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Heat and mass transfer in the unsteady hydromagnetic free-convection flow in a rotating binary fluid I

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Abstract

The paper studies the unsteady free-convection flow near a moving infinite flat plate in a rotating binary mixture of an incompressible fluid. Both Soret (thermal diffusion), Dufour (diffusion-thermo) and radiation effects are considered when there is no chemical reaction. By imposing a time dependent perturbation on the constant plate temperature and concentration and assuming a differential approximation for the radiative flux, the coupled non linear problem is solved for the temperature and the concentration. First a critical value for the Soret was determined as* 0.10 *and the effects of Dufour, Soret and radiation show that while both Dufour and Soret have no effect on the temperature field, they both affect the concentration field with the Dufour causing an overwhelming increase and the Soret just a slight decrease. Furthermore radiation decreases both the temperature and concentration field.

1.0 Introduction

The study of heat and mass transfer to unsteady free-convection hydromagnetic rotating flows have been carried out because of its vast application in MHD power generators and hall accelerators [1], in re-entry problems [2], in astrophysics, meteorology and engineering [3-8]. For example Bestman, Alabraba and Ogulu [2] investigated the fully developed hydromagnetic flow of a slightly rarefied gas with radiative heat and mass transfer in a vertical channel as a model for space shuttle re-entry. Recently Israel-Cookey and Alagoa [4] analyzed the effects of magnetic field, radiation, free convection and frequency on the rotating boundary layer flow. Also the three authors Alabraba, Bestman and Ogulu in two other papers [6,7] studied the hydromagetic thermally radiating flow of a binary mixture with attendant Dufour and Soret effects associated with mass transfer. The analysis was carried out with the fluid chemically inert [6] and then chemically reacting [7].

In another study Bestman and Adjepong [3] tackled the problem of three dimensional MHD free convection flow with radiative heat transfer past an infinite moving plate in a rotating incompressible viscous and optically transparent medium by making fairly realistic assumption. Due to high temperatures involved, its application in astrophysical studies cannot be overemphasized. What this research ignored was mass transfer with its attendant Dufour and Soret effects.

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This paper therefore complements the work of Bestman and Adjepong [3] by incorporating mass transfer in a hydrogen-air mixture as a non chemical reacting fluid pair.

Nomenclature

(u', v')	dimensional velocity components
(x', y', z')	dimensional Cartesian coordinates
k	thermal conductivity
g	gravitational acceleration
c_p	specific heat at constant pressure
D_m	mass diffusivity
T'	dimensional temperature
C'	dimensional concentration
T_∞	reservoir temperature
C_∞	reservoir concentration
T_w	constant plate temperature
C_w	constant plate concentration
T_m	mean temperature
$q'_{z'}$	radiative heat flux
q	complex velocity
k_B	Boltzmann constant
$H_0'^2$	constant transverse magnetic field
k_T	thermodiffusion constant
$k_r'^2$	constant associated with chemical reaction in the Arrhenius term
Pr	Prandtl number
R	radiation parameter
D_f	Dufour parameter

$i = \sqrt{-1}$	
E	rotation parameter
M^2	magnetic parameter
G_r	free convection parameter due to temperature
D_m	mass diffusivity
S_f	Soret parameter
Sc	Schmidt's number
Greek symbols	
σ_c	electrical conductivity
μ	magnetic permeability
υ	kinematic viscosity
β	coeff. of volume expansion for temp.
ζ	coeff. of volume expansion for conc.
ε	small parameter
η	constant exponent in the Arrhenius term
χ	concentration susceptibility
ε'	dimensional activation energy
$\bar{\varepsilon}$	dimensionless activation energy
Ω	plate angular velocity
ρ_∞	reservoir density
α	absorption coefficient
σ	Stefan Boltzmann constant

2.0 Mathematical formulation

The physical model for three dimensional incompressible unsteady flow past an infinite vertical heated flat plate which moves in its own plane along the positive x' direction with velocity U_0 and rotates about the z'-axis with angular velocity Ω as shown in Figure 1. The plate temperature and concentration are maintained at $T_w[1+\varepsilon f(t')]$ and $C_w[1+\varepsilon f(t')]$ respectively in which T_w the constant plate temperature is high enough to provoke radiative heat transfer, $f(t')$ is an arbitrary function of time which for this problem will be taken as a Heaviside step function $f(t') = H(t')$.

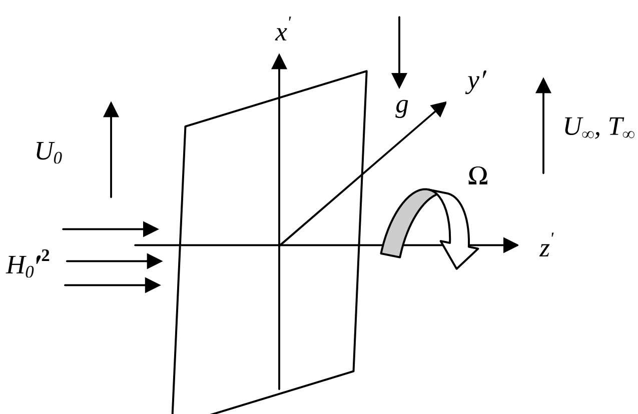

Figure 1: The physical model and coordinate system

A constant transverse magnetic field is applied in the z' direction and under the usual Boussinesq approximation [9], the basic equations governing the physics of the problem following the argument of Tokis [5] and Alabraba [6] are

