Journal of the Nigerian Association of Mathematical Physics
Volume 11 (November 2007), 549 - 556

Second law analysis of a reacting temperature dependent viscous flow through an inclined channel with isothermal walls.

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Abstract

In this paper, entropy generation during the flow of a reacting viscous fluid through an inclined Channel with isothermal walls are investigated. The coupled energy and momentum equations were solved numerically. Previous results in literature* (Adesanya et al 2006 [[17]) *showed both velocity and temperature have two solutions. We compute the entropy generation for each set of solutions. Tables and figures feature prominently.

1.0 Introduction

The study of a viscous flow through an inclined channel has many applications of technological significance; the improvement in thermal systems as well as energy utilization during the convection in any fluid is one of the fundamental problems of the engineering processes. Since implored thermal systems will provide better material processing, energy conservation and environmental effects Makinde (2004 [7]). Entropy generation is the measure of the destruction of the available work of a system and the second law analysis is one of the methods used for predicting the performance of the engineering processes. It is applied to investigate the irreversibility in terms of entropy generation rate which is important in upgrading system performance. The method was introduced by Bejan (1980, 1994, 1996 [1, 2, 3]), and later by Mahmud and Franser (2002 [5]), Makinde (2004 [7]), Makinde and Osalusi (2005 [8]), Narusawa (2001 [9]), Sahin (2002 [10]), Saoolin and Aiboud-Saouli (2004 [14]).

The latter investigated second law analysis of laminar falling liquid film along inclined heated plates they considered the upper surface of the liquid film free and adiabatic and the lower wall is fixed with constant heat flux. Recently Makinde and Gbolagade (2005 [16]) investigated the second law analysis of incompressible viscous flow through an inclined channel with isothermal walls by analytical method of solution, they assume that viscosity and density are constant and their result shows that the heat transfer irreversibility dominates along the channel centerline while an increase in Group Parameter may cause fluid friction irreversibility to dominate near the channel heated walls.

In this present study, we are interested in the steady state of a reacting flow of fluid whose viscosity depends on temperature in a channel; we also want to investigate the rate of entropy generation in the channel.

2.0 Mathematical model

Let us consider an inclined channel with fluid flowing steadily downstream in the + *x* – direction the plates are assumed to extend with gap h in the $\pm - x$ direction.

2.1 Momentum equation

$$\frac{d}{dy}\left(\mu \frac{du}{dy}\right) + \rho g \sin\theta = 0 \qquad (2.1)$$

2.2 Energy Equation

$$\frac{d}{dy}\left(k \frac{dT}{dy}\right) + Qe^{-\frac{E}{RT}} = 0 \qquad (2.2)$$

2.3 Entropy Generation (Makinde and Gbolagade 2005 [16])

$$E_G = \frac{K}{T_0^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T_0}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2.3)$$

The no-slip condition implies that $u(\pm h) = 0$ (2.4)

we also assume an inlet condition $T(-h) = T_0 = T(h) = 0$ (2.5)

where u = flow in horizontal direction, $\theta -$angle inclination to horizontal direction, T – absolute temperature, k – the thermal conductivity, μ – fluid dynamic viscosity, ρ – fluid density, c_p – specific heat capacity, E – Activation energy, R – universal gas constant, g – gravitational force, Q - heat per unit mass during reaction.

we also assume that:

$$\mu = \mu_0 e^{\alpha(T - T_0)} \qquad (2.6)$$

we now introduce the following dimensionless parameters

$$\bar{y} = \frac{y}{h}, \quad \theta = \frac{E(T - T_0)}{RT_0^2}, \quad \phi = \frac{u\mu_0}{\rho g h^2 \sin\theta} \qquad (2.7)$$

where *u*-is the axial flow velocity,

y – is the co-ordinate across flow,

h – is the distance between plate,

T_1 and T_o – are wall temperature.

