Journal of the Nigerian Association of Mathematical Physics
Volume **11 (November 2007), 557 - 570**

Pressure on the Cochlea as a Load on the basilar membrane: Its contribution to the mechanism of hearing

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Abstract

A Mathematical model of the pressure equation was obtained and solved to show the effect of pressure as a load on the basilar membrane deformation (motion) and that the basilar membrane motion is in opposite direction to pressure (load) application on it. This means that when pressure is exerted on the basilar membrane, it pushes it down and vice versa. This movement of the basilar membrane also results in the movement of the Oval and Round windows which also move in opposite direction relative to the one another as well as to the load exerted on the basilar membrane. As expected, it was seen that the pressure difference at any point inside the cochlea and thus the basilar membrane is a function of the location of interest since x_1 ***and*** x_3 ***are implicated in the expression for the pressure difference.***

1.0 Introduction

This paper presents a mathematical model on the mechanism of transmission of the noise in the ear through the fluid found in the inner ear. We define noise here as the sound that cause discomfort to the hearer but this definition is not scientifically enough as noise to one person may not be so for another, Chalupnik (1977 [5]). Therefore, put more scientifically, noise can be defined as sound that measures above 85db (db means decibel, the unit of measurement of sound). Most noise come as irregular vibrations. Every sound has two major aspects which are the frequency or pitch and the intensity or amplitude, Cheremisionoff, P.N and Cheremisionoff, P.P (1978 [6]). The frequency is determined by how rapidly the generated sound wave s vibrates. In terms of the sound intensity, the human ear has a wide range to which it can be exposed. The range is from one billionth (10^{-9}) Watt to 10^{7} Watt. Silence which we describe as the arbitrary threshold level of sound is represented by zero decibel while the faintest sound audible to the human ear is represented by 1db.

The greatest physiological effect of noise in a man is temporary deafness or permanent hearing loss, Rau and Woolen (1980 [19]) and increase in blood pressure. Other effects are discomfort, tiredness, stress and feeling of irritation, aplynopsys (inability to sleep), low blood resistance to diseases and many more others. Noise has been implicated as one of the causes of

Ulcer and the allergies like hives. It can also lead to somatic manifestations such as gastric acid problems. In particular,, temporary exposure to noise can lead to impairment in hearing which is termed auditory fatigue while exposure to noise for a very long time without enough time for recovery usually lead to permanent hearing loss.

In general therefore, we see noise as a multidimensional problem because of multiple ways it can be interpreted and understood. Hence, a proper mathematical model and analysis of noise involves associating a random variable with multidimensional physical processes causing the noise.

2.0 Mechanism of hearing

The details of the structure of the ear can be found in the work of Adagba [1], Burtons and Hopkin [4]. For a given sound to be heard there must be the presence of air medium. Thus when a sound is generated, it results in generation of waves as a result of the vibration of the object that produced the sound. These sound waves are collected by the pinnae and directed through the external auditory meateus to impinge on the tympanic membrane or the ear drum. This vibrates the eardrum and then passes the energy of vibration , which is the pressure, on to the middle ear. The middle ear is open to the throat through the Eustachian tube so that the air pressure through this tube equalizes the air pressure on both sides of the eardrum and then the transfer of the energy of vibration (the pressure) from the outside of the eardrum to the middle ear is possible. At the middle ear, we have a series of three bones which help in the transfer of the vibrations coming from the outer part of the eardrum to the inner part. These bones are the *malleaus, incus and stape*. These bones are connected such that they connect the tympanic membrane with the oval window of the cochlea which is the organ where the vibrations are again converted into energy impulse. In the cause of conducting the vibrations to the cochlea by these bones, the vibration is increased in strength or rather amplified. It is known that sound pressure received at the tympanic membrane are relatively lost as most is transmitted at the cochlea where they are amplified to a 22-fold greater pressure, Burtons and Hopkin (1983 [4]), Wegel et al (1932 [22]), Wever and Lawrence (1930b [23], 1950a [24]). the mechanical forces that are transferred by the bones of the middle ear are transformed into the hydraulic pressure when the stape strikes the oval window. Since the oval window is filled with fluid, this pressure applied to the oval window by the striking of the stape is transmitted through this fluid which eventually causes a vibration of the *basilar membrane*, a slight structure which extends from the cochlea to the auditory nerves and totally lying in the fluid. This region of the ear is called the inner ear. The basilar membrane has *Hair cells* on its surface such that the vibration of the fluid and thus the basilar membrane produces shearing movement between the hair cells and the tectoral membrane of the organ of Corti. This initiates wave impulse in the fibres of the auditory nerve. It is generally agreed that sound waves are analysed at the cochlea and that each frequency has its own place in the basilar membrane. The auditory nerve will then send this received impulse to the brain for interpretation and subsequent response to it. Mathematically we shall not go into how we respond to this noise here but rather we shall look at how this noise is received and how the ear structures respond to it.

In this paper therefore, we wish to draw a mathematical model on the pressure (Vibration) on the Cochlea as a load on the basilar membrane. We will show how this pressure deforms the basilar membrane which results in impulse generation and the subsequent transfer to the brain through the auditory nerves for interpretation.

3.0 The fluid mechanics of the Cochlea as a result of external pressure.

As stated earlier, the cochlea is the part of the inner ear which is a small fluid-filled chamber and contains the biological structures that converts mechanical signals into neural signals. In addition to the signal conversion, it does process signals. Thus, a clear understanding of the mechanism requires that we understand the cochlea fully as it relates to audition; Lesser and Berkkley (1972), Ranke (1950b [18]) and Lamb (1904 [12]).