$$\frac{\partial u'}{\partial t'} - 2\Omega v' = \upsilon \frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma_c \mu^2 H_0'^2 u'}{\rho_\infty} + g\beta(T' - T_\infty) + g\zeta(C' - C_\infty) \quad (2.1)$$

$$\frac{\partial v'}{\partial t'} + 2\Omega u' = \upsilon \frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma_c \mu^2 H_0'^2 v'}{\rho_\infty} \quad (2.2)$$

$$\rho_\infty c_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z'^2} - \nabla q'_{z'} + \frac{D_m k_T}{\chi c_p} \frac{\partial^2 C'}{\partial z'^2} \quad (2.3)$$

$$\frac{\partial C'}{\partial t'} = D_m \frac{\partial^2 C'}{\partial z'^2} - k_r'^2 T'^\eta \exp\left(-\frac{\varepsilon'}{k_B T'}\right) C' + \frac{D_m k_T}{T_m} \frac{\partial^2 T'}{\partial z'^2} \quad (2.4)$$

$$\frac{\partial^2 q'_{z'}}{\partial z'^2} - 3\alpha^2 q'_{z'} - 16\alpha\sigma T'^3 \frac{\partial T'}{\partial z'} = 0 \quad (2.5)$$

where on $z' = 0$: $u' = U_0$, $v' = 0$, $T' = T_w[1 + \varepsilon f(t')]$, $C' = C_w[1 + \varepsilon f(t')]$

$z' \to \infty$: $u' = v' = 0$, $T' = T_\infty$, $C' = C_\infty$ (2.6a,b)

Equation (2.5) is the differential approximation for radiation in one space coordinate z'. For optically thin medium with relatively low density where $\alpha \ll 1$, Equation (2.5) in the spirit of Bestman et al [3] becomes

$$\frac{\partial q'_{z'}}{\partial z'} = 4\sigma\alpha\left(T'^4 - T_\infty^4\right) \quad (2.7)$$

Equations (2.1) - (2.4) subject to equations (2.6) and (2.7) become

$$\frac{\partial q}{\partial t} + i2Eq = \frac{\partial^2 q}{\partial z^2} - M^2 q + Gr(\theta - 1) + Gc(C - 1) \quad (2.8)$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} - R\Pr(\theta^4 - 1) + D_f \frac{\partial^2 C}{\partial z^2} \quad (2.9)$$

$$Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial z^2} - k_r^2 \theta^\eta \exp\left(-\frac{\bar{\varepsilon}}{\theta}\right) C + S_f \frac{\partial^2 \theta}{\partial z^2} \quad (2.10)$$

where we have introduced the following dimensionless quantities

$$t = \frac{t' U_0}{\upsilon}, z = \frac{z' U_0}{\upsilon}, (u, v) = \frac{(u', v')}{U_0}, (\theta, \theta_w) = \frac{(T', T_w)}{T_\infty}, (C, C_w) = \frac{(C', C_w)}{C_\infty}, E = \frac{\Omega \upsilon}{U_0^2},$$

$$M^2 = \frac{\sigma_c \mu^2 H_0'^2 \upsilon}{\rho_\infty U_0^2}, Gr = \frac{g\beta T_\infty \upsilon}{U_0^3}, Gc = \frac{g\zeta C_\infty \upsilon}{U_0^3}, \Pr = \frac{\rho_\infty c_p \upsilon}{k}, R = \frac{4\sigma\alpha\upsilon T_\infty^3}{\rho_\infty c_p U_0^2}, D_f = \frac{D_m k_T C_\infty}{\chi c_p T_\infty k},$$

$$\bar{\varepsilon} = \frac{\varepsilon'}{k_B T_\infty}, Sc = \frac{\upsilon}{D_m}, k_r^2 = \frac{k_r'^2 T_\infty^\eta \upsilon^2}{D_m U_0^2}, S_f = \frac{k_T T_\infty}{C_\infty T_m}$$

Equations (2.8) - (2.10) are subject to the boundary condition

$z = 0$: $q = 1$, $\theta = \theta_w[1 + \varepsilon H(t)]$, $C = C_w[1 + \varepsilon H(t)]$

$z \to \infty$: $q = 0$, $\theta = 1$, $C = 1$ (2.11a,b)

$q = u + iv$

The heat and mass transfer problem therefore entails the solution of equations (2.9) and (2.10) subject to equation (2.11).

3.0 Method of solution

Equations (2.9) and (2.10) are highly nonlinear and so will involve a step by step numerical integration. However if ε in equation (2.11) is small, we can advance analytical solution by adopting regular perturbation expansion of the form

$$q = q^{(0)}(z) + \varepsilon q^{(1)}(z,t) + \ldots$$
$$\theta = \theta^{(0)}(z) + \varepsilon\theta^{(1)}(z,t) + \ldots \qquad (3.1)$$
$$C = C^{(0)}(z) + \varepsilon C^{(1)}(z,t)+\ldots$$