The dimensionless equations together with appropriate initial and Boundary conditions after dropping ((-)) can be written as

$$\frac{d}{dy}\left(e^{\lambda\theta} \frac{d\phi}{dy}\right) + 1 = 0 \qquad (2.8)$$

$$\phi(-1) = 0 = \phi(1) \qquad (2.9)$$

$$\frac{d^2\theta}{dy^2} + fe^{\frac{\theta}{1+\in\theta}} = 0 \qquad (2.10)$$

$$\theta(-1) = 0 = \theta(1) \qquad (2.11)$$

$$N = \left(\frac{d\theta}{dy}\right)^2 + ae^{\lambda\theta}\left(\frac{d\phi}{dy}\right)^2 \qquad (2.12)$$

where

$$f=\frac{QEh^2e^{\frac{E}{RT_0}}}{RkT_0^2}, \in=\frac{RT_0}{E}, a=\frac{\mu_0\rho^2g^2h^4\sin^2\theta}{T_0k\in^2}, b=\alpha\in T_0, N=\frac{E_G}{K\in^2}$$

We assume that $\in=\frac{RT_0}{E}$ is negligible. Equation (2.11) subject to (2.10) Solved by Adesanya et al (2006 [17]), when f = 0.5, gives two solutions

$$e^{\frac{\theta}{2}}=e^{0.156}\sec h\, 0.58y \qquad (2.13)$$

and

$$e^{\frac{\theta}{2}}=e^{1.438}\sec h\, 2.11y \qquad (2.14)$$

Setting $\lambda=\frac{1}{2}$ and using (2.13) in equation (2.8), we obtain

$$\frac{d}{dy}\left(e^{0.156}\sec h\, 0.58y\frac{d\phi}{dy}\right)+1=0 \qquad (2.15)$$

Integrating

$$e^{0.156}\sec h\, 0.58y\frac{d\phi}{dy}+y+c=0 \qquad (2.16)$$

where c is a constant of integration. Integrating again using maple soft ware, we have

$$\phi_1=-1.475102053y\sinh 0.58y+2.543279\cosh 0.58y-2.078845504 \qquad (2.17)$$

$$\phi(-1)=0=\phi(1)$$

The computation is in Table 1. And repeating the procedure for equation (2.14), we obtain

$$\phi_1=-0.1125128381y\sinh 2.11y+0.5332361995\cosh 2.11y-0.2340506389 \qquad (2.18)$$

$$\phi(-1)=0=\phi(1)$$

The computation is in Table 2

3.0 Entropy generation

The dimensionless entropy generation number may be defined by

$$N=(1.16)^2a\ \tanh^2 0.58y+\frac{yb}{e^{0.156}\sec h\, 0.58y} \qquad (3.1)$$

The computation is in Table 3 and the combine graph is Figure 2.

$$N=(4.22)^2a\ \tanh^2 2.11y+\frac{yb}{e^{1.438}\sec h\, 2.11y} \qquad (3.2)$$

The computation is in Table 4 and the combine graph is Figure 2.

Here $\frac{N_f}{N_y}$ gives the ratio of heat transfer entropy to fluid friction entropy. When $0\le\Phi<1$ means that the heat transfer entropy generation dominates the irreversibility ratio and $\Phi>1$ implies fluid friction dominates, $\Phi=1$ implies that both heat transfer and fluid friction have equal contribution to entropy generation to irreversibility ratio.

4.0 Discussion of result

Figure 1 agrees with previously obtained result by Adesanya et al in (2006 [17]). Like the temperature field, the velocity has two solutions Figure 2 shows that the entropy generation is minimum at $y = 0$ for each pair of solutions. Table 5 shows that that heat transfer entropy generation dominates the irreversibility ratio at higher temperature and fluid friction dominates at lower temperature.