To model the pressure on the Cochlea as a load on the basilar membrane which is inside the cochlea, the following points and assumptions are noted:

1. The model is a two dimensional model in an enclosed cavity containing a structure of spatial variable elastic properties.
2. The spiral cochlea is unwound
3. The central duct in the cochlea which contains the organ of Corti and which is enclosed by Reisner's membrane and basilar membrane will be represented by a single elastic partition.
4. The mechanical properties of each partition are represented by the assumption that each point acts as a damped harmonic oscillator point to point, coupling being only through the surrounding fluid. This assumption leads to representing the partition by a mechanical impedance $z(x_1 ,w)$, x_1 being the distance from the oval window along the partition.
5. The endolymph is considered incompressible for it has the same sound speed as water, which is likely, the wavelength of an acoustic signal at 500Hz (at high frequency for hearing) is about 30 cm while the cochlea is only 35cm.
6. The fluid flow will be considered inviscid though we shall regard this as a first step in an expansion procedure.
7. The endolymph is considered inviscid
8. One point that is needed to be remembered is the fact that the motion of the basilar membrane is small, a displacement of about 10^{-6}cm corresponds to normal amplitude of sound.
9. Man can detect sound corresponding to the basilar membrane and eardrum displacement of 10^{-10}cm. Also known is the fact that non-linearity does exist in mechanics, though many of these are to become noticeable over a long period of time. The works of Goldstein (1967 [8]), Goblick and Pfeiffer (1969) in electrophysiology include the presence of non-linearities in cochlea mechanics.
10. There exists non-linearity caused by eddy in the cochlea and this is different from the above linearity talked about. This eddy called the Bekesy eddy, is understood as resulting from the combination of viscous and non-linear effects.

From all these, we have that the flow pattern in cochlea model excited by an oscillatory disturbance, which is the original pressure exerted on the tympanic membrane by the sound that was generated, exhibits a steady streaming motion as well as motion typical of a fluid with a free surface. As the excitation is purely oscillatory, the steady motion must result from a non-linear interaction, Morse (1948 [16]), Pain (1976 [17]) and Rhode (1971 [20]).

4.0 The mathematical model.

To be able to get the required model that will describe the cochlea mechanics, we assume that the basilar membrane motion is caused by the pressure exerted on it which we considered as a load and is primarily controlled by potential flow. Physical measurements by Von Bekesy (1947 [21]) shows that the maximum basilar membrane slope is sufficiently small and this supports the linearization of the equations involved. Our model here is for an enclosed two-dimensional cavity and the basilar membrane appears in it as a thin plate immersed in the fluid in the cavity. We shall consider the linear short-time scale aspect of the cochlea behaviour. Thus, for the figure below, we assume a linearised two-dimensional potential flow, that is:

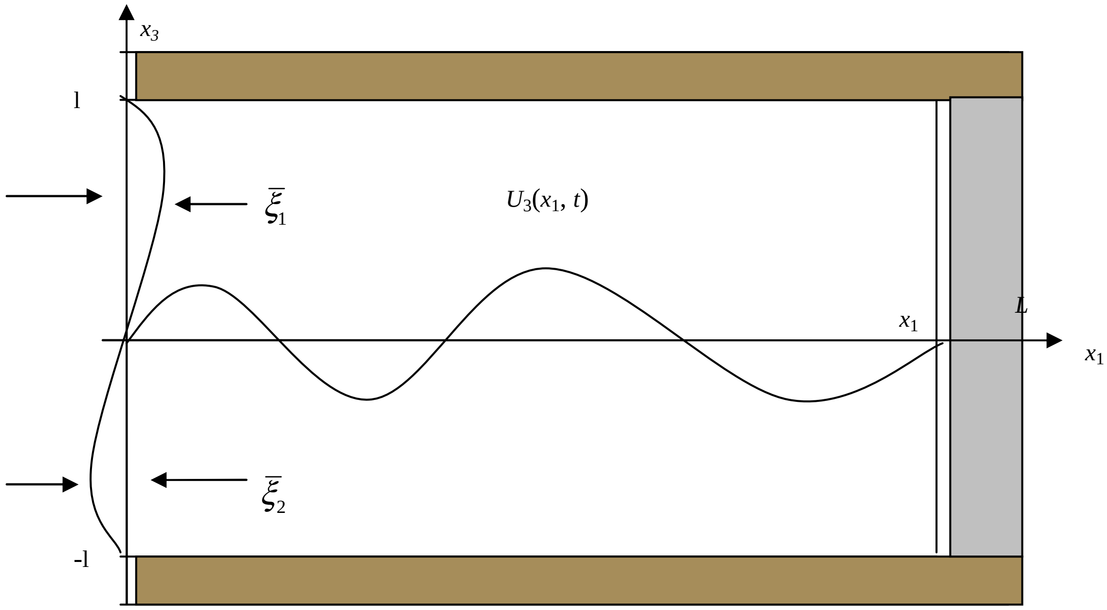


Figure 1: Potential flow model of the Cochlea

We denote the upper domain where $x_3 > 0$, with the subscript 1 while for $x_3 < 0$, we use the subscript 2.

Using the work of Green and Naghdi (1967), the equation of motion of such plate is given by:

$$\alpha_3 u_{3,\alpha\alpha} + F_3 = \rho \ddot{u}_3 \tag{4.1}$$

where α_3 is the elastic constant, ρ is the mass density of the membrane, u_3 is the velocity of the vibration of the membrane while F_3 is the load on the membrane which can be viewed as external forces acting on the membrane. The equation of motion characterizing the flow of a non-viscous incompressible fluid in such cavity is given as:

$$\frac{\partial U}{\partial x_1} + \frac{\partial U}{\partial x_3} = 0 \tag{4.2}$$

$$\rho\left\{\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x_1} + V\frac{\partial U}{\partial x_3}\right\} = -\frac{\partial P_1}{\partial x_1} \tag{4.3}$$

$$\rho\left\{\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x_1} + V\frac{\partial V}{\partial x_3}\right\} = -\frac{\partial P_2}{\partial x_3} \tag{4.4}$$

where ρ is the fluid density, P_i is the fluid pressure, $i = 0,1,2$ and the U and V are the velocities on both axes. Since the flow motion of interest is that motion with small amplitude, we neglect the product terms in equations (4.3) and (4.4) and we obtain

$$\rho\frac{\partial U}{\partial t} = -\frac{\partial P_1}{\partial x_1} \tag{4.5}$$

$$\rho\frac{\partial V}{\partial t} = -\frac{\partial P_{21}}{\partial x_3} \tag{4.6}$$

We shall assume a velocity potential Φ such that $\nabla\Phi = (\bar{U}, \bar{V})$ where $\bar{U}$ *and* $\bar{V}$ are the x_1 and x_3 fluid velocity components and equation (4.2) is satisfied.