Substituting equation (3.1) in (2.8) - (2.10), ignoring $0(\varepsilon^2)$ and simplifying, we get the zero order equations as

$$2iEq^{(0)} = \frac{d^2q^{(0)}}{dz^2} - M^2q^{(0)} + Gr\left(\theta^{(0)} - 1\right) + Gc\left(C^{(0)} - 1\right) \qquad (3.2)$$

$$0 = \frac{d^2\theta^{(0)}}{dz^2} - R.\Pr\left(\theta^{(0)4} - 1\right) + D_f \frac{d^2C^{(0)}}{dz^2} \qquad (3.3)$$

$$0 = \frac{d^2C^{(0)}}{dz^2} - k_r{}^2 \exp\left(-\frac{\bar{\varepsilon}}{\theta^{(0)}}\right)\theta^{(0)\eta}C^{(0)} + S_f \frac{d^2\theta^{(0)}}{dz^2} \qquad (3.4)$$

$$z = 0: q^{(0)} = 1,\ \theta^{(0)} = \theta_w,\ C^{(0)} = C_w$$
$$z\to\infty: q^{(0)} = 0,\ \theta^{(0)} = 1,\ C^{(0)} = 1 \qquad (3.5a,b)$$

and the first order equations as

$$\frac{\partial q^{(1)}}{\partial t} + 2iEq^{(1)} = \frac{\partial^2 q^{(10}}{\partial z^2} - M^2q^{(1)} + Gr\theta^{(1)} + GcC^{(1)} \qquad (3.6)$$

$$\Pr\frac{\partial\theta^{(1)}}{\partial t} = \frac{\partial^2\theta^{(1)}}{\partial z^2} - 4.R.\Pr\theta^{(0)3}\theta^{(1)} + D_f\frac{\partial^2C^{(1)}}{\partial z^2} \qquad (3.7)$$

$$Sc\frac{\partial C^{(1)}}{\partial t} = \frac{\partial^2 C^{(1)}}{\partial z^2} - k_r{}^2\exp\left(-\frac{\bar{\varepsilon}}{\theta^{(0)}}\right)\left\{\theta^{(0)\eta-2}\theta^{(1)}C^{(0)}\left(\bar{\varepsilon} + \eta\theta^{(0)}\right) + \theta^{(0)\eta}C^{(1)}\right\} + S_f\frac{\partial^2\theta^{(1)}}{\partial z^2} \qquad (3.8)$$

$$\left.\begin{array}{l} z = 0: q^{(1)} = 0, \theta^{(1)} = \theta_w, C^{(1)} = C_w \\ z\to\infty: q^{(1)} = \theta^{(1)} = C^{(1)} = 0 \end{array}\right\} t \succ 0$$

$$t = 0: q^{(1)} = \theta^{(1)} = C^{(1)} = 0,\ z \succ 0, \qquad (3.9a,b,c)$$

Thus by asymptotic expansion for the flow velocity, temperature and concentration, the problem is split into a steady flow on which is superimposed a first order transient component.

In the absence of chemical reaction equation (3.4) is rewritten as

$$\frac{d^2C^{(0)}}{dz^2} = -S_f\frac{d^2\theta^{(0)}}{dz^2} \qquad (3.10)$$

which when combined with equations (3.3) and (3.5) give the solution for $\theta^{(0)}$ as

$$z = \sqrt{\frac{5(1 - D_fS_f)}{2R\Pr}}\int_{\theta^{(0)}}^{\theta_w}\frac{d\zeta}{\sqrt{\left(\zeta^5 - 5\zeta + 4\right)}} \qquad (3.11)$$

Also integrating equation (3.10) twice and combining with equation (3.5) gives the solution for $C^{(0)}$ as

$$C^{(0)} = -S_f\theta^{(0)} + \left(C_w + S_f\theta_w\right) \qquad (3.12a)$$

Imposing the boundary condition equation (3.5b) on the last equation gives an expression for C_w as

$$C_w = 1 + S_f - S_f\theta_w \tag{3.12b}$$

The solution for $q^{(0)}$ can therefore be got from equation (3.2) as

$$q^{(0)} = \exp\left\{-\left(M^2+2iE\right)^{\frac{1}{2}} z\right\} - \left\{\frac{5(1-D_f S_f)}{2R\Pr}\right\}^{\frac{1}{2}} \times$$
$$\times \int_{\theta^{(0)}}^{\theta_w} \frac{\sinh\left\{\left(M^2+2iE\right)^{\frac{1}{2}}\left[z\left(\theta^{(0)}\right)-z(\zeta)\right]\right\}\left[Gr(\zeta-1)+Gc\left(-S_f\zeta+C_w+S_f\theta_w-1\right)\right]}{\left(\zeta^5-5\zeta+4\right)^{\frac{1}{2}}} d\zeta \tag{3.13}$$

To solve equations (3.6) - (3.8) we take Laplace transform with respect to time, representing the transformed variable by s and placing a bar over the transformed function, the equations satisfied by $\overline{q}^{(1)}$, $\overline{\theta}^{(1)}$ and $\overline{C}^{(1)}$ are

$$\frac{d^2\overline{q}^{(1)}}{dz^2} - \left(M^2+2iE\right)\overline{q}^{(1)} = \left(Gr\overline{\theta}^{(1)} + Gc\overline{C}^{(1)}\right) \tag{3.14}$$

$$\frac{d^2\overline{\theta}^{(1)}}{dz^2} - \Pr\left(4R\theta^{(0)3}+s\right)\overline{\theta}^{(1)} + D_f\frac{d^2\overline{C}^{(1)}}{dz^2} = 0 \tag{3.15}$$

$$Scs\overline{C}^{(1)} = \frac{d^2\overline{C}^{(1)}}{dz^2} + S_f\frac{d^2\overline{\theta}^{(1)}}{dz^2} \tag{3.16}$$

$$\left.\begin{aligned} &z=0: \overline{\theta}^{(1)} = \frac{\theta_w}{s}, \overline{C}^{(1)} = \frac{C_w}{s}, \overline{q}^{(1)} = 0 \\ &z\to\infty: \overline{\theta}^{(1)} = \overline{C}^{(1)} = \overline{q}^{(1)} = 0 \end{aligned}\right\} \tag{3.17a, b}$$