Table 1: Velocity profile for θ_1

y	ysinh 0.58y	cosh 0.58y	c	vel θ1
-1	-0.904341	2.9831864	-2.078846	-6.42E-10
-0.9	-0.724907	2.8977207	-2.078846	0.09396864
-0.8	-0.567418	2.8220057	-2.078846	0.17574187
-0.7	-0.430837	2.7557866	-2.078846	0.24610457
-0.6	-0.314256	2.6988405	-2.078846	0.30573925
-0.5	-0.2169	2.6509759	-2.078846	0.35522995
-0.4	-0.138121	2.6120317	-2.078846	0.39506541
-0.3	-0.077389	2.5818768	-2.078846	0.42564184
-0.2	-0.034299	2.5604098	-2.078846	0.44726511
-0.1	-0.00856	2.5475584	-2.078846	0.4601525
0	0	2.5432794	-2.078846	0.4644339
0.1	-0.00856	2.5475584	-2.078846	0.4601525
0.2	-0.034299	2.5604098	-2.078846	0.44726511
0.3	-0.077389	2.5818768	-2.078846	0.42564184
0.4	-0.138121	2.6120317	-2.078846	0.39506541
0.5	-0.2169	2.6509759	-2.078846	0.35522995
0.6	-0.314256	2.6988405	-2.078846	0.30573925
0.7	-0.430837	2.7557866	-2.078846	0.24610457
0.8	-0.567418	2.8220057	-2.078846	0.17574187
0.9	-0.724907	2.8977207	-2.078846	0.09396864
1	-0.904341	2.9831864	-2.078846	-6.42E-10

Table 2: Velocity profile for θ_2

y	ysinh 2.11y	cosh 2.11y	c	vel θ2
-1	-0.45719611	0.2231455	0.234051	-2.89264E-09
-0.9	-0.33059334	0.1820716	0.234051	0.085528938
-0.8	-0.23509619	0.1491339	0.234051	0.148088389
-0.7	-0.16348259	0.1228605	0.234051	0.193428557
-0.6	-0.11019554	0.1020773	0.234051	0.225932368
-0.5	-0.07098949	0.0858555	0.234051	0.248916645
-0.4	-0.0426573	0.0734703	0.234051	0.264863634
-0.3	-0.02282191	0.0643682	0.234051	0.275596953
-0.2	-0.00978045	0.0581425	0.234051	0.282412729
-0.1	-0.00239168	0.054515	0.234051	0.286174004
0	0	0.0533236	0.234051	0.287374259
0.1	-0.00239168	0.054515	0.234051	0.286174004
0.2	-0.00978045	0.0581425	0.234051	0.282412729
0.3	-0.02282191	0.0643682	0.234051	0.275596953
0.4	-0.0426573	0.0734703	0.234051	0.264863634
0.5	-0.07098949	0.0858555	0.234051	0.248916645
0.6	-0.11019554	0.1020773	0.234051	0.225932368
0.7	-0.16348259	0.1228605	0.234051	0.193428557
0.8	-0.23509619	0.1491339	0.234051	0.148088389
0.9	-0.33059334	0.1820716	0.234051	0.085528938
1	-0.45719611	0.2231455	0.234051	-2.89264E-09

Table 3: Values N_y, N_f for (θ_1, ϕ_1)

y	TANH 0.58Y	Ny	sech 0.58y	Nf
-1	-0.52266543	0.36759	0.8525379	1.0035439
-0.9	-0.47924214	0.309048	0.8776827	0.7895825
-0.8	-0.43333871	0.25268	0.9012311	0.6075665
-0.7	-0.38507106	0.199525	0.9228869	0.4542528
-0.6	-0.33460065	0.15065	0.94236	0.3268404
-0.5	-0.28213481	0.10711	0.9593748	0.2229471
-0.4	-0.22792531	0.069904	0.9736786	0.14059
-0.3	-0.172265	0.039931	0.9850506	0.0781689
-0.2	-0.11548249	0.017945	0.9933095	0.0344529
-0.1	-0.05793505	0.004516	0.9983204	0.00857
0	0	0	1	0
0.1	0.05793505	0.004516	0.9983204	0.00857
0.2	0.11548249	0.017945	0.9933095	0.0344529
0.3	0.172265	0.039931	0.9850506	0.0781689
0.4	0.22792531	0.069904	0.9736786	0.14059
0.5	0.28213481	0.10711	0.9593748	0.2229471
0.6	0.33460065	0.15065	0.94236	0.3268404
0.7	0.38507106	0.199525	0.9228869	0.4542528
0.8	0.43333871	0.25268	0.9012311	0.6075665
0.9	0.47924214	0.309048	0.8776827	0.7895825
1	0.52266543	0.36759	0.8525379	1.0035439