Now,

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial t}\left(\frac{\partial\overline{\Phi}}{\partial x_1}\right)=\frac{\partial}{\partial x_1}\left(\frac{\partial\overline{\Phi}}{\partial t}\right) \tag{4.7}$$

$$\frac{\partial V}{\partial t}=\frac{\partial}{\partial t}\left(\frac{\partial\overline{\Phi}}{\partial x_3}\right)=\frac{\partial}{\partial x_3}\left(\frac{\partial\overline{\Phi}}{\partial t}\right) \tag{4.8}$$

If we substitute $U=\frac{\partial\overline{\Phi}}{\partial x_1}$ *and* $V=\frac{\partial\overline{\Phi}}{\partial x_3}$ into equation (4.2), we get

$$\frac{\partial U}{\partial x_1}+\frac{\partial V}{\partial x_3}=\frac{\partial^2\overline{\Phi}}{\partial x_1^2}+\frac{\partial^2\overline{\Phi}}{\partial x_3^2}=\nabla_1^2\overline{\Phi}=0 \tag{4.9}$$

This implies that in the upper and lower chambers,

$$\nabla_{(1)}^2\overline{\Phi}_1=\nabla_{(1)}^2\overline{\Phi}_2$$

where

$$\nabla_{(1)}^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2} \tag{4.10}$$

Substituting equations (4.7) and (4.8) into equations (4.5) and (4.6) gives us :

$$\frac{\partial}{\partial x_1}\left(\frac{\partial\overline{\Phi}}{\partial t}\right)+\frac{1}{\rho}\frac{\partial\overline{P}_1}{\partial x_1}=\frac{\partial}{\partial x_1}\left(\frac{\partial\overline{\Phi}}{\partial t}+\frac{\overline{P}_1}{\rho}\right)=0 \tag{4.11}$$

and

$$\frac{\partial}{\partial x_3}\left(\frac{\partial\overline{\Phi}}{\partial t}\right)+\frac{1}{\rho}\frac{\partial\overline{P}_2}{\partial x_3}=\frac{\partial}{\partial x_3}\left(\frac{\partial\overline{\Phi}}{\partial t}+\frac{\overline{P}_2}{\rho}\right)=0 \tag{4.12}$$

Thus, we have the pressure equations in the upper and lower chambers as

$$\rho\frac{\partial\overline{\Phi}_1}{\partial t}+\overline{P}_1=0 \tag{4.13}$$

$$\rho\frac{\partial\overline{\Phi}_1}{\partial t}+\overline{P}_2=0 \tag{4.14}$$

Having derived these equations, let us then solve the equations to be able to carry out some analysis and discussions of the results.

5.0 Solutions of the pressure equation and the equation of motion of the fluid

The equations of the motion of the fluid in both chambers was given as

$$\nabla_{(1)}^2\overline{\Phi}=\frac{\partial^2\overline{\Phi}}{\partial x_1^2}+\frac{\partial^2\overline{\Phi}}{\partial x_3^2}=0$$

We therefore seek for its solution by noting that this is a well known Laplacian equation and that the field variables will be proportional to $e^{iwt}=e^{st}$. In this case therefore, we write:

$$\Phi=\mathrm{Re}\left(\overline{\Phi}e^{st}\right)\quad u_3=\mathrm{Re}\left(\overline{u}_3e^{st}\right),\quad P_i=\mathrm{Re}\left(\overline{P}_ie^{st}\right),\quad \xi=\mathrm{Re}\left(\overline{\xi}\,e^{st}\right)$$

so that the equations of motion as well as the pressure equations become:

$$\nabla_{(1)}^2\Phi_1=0,\quad \nabla_{(1)}^2\Phi_2=0,\ P_1+\rho s\Phi_1=0,\quad P_2+\rho s\Phi_2=0$$

Thus the solutions of the Laplace's equations are known to be of the forms :

$$\overline{\Phi}_1(x_1,x_3)=\left(k_1Cos\lambda x_1+k_2Sin\lambda x_1\right)\left(A_1Cosh\lambda x_3+B_1Sinh\lambda x_3\right) \tag{5.1}$$

$$\overline{\Phi}_2(x_1,x_3)=\left(\overline{k}_1Cos\lambda x_1+\overline{k}_2Sin\lambda x_1\right)\left(A_2Cosh\lambda x_3+B_2Sinh\lambda x_3\right) \tag{5.2}$$

These solutions and their derivatives with respect to x_1 are finite and continuous at all points except possibly at some points on the boundary of the field. Thus, the smoothness of the velocity distribution is ensured at all points of the fluid except at these points stated; Barbel et al (2002 [2]), Bell and Holmes (1986b [3]), Gupta (1987 [10]), Harold (1982 [11]).

Now the boundary conditions on $\overline{\Phi}$ at $x_1 = L$ and $x_3 = l$ is:

$$At \quad x_1 = L, \qquad \frac{\partial \overline{\Phi}_1}{\partial x_1} = \frac{\partial \overline{\Phi}_2}{\partial x_1} = 0$$

$$At \quad x_3 = l, \qquad \frac{\partial \overline{\Phi}_1}{\partial x_3} = 0$$

$$At \quad x_3 = -l, \qquad \frac{\partial \overline{\Phi}_2}{\partial x_3} = 0$$

Now $\frac{\partial \overline{\Phi}_1}{\partial x_1} = (-\lambda k_1 Sin\lambda x_1 + \lambda k_2 Cos\lambda x_1)(A_1 Cosh\lambda x_3 + B_1 Sinh\lambda x_3)$. So that at $x_1 = L$, we then have

$$0 = -\lambda k_1 Sinh\lambda L + \lambda k_2 Cosh\lambda L \text{ and for } \lambda \neq 0,$$

we get that $$k_2 = \frac{k_1 Sin\lambda L}{Cos\lambda L} s \qquad (5.3)$$

Similarly, on $x_3 = l$, we have

$$\frac{\partial \overline{\Phi}_1}{\partial x_3} = \left(\bar{k}_1 Cos\lambda x_1 + \bar{k}_2 Sin\lambda x_1\right)\left(\lambda A_1 Sinh\lambda x_3 + \lambda B_1 Cosh\lambda x_3\right)$$

$$\Rightarrow \qquad 0 = A_1 Sinh\lambda l + B_1 Cosh\lambda l$$

that is, $$B_1 = \frac{-A_1 Sinh\lambda l}{Cosh\lambda l} \qquad (5.4)$$

Substituting equations (5.3) and (5.5) into equation (5.1), we get

$$\overline{\Phi}_1(x_1, x_3) = \frac{\beta Cos\lambda(L - x_1) Cosh\lambda(L - x_3)}{Cos\lambda L Cosh\lambda l} \qquad for\ \beta = k_1 A_1 \qquad (5.5)$$