We first consider solution for $\overline{\theta}^{(1)}$ which is possible if $\theta_w - 1$ is of order 0(1), that is the difference between T_w and T_∞ is small, then $\theta^{(0)} = 1$ and the equations (3.15) and (3.16) give rise to a quartic equation in $\overline{\theta}^{(1)}$ as

$$aD^4\overline{\theta}^{(1)} - bD^2\overline{\theta}^{(1)} + c\overline{\theta}^{(1)} = 0$$

with solution as

$$\overline{\theta}^{(1)} = A_1^{(+)}e^{\omega_1 z} + A_1^{(-)}e^{-\omega_1 z} + A_2^{(+)}e^{\omega_2 z} + A_2^{(-)}e^{-\omega_2 z} \tag{3.18}$$

where $\omega^2_{1,2} = \dfrac{b\pm\sqrt{b^2-4ac}}{2a}$ which translates to

$$\left.\begin{aligned} \omega_1^2 &= \gamma_1 s + \left(\gamma_2 s^2 + \gamma_3 s + \gamma_4\right)^{\frac{1}{2}} + \gamma_5 \\ \omega_2^2 &= \gamma_1 s - \left(\gamma_2 s^2 + \gamma_3 s + \gamma_4\right)^{\frac{1}{2}} + \gamma_5 \end{aligned}\right\} \tag{3.19}$$

We can write $\left(\gamma_2 s^2 + \gamma_3 s + \gamma_4\right)^{\frac{1}{2}} = \left[(\sqrt{\gamma_2}s + \sqrt{\gamma_4})^2\right]^{\frac{1}{2}} = \sqrt{\gamma_2}s + \sqrt{\gamma_4}$ provided Sc*D_f* S_f << (Pr + Sc). Equation (3.19) can then be reduced to the form

$$\omega_1^2 = R_1 s + R_2 \text{ and } \omega_2^2 = R_3 s \tag{3.20}$$

From equation (3.15) we have $D^2\overline{C}^{(1)} = \frac{1}{D_f}\left\{\Pr(4R+s) - D^2\right\}\overline{\theta}^{(1)}$

$$= \Omega_1\left(A_1^{(+)}e^{\omega_1 z} + A_1^{(-)}e^{-\omega_1 z}\right) + \Omega_2\left(A_2^{(+)}e^{\omega_2 z} + A_2^{(-)}e^{-\omega_2 z}\right)$$

where $\Omega_j = \frac{\left\{\Pr(4R+s) - \omega_j^2\right\}}{D_f} = \frac{\gamma + \Pr s - \omega_j^2}{D_f}$ and can be reduced by substituting γ and equation (3.20) to

$$\left.\begin{aligned}\Omega_1 &= R_4 s + R_5\\ \Omega_2 &= R_6 s + R_7\end{aligned}\right\} \tag{3.21}$$

Integrating the $\overline{C}^{(1)}$ equation twice and without loss of generality we get

$$\overline{C}^{(1)} = \frac{\Omega_1}{\omega_1^2}\left(A_1^{(+)}e^{\omega_1 z} + A_1^{(-)}e^{-\omega_1 z}\right) + \frac{\Omega_2}{\omega_2^2}\left(A_2^{(+)}e^{\omega_2 z} + A_2^{(-)}e^{-\omega_2 z}\right) \tag{3.22}$$

Equation (3.18) subject to equation (3.17) gives

$$\left.\begin{aligned}A_1^{(+)} = A_2^{(+)} &= 0\\ A_2^{(-)} &= \frac{\theta_w}{s} - A_1^{(-)}\end{aligned}\right\} \tag{3.23a,b}$$

Equation (3.22) subject to equation (3.17) and equation (3.23) gives

$$A_1^{(-)} = \frac{\left(\frac{C_w}{s} - \frac{\Omega_2}{\omega_2^2}\frac{\theta_w}{s}\right)}{\left(\frac{\Omega_1}{\omega_1^2} - \frac{\Omega_2}{\omega_2^2}\right)} \tag{3.24}$$

Substituting equations (3.22a,b) and (3.24) in equations (3.18) and (3.22) results in

$$\overline{\theta}^{(1)} = \frac{\omega_1^2\left(\omega_2^2 C_w - \Omega_2\theta_w\right)}{s\left(\Omega_1\omega_2^2 - \Omega_2\omega_1^2\right)}e^{-\omega_1 z} + \frac{\omega_2^2\left(\Omega_1\theta_w - \omega_1^2 C_w\right)}{s\left(\Omega_1\omega_2^2 - \Omega_2\omega_1^2\right)}e^{-\omega_2 z}$$

and

$$\overline{C}^{(1)} = \frac{\Omega_1\left(\omega_2^2 C_w - \Omega_2\theta_w\right)}{s\left(\Omega_1\omega_2^2 - \Omega_2\omega_1^2\right)}e^{-\omega_1 z} + \frac{\Omega_2\left(\Omega_1\theta_w - \omega_1^2 C_w\right)}{s\left(\Omega_1\omega_2^2 - \Omega_2\omega_1^2\right)}e^{-\omega_2 z}$$

