and n = 100

Iteration	α	β	γ	σ^2
0	0.8200	0.2600	0.1200	1.4000
1	0.7936	0.4443	0.0771	2.2182
2	0.7913	0.4024	0.0693	1.4551
3	0.7920	0.3355	0.0675	1.3365
4	0.7972	0.3081	0.0665	1.3096
5	0.7978	0.3052	0.0663	1.3070
6	0.7978	0.3052	0.0663	1.3070
7	0.7978	0.3052	0.0663	1.3070

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