In a similar way , equation (5.2) will give

$$\overline{\Phi}_2(x_1, x_3) = \frac{\gamma Cos\lambda(L - x_1) Cosh\lambda(L + x_3)}{Cos\lambda L Cosh\lambda l} \qquad for\ \gamma = \bar{k}_1 A_2 \qquad (5.6)$$

λ can be evaluated explicitly to give $\lambda = \omega\sqrt{\frac{\rho}{\alpha_3}}$ so that

$$\overline{\Phi}_1(x_1, x_3, t) = \frac{\beta Coswt Cos\omega\sqrt{\frac{\rho}{\alpha_3}}(L - x_1) Cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(L - x_3)}{Cos\omega\sqrt{\frac{\rho}{\alpha_3}} L Cosh\omega\sqrt{\frac{\rho}{\alpha_3}} l} \qquad for\ \beta = k_1 A_1 \qquad (5.7)$$

and $$\overline{\Phi}_2(x_1, x_3, t) = \frac{-\beta Coswt Cos\omega\sqrt{\frac{\rho}{\alpha_3}}(L - x_1) Cosh\omega\sqrt{\frac{\rho}{\alpha_3}}(L + x_3)}{Cos\omega\sqrt{\frac{\rho}{\alpha_3}} L Cosh\omega\sqrt{\frac{\rho}{\alpha_3}} l} \qquad for\ -\beta = \gamma \qquad (5.8)$$

Equations (5.7) and (5.8) are the solutions of the fluid equation in potential form where the velocities can be determined very easily. The term Coswt is to take care of the fluctuation with respect to time. If the noise is not a function of time in which case t can be assumed to varnish, then $Cos0 = 1$ and we have our equations (5.5) and (5.6) still describing our potentials at both sides or chambers with appropriate substitution for λ. Now for the pressure as a load on the Basilar membrane, we solve to get the expression for the pressure equations from the equations:

$$P_1 + \rho s\Phi_1 = 0, \quad P_2 + \rho s\Phi_2 = 0\,. \tag{5.9}$$

From this equation, we get that $\overline{P}_1 - \overline{P}_2 = \rho s\left(\overline{\Phi}_1 - \overline{\Phi}_2\right)$.

Substituting using equations (5.5) and (5.6) and then simplifying, we obtain:

$$\overline{P}_2 - \overline{P}_1 = \rho s\left(\overline{\Phi}_1 - \overline{\Phi}_2\right) = \frac{\rho \mathrm{s}\left[\beta \mathrm{Cos}\lambda(\mathrm{L - x_1})Cosh\lambda(l - x_3) - \gamma \mathrm{Cos}\lambda(\mathrm{L - x_1})Cosh\lambda(l + x_3)\right]}{Cos\lambda L Cosh\lambda l} \tag{5.10}$$

Thus, on $x_3 = 0$, we get that

$$\overline{P}_2 - \overline{P}_1 = \rho s\left(\overline{\Phi}_1 - \overline{\Phi}_2\right) = \frac{\rho \mathrm{s}\,(\beta - \gamma)\mathrm{Cos}\lambda(\mathrm{L - x_1})}{Cos\lambda L}\,. \tag{5.11}$$

The pressure difference at any point inside the cochlea is given by:

$$\overline{P}_2 - \overline{P}_1 = \rho s\left(\overline{\Phi}_1 - \overline{\Phi}_2\right) = \frac{\rho \mathrm{s}\,(-\overline{P}_o + \overline{P}_1(0, x_3) - \overline{P}_2(0, \mathrm{x}_3))\mathrm{Cos}\lambda(\mathrm{L - x_1})}{z(x_3, w)\lambda Sin\lambda L}\,. \tag{5.12}$$

Now using the fact that $\overline{P}_2 = \overline{P}_0 - \overline{P}_1$, the problem as presented can be simplified by redefining the arbitrary time function in the introduction of the velocity potential. Thus, for region 2, where $-l < x_3 < 0$, we have:

$$\overline{\Phi}_1 = \overline{\Phi}_2 - \frac{\overline{P}_0}{\rho s}\,. \tag{5.13}$$

Substituting this in equation (5.9) we have that:

$$\rho s\left(\overline{\Phi}_1 + \overline{\Phi}_1 + \frac{\overline{P}_0}{\rho s}\right) = \rho s\left(2\overline{\Phi}_1 + \frac{\overline{P}_0}{\rho s}\right) = \overline{P}_0 - 2\overline{P}_1 \tag{5.14}$$

Allowing $\overline{P}_0 = 0$, we obtain $-\rho s\overline{\Phi}_1 = \overline{P}_1$ so that $\overline{P}_1$ valid at $x_1 = 0$ gives:

$$\overline{P}_1(0, x_0) = -\frac{\beta s\rho Coshw\sqrt{\frac{\rho}{\alpha_3}}(l - x_3)}{Coshw\sqrt{\frac{\rho}{\alpha_3}}l} \tag{5.15}$$

For the vibrations in the Oval and Round windows, we had earlier obtained the expressions, Mbah and Adagba (2006 [15]),:

$$\xi_1 = \frac{-\beta w\sqrt{\frac{\rho}{\alpha_3}}\tan w\sqrt{\frac{\rho}{\alpha_3}}LSinwtCoshw\sqrt{\frac{\rho}{\alpha_3}}(l - x_3)}{wCosw\sqrt{\frac{\rho}{\alpha_3}}l}$$
$$= \frac{-\beta\rho Coshw\sqrt{\frac{\rho}{\alpha_3}}(l - x_3)\left\{r_o Coswt + (m_o w - \frac{k_o}{w})Sinwt\right\}}{\left\{r_o^2 + (m_o w - \frac{k_o}{w})^2\right\}Coshw\sqrt{\frac{\rho}{\alpha_3}}l} \tag{5.16}$$

for the upper chamber and

$$\xi_2 = \frac{-\beta w\sqrt{\frac{\rho}{\alpha_3}}\tan w\sqrt{\frac{\rho}{\alpha_3}}LSinwtCoshw\sqrt{\frac{\rho}{\alpha_3}}(l + x_3)}{wCosw\sqrt{\frac{\rho}{\alpha_3}}l}$$
$$= \frac{-\beta\rho Coshw\sqrt{\frac{\rho}{\alpha_3}}(l + x_3)\left\{r_o Coswt + (m_o w - \frac{k_o}{w})Sinwt\right\}}{\left\{r_o^2 + (m_o w - \frac{k_o}{w})^2\right\}Coshw\sqrt{\frac{\rho}{\alpha_3}}l} \tag{5.17}$$

for the lower chamber.