These two equations after substituting equation (3.20) and equation (3.21) and applying partial fraction give

$$\overline{\theta}^{(1)} = \left(\frac{\beta_1}{s} + \frac{\beta_2}{s+a_3} + \frac{\beta_3}{(s+a_3)^2}\right)e^{-k_x\sqrt{(s+a_1)}} - \left(\frac{\beta_4}{s+a_3} + \frac{\beta_5}{(s+a_3)^2}\right)e^{-k_y\sqrt{s}} \tag{3.25}$$

$$\overline{C}^{(1)} = \left(\frac{\chi_1}{s} + \frac{\chi_2}{s+a_3} + \frac{\chi_3}{(s+a_3)^2}\right)e^{-k_x\sqrt{s+a_1}} - \left(\frac{\chi_4}{s} + \frac{\chi_5}{s+a_3} + \frac{\chi_6}{(s+a_3)^2}\right)e^{-k_y\sqrt{s}} \tag{3.26}$$

By the first shifting theorem and inverse Laplace transform we deduce $\theta^{(1)}$ and $C^{(1)}$ as

$$\theta^{(1)} = e^{-a_1 t}\left\{\beta_1 L^{-1}\left(\frac{e^{-k_X\sqrt{s}}}{s-a_1}\right) + \beta_2 L^{-1}\left(\frac{e^{-k_X\sqrt{s}}}{s-a_4}\right) + \beta_3 L^{-1}\left(\frac{e^{-k_X\sqrt{s}}}{(s-a_4)^2}\right)\right\} -$$

$$-\left\{\beta_4 L^{-1}\left(\frac{e^{-k_y\sqrt{s}}}{s+a_3}\right)+\beta_5 L^{-1}\left(\frac{e^{-k_y\sqrt{s}}}{(s+a_3)^2}\right)\right\} \tag{3.27}$$

$$C^{(1)}=e^{-a_1 t}\left\{\chi_1 L^{-1}\left(\frac{e^{-k_x\sqrt{s}}}{s-a_1}\right)+\chi_2 L^{-1}\left(\frac{e^{-k_x\sqrt{s}}}{s-a_4}\right)+\chi_3 L^{-1}\left(\frac{e^{-k_x\sqrt{s}}}{(s-a_4)^2}\right)\right\}- \\ -\left\{\chi_4 L^{-1}\left(\frac{e^{-k_y\sqrt{s}}}{s}\right)+\chi_5 L^{-1}\left(\frac{e^{-k_y\sqrt{s}}}{s+a_3}\right)+\chi_6 L^{-1}\left(\frac{e^{-k_y\sqrt{s}}}{(s+a_3)^2}\right)\right\} \tag{3.28}$$

such that L^{-1} denotes the inverse Laplace transform. We can show that by writing

$$\left.\begin{array}{l}\frac{1}{s-a_1}=\frac{1}{s-a_4}=\frac{1}{s\pm a_3}=\frac{1}{s} \\ \frac{1}{(s\pm a_3)^2}=\frac{1}{(s-a_4)^2}=\frac{1}{s^2}\end{array}\right\}, s\to\infty$$

Equations (3.27) and (3.28) become

$$\left.\begin{array}{l}\theta^{(1)}=e^{-a_1 t}\left\{\beta_1 erfc\left(\frac{k_x}{2\sqrt{t}}\right)+\beta_2 erfc\left(\frac{k_x}{2\sqrt{t}}\right)-4t\beta_3 erfc\left(\frac{k_x}{2\sqrt{t}}\right)\right\}- \\ -\beta_4 erfc\left(\frac{k_y}{2\sqrt{t}}\right)+4t\beta_5 erfc\left(\frac{k_y}{2\sqrt{t}}\right) \\ C^{(1)}=e^{-a_1 t}\left\{\chi_1 erfc\left(\frac{k_x}{2\sqrt{t}}\right)+\chi_2 erfc\left(\frac{k_x}{2\sqrt{t}}\right)-4t\chi_3 erfc\left(\frac{k_x}{2\sqrt{t}}\right)\right\}- \\ -\chi_4 erfc\left(\frac{k_y}{2\sqrt{t}}\right)-\chi_5 erfc\left(\frac{k_y}{2\sqrt{t}}\right)+4t\chi_6 erfc\left(\frac{k_y}{2\sqrt{t}}\right)\end{array}\right\},.t\to 0^+ \tag{3.29}$$

Also substituting $\xi^2=s+M^2+2iE$ in equation (3.14), we get

$$\frac{d^2\bar{q}^{(1)}}{dz^2}-\xi^2\bar{q}^{(1)}=-\left(Gr\bar{\theta}^{(1)}+Gc\bar{C}^{(1)}\right)$$ This equation with Wronskian as 2ξ gives the solution as $\bar{q}^{(1)}=\frac{1}{2}\int_0^z\frac{e^{-\xi(z-\bar{z})}}{\xi}\left(Gr\bar{\theta}^{(1)}+Gc\bar{C}^{(1)}\right)d\bar{z}$

$$=\frac{1}{2}\left\{Gr\int_0^z\frac{e^{-\xi(z-\bar{z})}}{\xi}\bar{\theta}^{(1)}d\bar{z}+Gc\int_0^z\frac{e^{-\xi(z-\bar{z})}}{\xi}\bar{C}^{(1)}d\bar{z}\right\}$$