Table 4: Values N_y, N_f for (θ_2, ϕ_2)

y	TANH 2.11Y	Ny	sech 2.11y	Nf
-1	-0.9710286	16.79148	0.2389635	0.9934659
-0.9	-0.9561518	16.28091	0.2928716	0.6565869
-0.8	-0.933892	15.53167	0.3575552	0.4249339
-0.7	-0.9009044	14.45381	0.4340176	0.2680238
-0.6	-0.8527098	12.94874	0.5223849	0.163605
-0.5	-0.7837427	10.93886	0.6210857	0.0955593
-0.4	-0.6879219	8.427586	0.7257847	0.0523355
-0.3	-0.5601145	5.586998	0.8284152	0.0257916
-0.2	-0.398614	2.829632	0.9171188	0.0103543
-0.1	-0.2079235	0.769896	0.9781451	0.0024271
0	0	0	1	0
0.1	0.20792347	0.769896	0.9781451	0.0024271
0.2	0.39861399	2.829632	0.9171188	0.0103543
0.3	0.56011449	5.586998	0.8284152	0.0257916
0.4	0.68792192	8.427586	0.7257847	0.0523355
0.5	0.78374267	10.93886	0.6210857	0.0955593
0.6	0.85270983	12.94874	0.5223849	0.163605
0.7	0.9009044	14.45381	0.4340176	0.2680238
0.8	0.933892	15.53167	0.3575552	0.4249339
0.9	0.95615177	16.28091	0.2928716	0.6565869
1	0.97102855	16.79148	0.2389635	0.9934659

Table 5: Values for $N_f \ / \ N_f$ for (θ_1, ϕ_1)

y	Nf/Ny for (Φ1,θ1)	Nf/Ny for (Φ2,θ2)
-1	2.73006409	0.059164892
-0.9	2.554886128	0.040328648
-0.8	2.404489606	0.027359186
-0.7	2.276669107	0.018543469
-0.6	2.169532554	0.012634819
-0.5	2.081480967	0.008735769
-0.4	2.011190951	0.006210027
-0.3	1.957599896	0.004616368
-0.2	1.919893879	0.003659224
-0.1	1.897498247	0.003152458
0	> 0	< 0
0.1	1.897498247	0.003152458
0.2	1.919893879	0.003659224
0.3	1.957599896	0.004616368
0.4	2.011190951	0.006210027
0.5	2.081480967	0.008735769
0.6	2.169532554	0.012634819
0.7	2.276669107	0.018543469
0.8	2.404489606	0.027359186
0.9	2.554886128	0.040328648
1	2.73006409	0.059164892