The motion of the basilar membrane which is equivalent to the basilar membrane deformation, equation (4.1), as a result of the exerted pressure (which we call the load) is given as, Mbah and Adagba (2006 [15]):

$$U_3(x_1,t) = \frac{-\beta w\sqrt{\frac{\rho}{\alpha_3}}\tanh w\sqrt{\frac{\rho}{\alpha_3}}l\, Sinwt\, Cosw\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)}{wCosw\sqrt{\frac{\rho}{\alpha_3}}L} \tag{5.18}$$

which can be simplified further as:

$$U_3(x_1,t) = \frac{-\beta w\sqrt{\frac{\rho}{\alpha_3}}\tanh w\sqrt{\frac{\rho}{\alpha_3}}l\left[Sin(wt + w\sqrt{\frac{\rho}{\alpha_3}}(L-x_1)) - Sin(wt - w\sqrt{\frac{\rho}{\alpha_3}}(L-x_1))\right]}{2wCosw\sqrt{\frac{\rho}{\alpha_3}}L}$$

$$= \frac{-\beta w\sqrt{\frac{\rho}{\alpha_3}}\tanh w\sqrt{\frac{\rho}{\alpha_3}}l\left[Sinw(t + x_1\sqrt{\frac{\rho}{\alpha_3}} + L\sqrt{\frac{\rho}{\alpha_3}}) - Sinw(t + x_1\sqrt{\frac{\rho}{\alpha_3}} - L\sqrt{\frac{\rho}{\alpha_3}})\right]}{2wCosw\sqrt{\frac{\rho}{\alpha_3}}L} \tag{5.19}$$

This is oscillatory with two traveling waves, one in the positive direction and the other in the negative direction but with the same speed, Jeffery and Jeffery (1956), Milkilin (1970), Murray (1977). However, the fluid variables U_3 is unbounded if

$$Cosw\sqrt{\frac{\rho}{\alpha_3}}L = 0 \qquad \Rightarrow \quad w = \frac{(2n-1)\pi}{2L}\sqrt{\frac{\rho}{\alpha_3}}$$

This means then that the solution for the basilar membrane equation exists only if

$$w \neq \frac{(2n-1)\pi}{2L}\sqrt{\frac{\rho}{\alpha_3}}$$

Therefore, if we consider the maximum deflection with respect to the distance to be at the helicotrema ($x_1 = L$), which is the requirement for the place principle at low frequencies, Furness and Hackey (1985), Furukawa and Matura (1985), then we have

$$U_3(x_1,t) = \frac{-\beta w\sqrt{\frac{\rho}{\alpha_3}}\tanh w\sqrt{\frac{\rho}{\alpha_3}}l\, Sinwt}{Cosw\sqrt{\frac{\rho}{\alpha_3}}L} \tag{5.20}$$

6.0 Analysis and discussion of the results

We have seen how the pressure exerted on the tympanic membrane as a result of the noise is transferred to the cochlea which then formed the load on the basilar membrane that eventually enables the transmission of the noise to the Brain for interpretation. Equation (5.1) is the net pressure on the basilar membrane so that if $-\beta = \lambda$, then the equation becomes:

$$\overline{P} = \overline{P}_2 - \overline{P}_1 = \rho s(\overline{\Phi}_1 - \overline{\Phi}_2) = \frac{2\rho s\, \beta\, \mathrm{Cos}\lambda(\mathrm{L - x_1})}{Cos\lambda L}.$$

Recalling that $P(x_1,t) = \mathrm{Re}\left(\overline{P}(x_1,t)e^{st}\right)$, we then have that:

$$P(x_1,t) = \frac{2\rho s\, \beta\, \mathrm{Cos}\lambda(\mathrm{L - x_1})e^{st}}{Cos\lambda L} = \frac{2i\rho s\, \beta w\, \mathrm{Cos}\lambda(\mathrm{L - x_1})\{Coswt + iSinwt\}}{Cos\lambda L}$$

$$= \frac{-2\rho s\, \beta w Sinwt\, \mathrm{Cos}\lambda(\mathrm{L - x_1})}{Cos\lambda L} \tag{6.1}$$

This is the net pressure (Load) on the basilar membrane and it pushes the basilar membrane downwards thereby increasing the pressure in the scalar tympani and bulges the round window outwards as seen in the Figures 2 and 7 respectively. We can also see from these figures that a negative pressure draws the basilar membrane upwards such that these alternations cause the vibration.

In terms of this pressure, equation (6.1), we can write the deformation of the basilar membrane as:

$$U_3(x_1,t) = -w\sqrt{\frac{\rho}{\alpha_3}}\tanh w\sqrt{\frac{\rho}{\alpha_3}}l\,\frac{P(x_1,t)}{2\rho w} = -\sqrt{\frac{\rho}{\alpha_3}}\tanh w\sqrt{\frac{\rho}{\alpha_3}}l\,\frac{P(x_1,t)}{2\rho} \qquad (6.2)$$

This shows that if the net pressure is negative, the deformation of the Basilar membrane, $u_3(x_1,t)$ is positive. This means that the basilar membrane is pushed upwards if the pressure from below the basilar membrane exceeds the pressure from above and vice versa. See the basilar membrane motions as a result of the pressure exerted on it as shown in Figures 1, 2 and 4. Figures 3 and 5 show the pressure effect on the cochlea.