By applying the shifting theorem and later convolution gives

$$q^{(1)}=\frac{Gr}{2\sqrt{\pi}}\int_0^z d\bar{z}\int_0^t\frac{1}{\sqrt{\tau}}e^{-\left(M^2+2iE\right)\tau}e^{-\frac{(z-\bar{z})^2}{4\tau}}\theta^{(1)}(t-\tau)d\tau \\ +\frac{Gc}{2\sqrt{\pi}}\int_0^z d\bar{z}\int_0^t\frac{1}{\sqrt{\tau}}e^{-\left(M^2+2iE\right)\tau}e^{-\frac{(z-\bar{z})^2}{4\tau}}C^{(1)}(t-\tau)d\tau \tag{3.30}$$

The solution is now complete. ■

4.0 Results and discussion

Asymptotic solutions have been obtained for the temperature and concentration fields of the problem of three dimensional unsteady MHD free convection flow near a moving infinite vertical flat plate in a rotating hydrogen-air mixture as a non chemical reacting fluid pair. By invoking the differential approximation for the radiative flux in the optically thin limit, the non linear problem is tackled by asymptotic approximation resulting to a steady flow on which is superimposed a first order transient flow.

We have used for the numerical computation Pr = 0.71, Sc = 2.0, R = 1.0 and 2.0, $\theta_w = 10$, t = 0.01, ε = 0.1. Equations (3.11), (3.12a) and (3.13) gives solution for the steady state component of temperature $\theta^{(0)}$, concentration $C^{(0)}$ and velocity $q^{(0)}$ with $\theta^{(0)}$ evaluated by numerical integration. Figure 2 shows the influence of D_f, S_f on the concentration and also determines an acceptable range for S_f. The result shows that the concentration is overwhelmingly sensitive to Dufour than to Soret. The acceptable limit of the Soret is 0.1, hence we have chosen a range of D_f and S_f to be between 0.01 to 0.1.

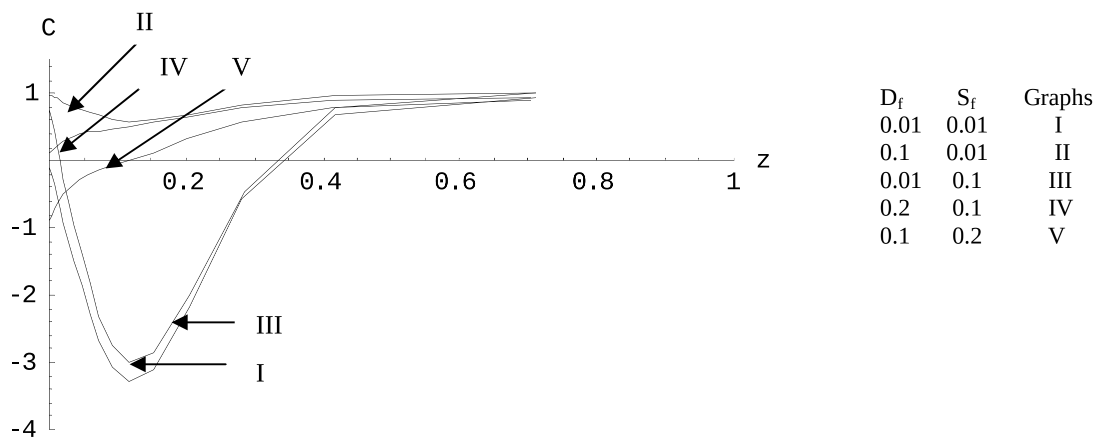

Figure 2: Concentration profile against boundary layer z.

In Figure 3 the temperature profile is depicted for various values of D_f, S_f and R. The result show that D_f and S_f have no effect on the temperature while increase in R causes a decrease in

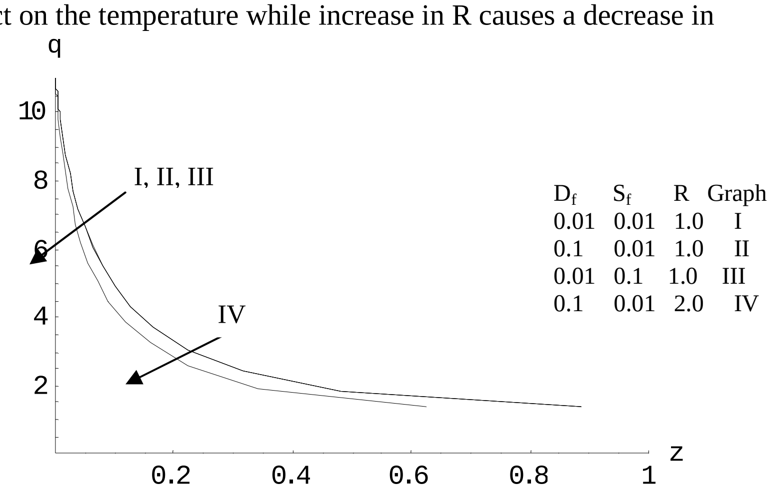

Figure 3: Temperature profile θ against boundary layer z for different D_f , S_f and R

temperature which is in good agreement with the result of Israel-Cookey [10].

Finally in Figure 4 we also notice that increase in R like the temperature causes a decrease in the concentration field.

