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Solving microwave heating model in a slab using shooting technique

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Abstract

We employ shooting technique to explicitly construct the approximate solution of steady state reaction – diffusion equations with source term that arise in modeling microwave heating in an infinite slab with isothermal walls. In particular, we consider the case where the source term decreases spartially and increases with temperature. The important properties of the temperature fields and thermal criticality are discussed.

Keywords: Microwave heating model; shooting technique; thermal critically.

1.0 Introduction

Microwaves are a form of electromagnetic radiation; that is, they are waves of electrical and magnetic energy moving together through space. Electromagnetic radiation ranges from the energetic X-ray to the less energetic radio frequency waves used in broadcasting. Microwave fall into the radio frequency band of electromagnetic radiation. Microwaves have three characteristics that allow them to be use in cooking: they are reflected by metal; they pass through glass, paper, plastic, and similar materials; and they are absorbed by goods, **Hill** and Marchant [4]. This technology has found new applications in many industrial processes, such as those involving melting, smelting, sintering, dying, and joining, kriegsmann [8]. For example, an electric kiln was retrofit for a feasibility firing study of an ultra low weight refractory brick and **clay** slab, Itaya et al [4]. The dynamically control microwave drying technology was chosen due to the heating characteristics of the brick and the need for uniform shrinkage (see Makinde [7]).

Heating by microwave radiation constitutes a reaction diffusion problem with radiative heat source term and the long – timer behaviour of the solutions in space may lead to appearance of hotspots in the system, that is, isolated regions of excessive heating, coleman [1]. In order to predict the occurrence of such phenomena, it is necessary to analyze a simplified mathematical model from which insight might be gleaned into an inherently complex physical process.

The theory of reaction diffusion equations is quite elaborate and their solution in rectangular, cylindrical and spherical coordinate remains and extremely important problem of practical relevance in the engineering sciences. Several numerical approaches have been developed in the last few decades e.g. finite difference, spectral method, Hermite–padé approximation technique, etc, to tackle this problem [2, 6, 10, 8, 7].

In this paper, we intend to construct approximate solution for a steady state reaction diffusion equation that models microwave heating in an infinite slab with isothermal walls using shooting method with Runge Kutta technique (see Olanrewaju [8]). The work of Makinde [7] motivated this research work whether we can get similar results with different methods of solution which is faster in term of time.

2.0 Mathematical formulation

We now consider a microwave heating in an infinite slab with isothermal walls a shown in Fig. 1. It is assumed that energy dissipation has negligible effect on the electromagnetic field and so that temperature distribution can be and so the temperature distribution can be investigated in isolation. For simplest microwave heating model, the steady state nonlinear reaction diffusion equation with source term that describes the thermal behaviour can be written in the form [3]

$$\alpha \frac{d^2T}{dy^2} + Ee^{-\beta y}T^m = 0 \tag{2.1}$$

with the following boundary conditions

$$\frac{dT}{dy}(0) = \sigma, \qquad T(a) = T_0 \tag{2.2}$$

where T = temperature
T_0 = wall temperature
m = thermal absorptivity index
α = constant thermal conductivity of the material
σ = positive constant
E = amplitude of the incident radiation
β = electric field amplitude decay rate
(x, y) = distances measured in the axial and normal direction
a = slab half width.

Remark: it is important to note that the value of m depends upon; the material under consideration, the temperature ranges under investigation and the frequencies (say 10^{10} HZ), material such as fused glass are very accurately approximated by linear and quadratic power (m = 1, 2). At lower frequencies, higher values of thermal absorptivity index m > 2 will be required [5].

Introducing the following dimensionless variables into equations (2.1) and (2.2):

$$\lambda = \frac{Ea^2T_0^{m-1}}{\alpha}, \; k = \alpha\beta, \; y = \frac{\bar{y}}{a}, \; \bar{T} = \frac{T}{T_0} \tag{2.3}$$

We obtain the dimensionless governing equation together with the corresponding boundary conditions as (neglecting the bar symbol for clarity):

$$\left.\begin{aligned} &\frac{d^2T}{dy^2} + \lambda e^{-ky}T^m = 0 \\ &\text{with} \\ &\frac{\mathrm{dT}}{\mathrm{dy}}(0) = \sigma, \; T(1) = 1 \end{aligned}\right\} \tag{2.4}$$

where λ and k represent, the thermal absorptivity and electric field decay rate parameters respectively.