For the motion of the Oval and Round windows shown as equations (5.16) and (5.17), we can see that we can rewrite them in terms of the Pressure equation. Their motion as earlier said will be such that ξ_1 will be in opposite direction with basilar membrane motion while ξ_2 will go in the same direction, see Figures 6 and 7

The major results obtained from this study therefore are:

1. We obtained the expression for the load on the basilar membrane which we called the net pressure.
2. We showed that if the basilar membrane deformation expression $u_3(x_1,t)$ is written in terms of the pressure (load) expression, then the basilar membrane motion is in opposite direction to the manner of exertion of pressure on the cochlea.
3. We showed that the Oval window motion ξ_1 moves in opposite direction to the pressure exertion while the round window moves in the same direction as the pressure is exerted.
4. An expression for the pressure difference at any point inside the cochlea was also obtained and it is found to be dependent on the location of the point

Concluding therefore, we may say that noise effect in the ear is transmitted effectively in the Cochlea through the fluid in which the exerted pressure due to the noise impinges on the basilar membrane as a load. The basilar membrane is immersed in the fluid and the rocking of the foot of the *stape* on the Cochlea transmits the noise to the fluid in the cavity in the form of vibration. It is this vibration that sets up this motion in the fluid as well as the basilar membrane deformation. Therefore, whether the hearing ability in an individual is lost or not does not in any way affect the establishment of fluid motion in the Cochlea and thus load on the basilar membrane once there is noise or even sound entering the ear. However, the load level on the basilar membrane affects the elasticity of the microvilli, the HC and in general, the basilar membrane performance of its function in audition.

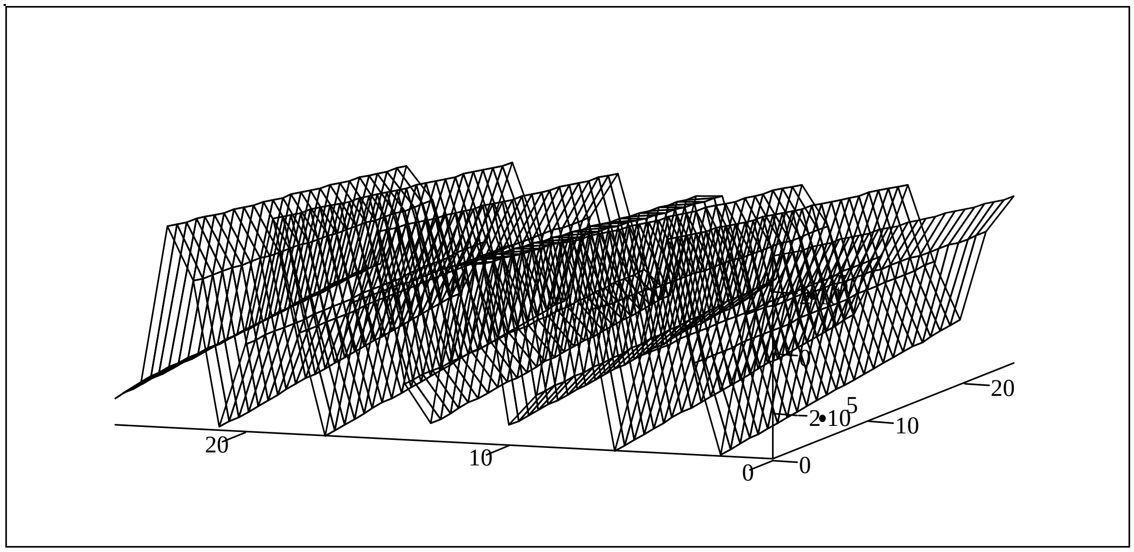
6.1 Deformation of the Basilar membrane

Choosing values for the variables as:

$$\alpha_3 := 2,\ L := 35,\ \rho := 1,\ m_0 := 0.05,\ \omega := 10^4,\ s := 10^4,\ 1 := 1,\ \beta := 1,\ N := 25,\ i := 0,\cdots,N,$$
$$j := 0,\cdots,N,\ x_i := 1\cdot 5 + 0\cdot 5,\ it_j := 1\cdot 5 + 0\cdot 15j$$

$$u_3(x,t) = \frac{-\left[\beta\omega\left(\frac{\rho}{\alpha_3}\right)^{0\cdot5}\right]\tanh\left[\omega\left(\frac{\rho}{\alpha_3}\right)^{0\cdot5}.1\right]\sin(\omega,t)\cos\left[\left(\frac{\rho}{\alpha_3}\right)^{0\cdot5}\right](L-x)}{\cos\left[\left[\omega\left(\frac{\rho}{\alpha_3}\right)^{0\cdot5}\right]L\right]}$$

$$M_{i,j} = u_3(x_i, t_j)$$



M

Figure 1: Deformation of the membrane at different times.

6.2 Load on the basilar membrane (Net Pressure)

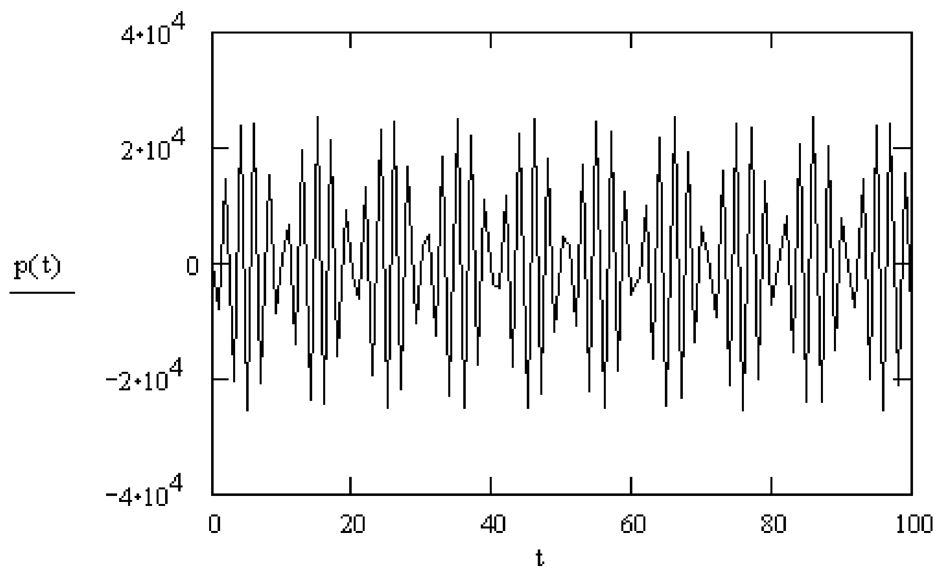


Figure 2: The pressure on Basilar membrane over time

6.3 Component pressure in the Cochlea

$$P_2(x,t)=\frac{1}{2}\sin\left[\omega\left(t-x\sqrt{\frac{\rho}{\alpha_3}}+L\sqrt{\frac{\rho}{\alpha_3}}\right)\right],\; M_{ij}:=P_2(x_i,t_j)$$