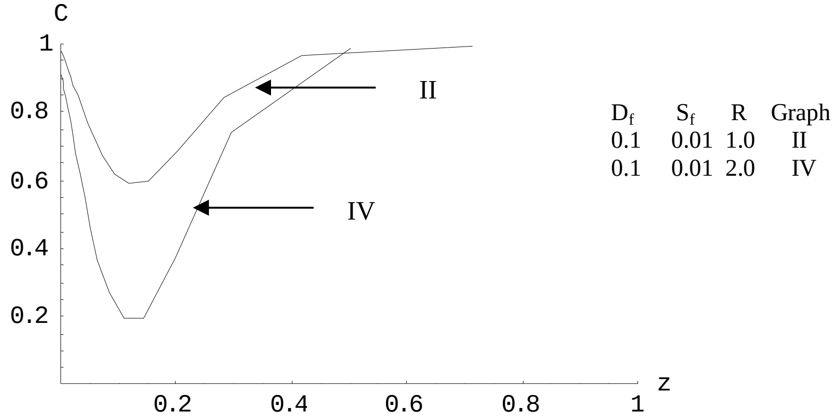

Figure 4: Concentration profile C against boundary layer z for different R

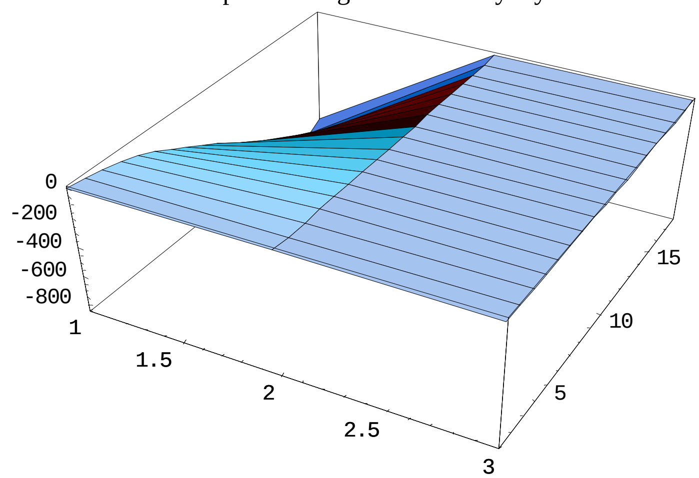

Figure 5: Three-dimensional plot of Concentration *C* against boundary layer thickness z and time *t* for $D_f = S_f = 0.01$, Sc= 2.0, R=1.0, $\theta_w = 10.0$

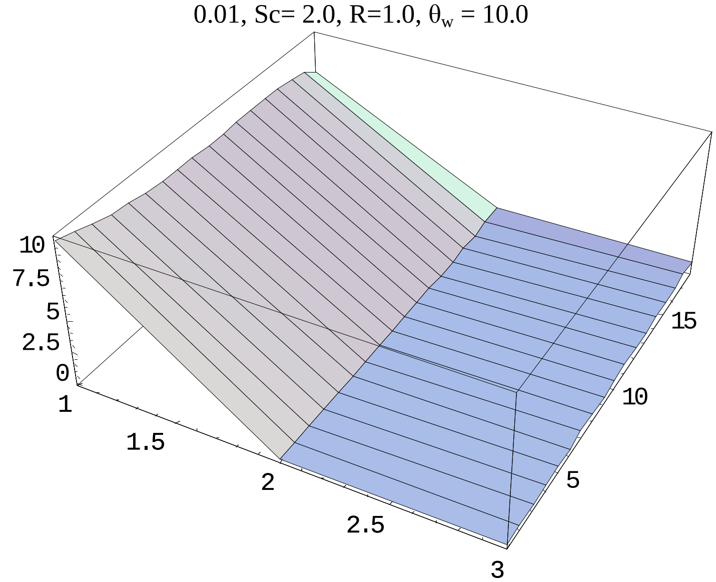

Figure 6: Three-dimensional plot of Temperature θ against boundary layer thickness z and time t for $D_f = S_f = 0.01$, Sc= 2.0, R=1.0, $\theta_w = 10.0$

5.0 Conclusions

While for fully developed two dimensional laminar flow of electrically conducting binary fluid in a vertical channel [6], the following holds.

(i) The concentration field (C) follows the same pattern as the temperature field (θ) and increase in D_f and S_f cause a corresponding increase in C and θ.

(ii) Increase in radiation parameter cause a corresponding increase in θ and C.

For the unsteady three dimensional flow near a moving infinite vertical plate in a rotating binary fluid, the following is obtained.

(i) Though the flow patterns for the transient component equation (3.29) follow the same patterns for θ and C, the steady state component equations (3.11) and (3.12a) are in opposite direction. D_f and S_f have no effect on the temperature field but has effect on the concentration field with the Dufour being overwhelming.

(ii) Increase in radiation parameter causes a decrease in the temperature and concentration fields.