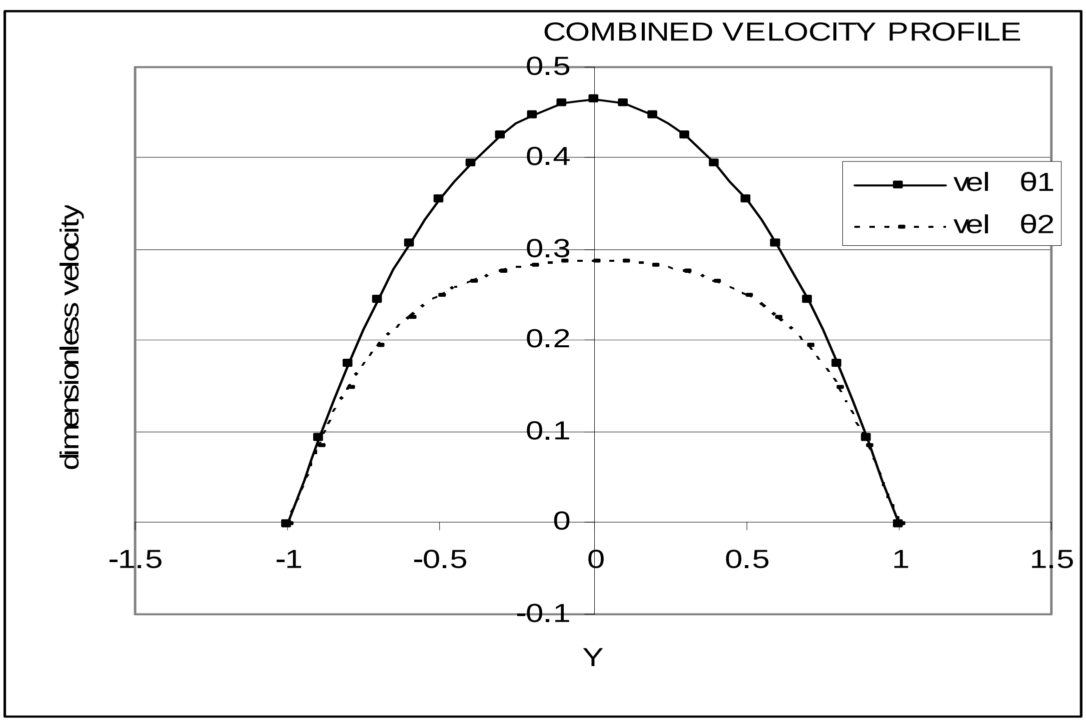

Figure 1

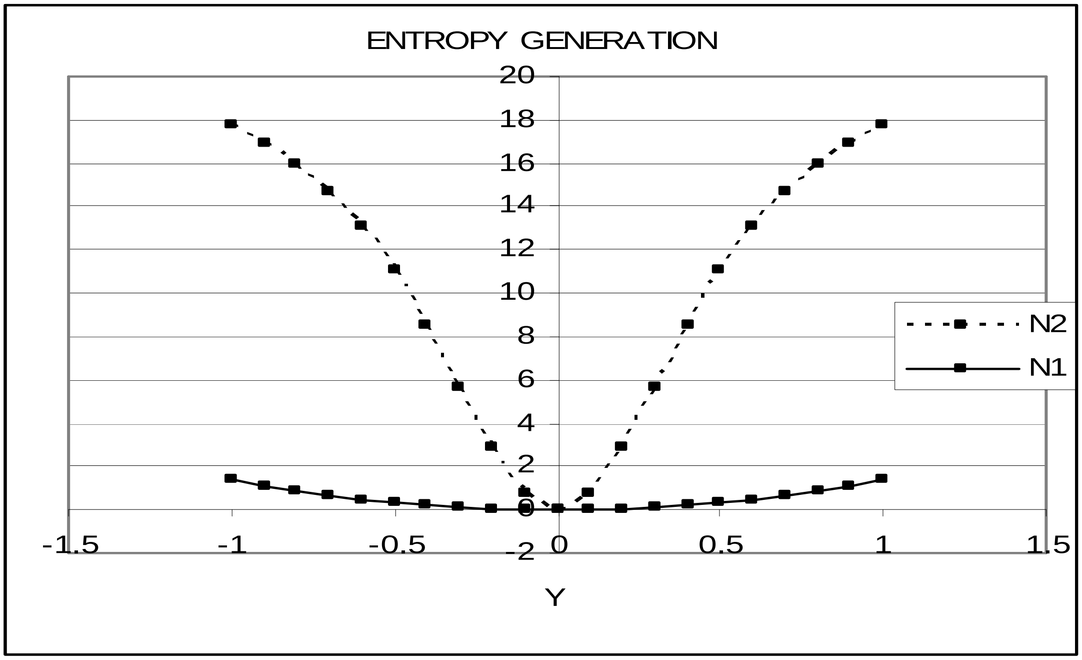

Figure 2

5.0 Conclusion

Here we have considered fluid whose viscosity increases with temperature. For fluids, for example engine oil, where viscosity decreases with temperature a similar phenomenon is expected. However some fluids depends on both temperature and pressure (e.g. lubricants at high pressure). We shall consider such fluid in another paper.

Nomenclature

U - axial velocity,
t - Time,
T - fluid temperature,
p - Pressure,
g - gravitational force,
k - Thermal conductivity,
$\mu -$ Fluid dynamic viscosity,
$\rho -$ Fluid density,
Q - Heat per unit mass during reaction,
R - Universal gas constant,
E - Activation energy.
N_x – Entropy generation by heat transfer due to axial conduction
N_y – Entropy generation by heat transfer due to transverse heat conduction.
N_f – Entropy generation due to fluid friction.
y – is the co-ordinate across flow,
h – is the distance between plate,
T_1 and T_o – are wall temperature.

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