3.0 Method of solution

For m = 1, equation (2.4) becomes a linear boundary value problem and can be easily solved. The exact solution is

$$T(y) = \frac{J_1\left(2\frac{\sqrt{\lambda}}{k}\right) Y_0\left(-\frac{2\sqrt{\lambda}}{k\sqrt{e^{ky}}}\right)}{J_1\left(2\frac{\sqrt{\lambda}}{k}\right) Y_0\left(-2\frac{e^{-\frac{k}{2}\sqrt{\lambda}}}{k}\right) + Y_1\left(-2\frac{\sqrt{\lambda}}{k}\right) J_0\left(2\frac{e^{-\frac{k}{2}\sqrt{\lambda}}}{k}\right)}$$

$$+ \frac{Y_1\left(-2\frac{\sqrt{\lambda}}{k}\right) J_0\left(2\frac{\sqrt{\lambda}}{k\sqrt{e^{ky}}}\right)}{J_1\left(2\frac{\sqrt{\lambda}}{k}\right) Y_0\left(-2\frac{e^{-\frac{k}{2}\sqrt{\lambda}}}{k}\right) + Y_1\left(-2\frac{\sqrt{\lambda}}{k}\right) J_0\left(2\frac{e^{-\frac{k}{2}\sqrt{\lambda}}}{k}\right)} \qquad (3.1)$$

where J_0, J_1 are Bessel functions of first kind and Y_o, Y_1 are Bessel functions of second kind (See Makinde, [7]).

For $n > 1$, the problem becomes nonlinear and it is convenient to use shooting method in order to transform the boundary value problem to an initial value problem by letting

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} y \\ T \\ T' \end{pmatrix} \qquad (3.2)$$

So

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} 1 \\ y_3 \\ -\lambda e^{-ky_1} y_2^m \end{pmatrix} \qquad (3.3)$$

Satisfying

$$\begin{pmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \beta \end{pmatrix} \qquad (3.4$$

Here, we look for the value of β so as to satisfy the condition $T(1) = 1$.

4.0 Results and conclusions

Figure 1 shows the graph of temperature against position y for fixed values of $\lambda = 0.4$ and various values of k. We observed that increase in the electric field decay rate leads to a decrease in temperature and at the same time decrease in the temperature gradient.

Figure 2 shows the curve of temperature against position y for fixed values of k = 0.2 and various values of λ. It is shown that as λ (thermal absorptivity) increases the temperature also increases and also leads to an increase in the temperature gradient.

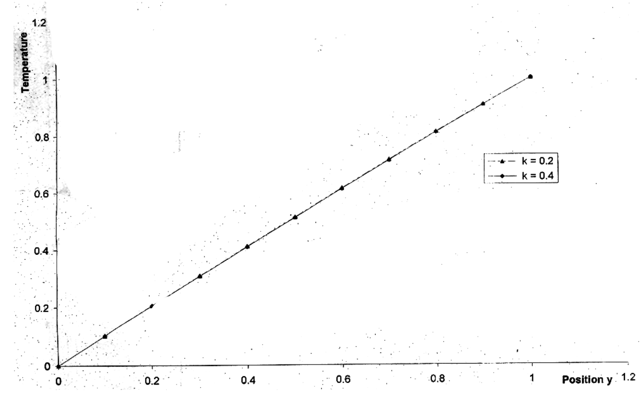

Figure 1: Graph of temperature against position y for fixed values of $k = 0.4$ and various values of lamda.

(where λ = lamda

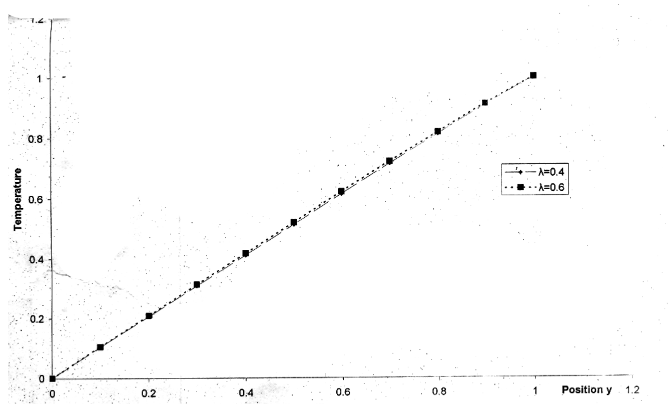

Figure 2: Graph of temperature against position y for fixed values of k = 0.2 and various values of lamda.

5.0 Conclusions

We employed shooting method to solve microwave heating model and we established that the thermal absorptivity and electric field decay rate parameters have significant influence in microwave heting model which agreed with Makinde [7]. It is established that shooting technique can also be used to solve microwave heating model and it is easy and faster. Researcher will look for easier and fastest method to solve any given model.

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