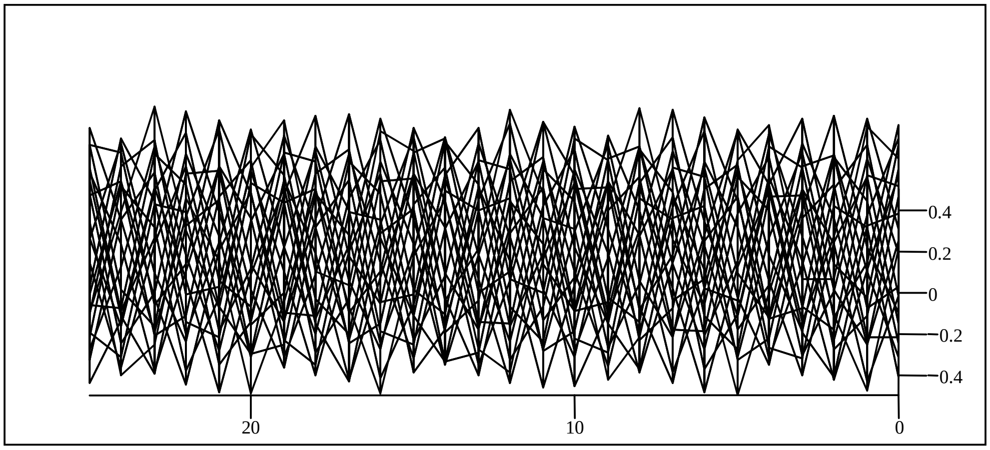


Figure 3: Pressure on the Cochlea over time

6.4 Deformation of the Basilar membrane along the *x*-axis

Choosing values for the variables as below, we obtain the graph:

$\alpha_3 := 2,\ L := 35,\ \rho := 1,\ m_0 := 0.05,\ \omega := 10^4,\ s := 10^4,\ 1 := 1,\ \beta := 1,\ N := 25,\ i := 0,\cdots,N,$
$j := 0,\cdots,N,\ x_i := 1\cdot 5 + 0\cdot 5,\ it_j := 1\cdot 5 + 0\cdot 15j$

$$u_3(x,t)=\frac{-\left[\beta\omega\left(\frac{\rho}{\alpha_3}\right)^{0\cdot 5}\right]\tanh\left[\omega\left(\frac{\rho}{\alpha_3}\right)^{0\cdot 5}.1\right]\cos\left[\omega\left(\frac{\rho}{\alpha_3}\right)^{0\cdot 5}\right](L-x)}{\omega\cos\left[\omega\left(\frac{\rho}{\alpha_3}\right)^{0\cdot 5}L\right]}$$

$$M_{i,j}=u_3(x_i,t_j)$$

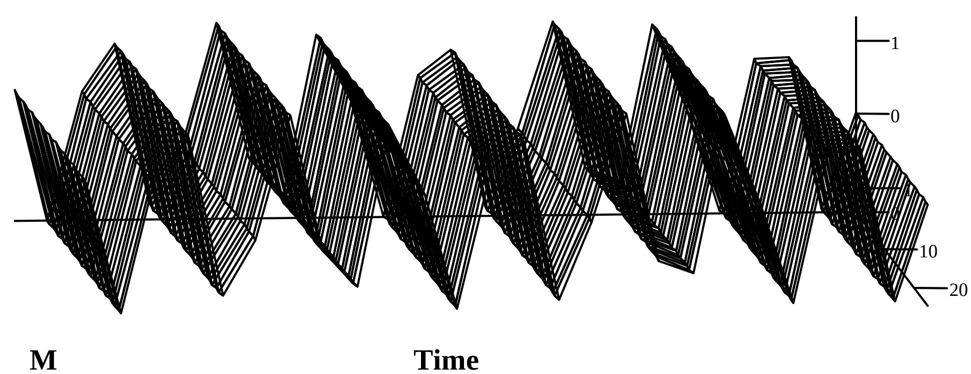


Figure 4: Pattern of membrane deformation

6.5 Net pressure in Cochlea

We demonstrate the pattern of pressure exertion on the Cochlea using the values given below for the variables:

$$\alpha_3 := 2,\ L := 35,\ \rho := 1,\ m_0 := 0.05,\ \omega := 10^4,\ s := 10^4,\ 1 := 1,\ \beta := 1,\ N := 25,\ i := 0,\cdots,N,$$
$$j := 0,\cdots,N,\ x_i := 1\cdot 5 + 0\cdot 5,\ it_j := 1\cdot 5 + 0\cdot 15 j$$

$$p(x,t) := \frac{-2\rho\omega\sin(\omega,t)\cos\left[\omega\sqrt{\frac{\rho}{\alpha_3}}(L-x)\right]}{\cos\left(\omega\sqrt{\frac{\rho}{\alpha_3}}L\right)} \qquad M_{i,j} = u_3(x_i, t_j)$$