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Appendix A

The following constants have been used

$a = 1 - D_f S_f$

$b = Scs + Pr(4R + s)$

$c = PrSc(4Rs + s^2)$

$\gamma = 4PrR$

$$\gamma_1 = \frac{\Pr + Sc}{2(1 - D_f S_f)}$$

$$\gamma_2 = \gamma_1^2 - \frac{\Pr Sc}{1 - D_f S_f}$$

$$\gamma_3 = \frac{(\gamma_1 - Sc)\gamma}{1 - D_f S_f}$$

$$\gamma_4 = \left(\frac{\gamma}{2(1 - D_f S_f)}\right)^2$$

$$\gamma_5 = \sqrt{\gamma_4}$$

$$R_1 = \gamma_1 + \sqrt{\gamma_2}$$

$$R_2 = \gamma_5 + \sqrt{\gamma_4}$$

$$R_3 = \gamma_1 - \sqrt{\gamma_2}$$

$$R_4 = \frac{\Pr - R_1}{D_f}$$

$$k_x = \sqrt{R_1}\, z$$

$$k_y = \sqrt{R_3}\, z$$

$$N_1 = R_1(R_6\theta_w - R_3 C_w)$$

$$N_2 = (R_2 R_6 + R_1 R_7)\theta_w - R_2 R_3 C_w$$

$$N_3 = R_2 R_7 \theta_w$$

$$N_4 = R_3(R_4\theta_w - R_1 C_w)$$

$$N_5 = R_3(R_5\theta_w - R_2 C_w)$$

$$Q_1 = R_4(R_6\theta_w - R_3 C_w)$$

$$Q_2 = (R_5 R_6 + R_4 R_7)\theta_w - R_3 R_5 C_w$$

$$Q_3 = R_5 R_7 \theta_w$$

$$Q_4 = R_6(R_4\theta_w - R_1 C_w)$$

$$Q_5 = (R_4 R_7 + R_5 R_6)\theta_w - (R_1 R_7 + R_2 R_6)C_w$$

$$R_5 = \frac{\gamma - R_2}{D_f}$$

$$R_6 = \frac{\Pr - R_3}{D_f}$$

$$R_7 = \frac{\gamma}{D_f}$$

$$a_1 = \frac{R_2}{R_1}$$

$$a_3 = \sqrt{\frac{\lambda_2}{\lambda_1}}$$

$$a_4 = a_1 - a_3$$

$$\lambda_1 = R_1 R_6 - R_3 R_4$$

$$\lambda_2 = R_2 R_7$$

$$\lambda_3 = 2\sqrt{\lambda_1 \lambda_2}$$

$$\chi_4 = \frac{Q_6}{\lambda_2}$$

$$\chi_6 = \frac{Q_5}{\lambda_1} - \frac{2Q_6}{\lambda_3} - \frac{Q_4 a_3}{\lambda_1}$$

$$Q_6 = R_7\left(R_5\theta_w - R_2 C_w\right)$$

$$\beta_1 = \frac{N_3}{\lambda_2} = \theta_w$$

$$\beta_2 = \frac{N_1}{\lambda_1} - \beta_1$$

$$\beta_3 = \frac{N_2}{\lambda_1} - \frac{2N_3}{\lambda_3} - \frac{N_1 a_3}{\lambda_1}$$

$$\beta_4 = \frac{N_4}{\lambda_1}$$

$$\beta_5 = \frac{N_5}{\lambda_1} - \frac{N_4 a_3}{\lambda_1}$$

$$\chi_1 = \frac{Q_3}{\lambda_2}$$

$$\chi_2 = \frac{Q_1}{\lambda_1} - \frac{Q_3}{\lambda_2}$$

$$\chi_3 = \frac{Q_2}{\lambda_1} - \frac{2Q_3}{\lambda_3} - \frac{Q_1 a_3}{\lambda_1}$$

$$\chi_5 = \frac{Q_4}{\lambda_1} - \frac{Q_6}{\lambda_2}$$

References

[1] P.C. Ram, S.S. Singh, R.K. Jain, Heat and mass transfer of a viscous heat generating fluid with hall current, Astrophys. Space Sci. 168 (1990) 209 – 216.

[2] A. R. Bestman , M. A. Alabraba , A. Ogulu , Radiative heat transfer to hydromagnetic flow of a slightly rarefied binary gas in a vertical channel , Astrophys. Space Sci. 189 (1992) 303 – 308.

[3] A. R. Bestman , S. K. Adjepong , Unsteady hydromagnetic free-convection flow with radiative heat transfer in a rotating fluid , Astrophys. Space Sci. 143 (1988) 73 – 80.

[4] C. Israel Cookey, K. D. Alagoa, Effects of magnetic field, radiation and free convection on the unsteady boundary layer M.H.D flow past a heated porous plate in a rotating fluid with time dependent suction, AMSE Modell., Simul. Control B 72 (3) (2003) 13 – 24.

[5] J. N. Tokis Unsteady three dimensional free-convection flow near an infinite vertical plate moving in a rotating fluid in the presence of a transverse magnetic field, Astrophys. Space Sci. 119 (1986) 305 – 313.

[6] M. A. Alabraba, A. R. Bestman, A. Ogulu, Laminar convection in binary mixture of hydromagnetic flow with radiative heat transfer I , Astrophys. Space Sci. 195 (1992) 431 – 439.

[7] A. Ogulu, A. R. Bestman, M. A. Alabraba , Laminar convection in binary mixture of hydromagnetic flow with radiative heat transfer II , Astrophys. Space Sci. 195 (1992) 441 – 445.

[8] C. Israel-Cookey , P. Mebine , A Ogulu , MHD free-convection and mass transfer flow on a porous medium in a rotating fluid due to radiative heat transfer , AMSE Modell. , Simul. Control B 7 (1) (2002) 1 – 7.

[9] A. Ogulu, E. Amos , Asymptotic approximation for the flow field in a free- convective flow of a non-Newtonian fluid past a vertical porous plate , Int. Comm. Heat Mass Transfer 23 (2005) 974 – 982.

[10] C. Israel-Cookey, A. Ogulu, V. B. Omubo-Pepple , Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction , Int. J. Heat Mass Transfer 46 (2003)