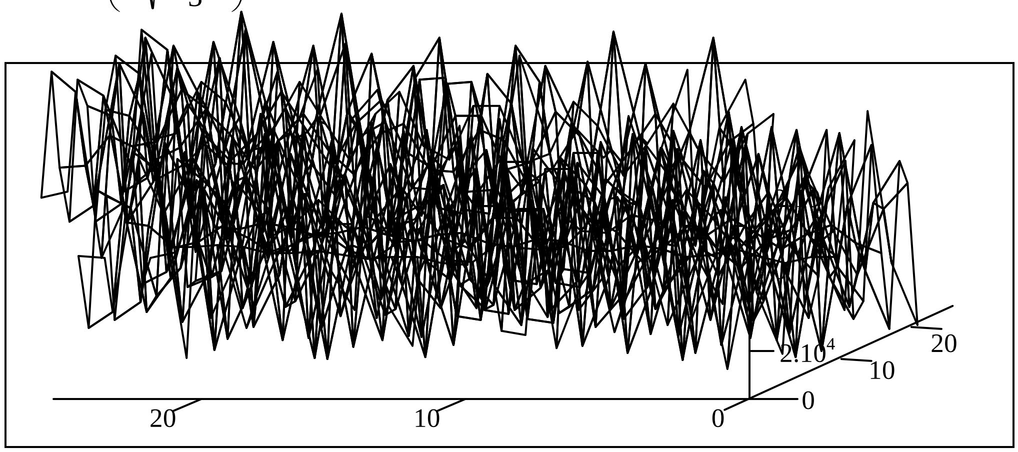


Figure 5: Pattern of pressure exertion on the Cochlea

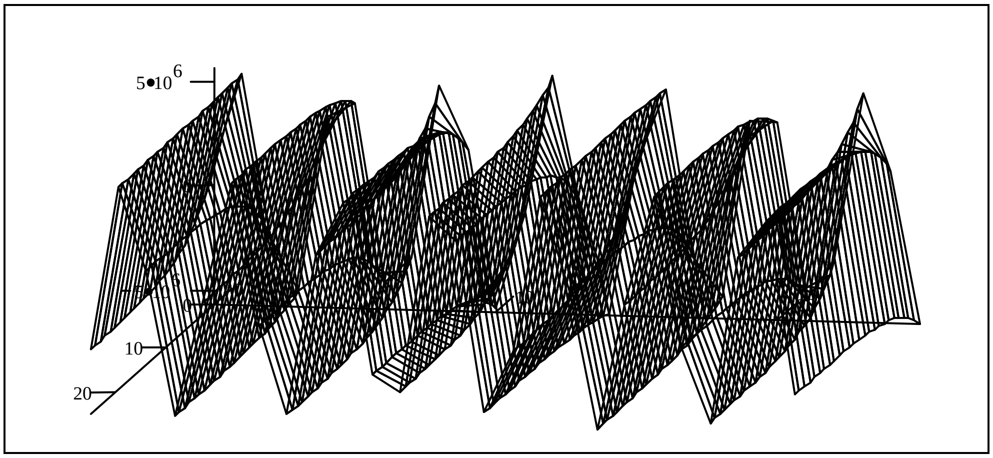
6.6 Displacement at the oval window

Using the values for the variables as given below, we obtain the graph for the displacement at the Oval window over time as:

$$\alpha_3 := 2,\ L := 35,\ \rho := 1,\ m_0 := 0.05,\ \omega := 10^4,\ s := 10^4,\ 1 := 1,\ \beta := 1,\ N := 25,\ i := 0,\cdots,N,$$
$$j := 0,\cdots,N,\ x_i := 1\cdot 5 + 0\cdot 5,\ it_j := 1\cdot 5 + 0\cdot 15 j$$

$$\xi_1(x,t) := \frac{-\left(\frac{\rho s^2}{\alpha_3}\right)^{0\cdot 5}\sin\left[\left(\frac{\rho s^2}{\alpha_3}\right)^{0\cdot 5}L\right]\left[(3000\exp(-1\cdot 5x)\sin(\omega,t)) + \left(m_0\,\omega - \frac{10^7\exp(-1\cdot 5x)}{\omega}\right)\cos(\omega,t)\right]}{\rho\cos\left[\left(\frac{\rho s^2}{\alpha_3}\right)^{0\cdot 5}L\right]}$$

$$Y_{i,j} = \xi_1(x_i, t_j)$$



Y **Time**

Figure 6: Pattern of displacement at the oval window over a period of time

6.7 Displacement at the round window

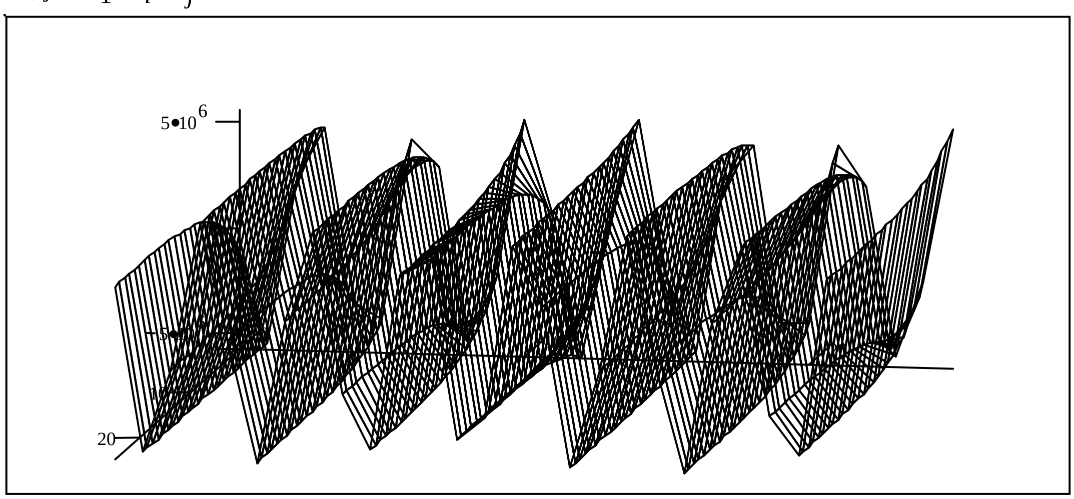
Using the values for the variables as given below, we obtain the graph for the displacement at the round window over time as:

$$\alpha_3 := 2,\ L := 35,\ \rho := 1,\ m_0 := 0.05,\ \omega := 10^4,\ s := 10^4,\ 1 := 1,\ \beta := 1,\ N := 25,\ i := 0, \cdots, N,$$

$$j := 0, \cdots, N,\ x_i := 1\cdot 5 + 0\cdot 5,\ it_j := 1\cdot 5 + 0\cdot 15 j$$

$$\xi_1(x,t) := \frac{-\left(\frac{\rho s^2}{\alpha_3}\right)^{0\cdot 5} \sin\left[\left(\frac{\rho s^2}{\alpha_3}\right)^{0\cdot 5} L\right]\left[(3000\exp(-1\cdot 5x)\sin(\omega,t)) + \left(m_0\,\omega - \frac{10^7\exp(-1\cdot 5x)}{\omega}\right)\cos(\omega,t)\right]}{\rho\cos\left[\left(\frac{\rho s^2}{\alpha_3}\right)^{0\cdot 5} L\right]}$$

$$Y_{i,j} = \xi_1(x_i, t_j)$$



Y Time

Figure 7: Pattern of displacement at the round window over a period of time.

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