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Effect of viscous damping on the response of a finite beam resting on a tensionless Pasternak Foundation subjected to a harmonic load

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Abstract

In this work we present results for the influence of viscous damping on the response if a finite beam resting on a Pasternak foundation using Galerkin weighted residual method. Results obtained show that the vibration amplitude reduces with increase in the damping term.

1.0 Introduction

Beams supported along their length are very common in structural configurations. This type of beam problem have beam of most interest to numerous researchers. In most studies the foundation is treated as a one parameter (Winkler) mode. The work of Hetenyi (1946) [8] provides a thorough treatment of the Winkler mode for elastic foundation. Ding (1993) [4] presented a general solution to vibration of beams on variable Winkler elastic foundation. Yokoyama (1991) [6] studies the vibration of beam column on a two parameter elastic model using finite element method. Coskun (2000) [2] studies the non-linear vibrations of beam on a non-linear tensionless Winkler foundation. The results of the above authors mostly include the determination of the extent of the lift off regions. Coskun (2003) [1] studied the harmonic vibrations of a finite bean resting on a Pasternak foundation (a two parameter elastic model) but neglected the impact of viscous damping.

In this paper we investigate the impact of viscous damping on the vibration of a finite beam under the action of a harmonic load resting on a Pasternak foundation.

2.0 Problem formulation

Consider a finite beam of length 2L, resting on a tensionless elastic foundation, and subjected to a central concentrated load force $\rho_o \cos\Omega t$ as shown below. Following the work of Coskun (2003) [1] and incorporating the viscous term we have

$$WIE_t^w - K_G W_1^n + (M_b + M_f)\frac{\partial^2 w_1}{\partial t^2} + 2M_b w_b \frac{\partial w_1}{\partial t} + kw_1 = p_o \cos\Omega t \quad 0 < x < L \qquad (2.1)$$

$$K_G \frac{\partial^2 w_2}{\partial x^2} - m_f \frac{\partial^2 w_2}{\partial t^2} - 2M_b w_b k w_1 \frac{\partial w_2}{\partial t} - kw_2 = 0 \quad < x < \infty \qquad (2.2)$$

where K_G = the shear foundation modulus
K = the Winkler foundation modulus

$W_1(x, t)$ = the vertical deflection of the beam axis in $0 < x < L$
$W_2(x, t)$ = the vertical deflection of the beam axis in $< x < \infty$
EI = the beam flexural rigidity
δ = the dirac delta function
P_o = the forcing amplitude
Ω = the forcing frequency
L = the contact length
M_f = the mass of foundation/unit length
M_b = the mass/unit length of the beam
w_b = circular frequency of damping

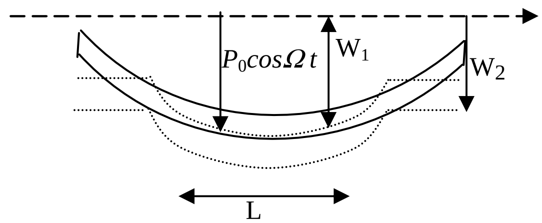


Figure 1: Beam resting on a tensionless pasternale foundation

3.0 Method of solution

Equations (2.1) and (2.2) will be solved using the method of Galerkin weighed residual method (WRM) while the resulting system of initial value problem shall be solved using the laplace transform. Subject to the following initial and boundary conditions:

$$\left.\begin{aligned} W_1(0,t) &= 0 = W_2(0,t) \\ W_1(L,t) &= 0 = W_2(L,0) \\ W_1''(0,t) &= W_2''(0,t) = 0 \\ W_1''(L,t) &= W_2''(L,t) = 0 \\ W_1(x,0) &= W_2(x,0) = V \end{aligned}\right\}_o \tag{3.1}$$

Assuming the solution

$$W_1 = \sum_{j=1}^{\wedge} q_j(t) \sin\ j\frac{\pi x}{L} \tag{3.2}$$

Putting (3.1) into (2.1) and carrying out the necessary differentiation we have

$$\frac{j^4\pi^4}{j^4} EI \sum_{j=1}^{\wedge} q_j(t) \sin\frac{j\pi x}{L} + K\frac{j^2\pi^2}{j^2} \sum_{j=1}^{\wedge} q_j(t)\sin\frac{j\pi x}{L} + (m_b + mf)E \sum_{j=1}^{\wedge} \ddot{q}_j(t)\sin\frac{j\pi x}{L}$$
$$+ 2M_b w_b \sum_{j=1}^{\wedge} \dot{q}_j(t)\sin\frac{j\pi x}{L} + K\sum_{j=1}^{\wedge} q_j(t)\sin\frac{j\pi x}{L} - P_o\cos\Omega t = 0 \tag{3.3}$$

It is required that the residual be orthogonal to the base function. Such that

$$\int_o^L\left[\frac{j^4\pi^4}{L^4} EI \sum_{j=1}^{\wedge} q_j(t)\sin\frac{j\pi x}{L}\sin\frac{j\pi x}{L}\right]dx + \int_o^L\left[K_G\frac{j^2\pi^2}{L^4}\sum_{j=1}^{\wedge} q_j(t)\sin\frac{j\pi x}{L}\sin\frac{j\pi x}{L}\right]dx +$$
$$\int_o^L\left[(m_b - m_f)\sum_{j=1}^{\infty} \ddot{q}_j(t)\sin\frac{j\pi x}{L}\sin\frac{j\pi x}{L}\right]dx + \int_o^L\left[2M_b w_b \sum_{j=1}^{\wedge} \dot{q}_j(t)\sin\frac{j\pi x}{L}\sin\frac{j\pi x}{L}\right]dx +$$

$$\int_o^L K \sum_{j=1}^{\wedge} q_j(t)\sin\frac{j\pi x}{L}\sin\frac{j\pi x}{L}dx - P_o\cos\Omega t\int_o^L \sin\frac{j\pi x}{L}dx = 0 \tag{3.4}$$

Using the fundamental mode of vibration for $n = k = 1$ we

$$\frac{j^4\pi^4}{L^4}EI\cdot\frac{L}{2}q(t) + K_G\frac{j^2\pi^2}{L^4}\cdot\frac{L^2}{2}q(t) + (m_b + m_f)\cdot\frac{L}{2}\ddot{q}(t) + 2M_b w_b\cdot\frac{L}{2}\dot{q}(t) + \frac{KL}{2}q(t)$$
$$-P_o\cos\ \Omega t\left[\frac{L}{\pi}(1-\cos\pi)\right] = 0 \tag{3.5}$$

on simplification we have

$$q(t) + \frac{K_G L^2}{EI\pi^2}q(t) + \frac{\mu L^4}{EI\pi^4}\ddot{q}(t) + \frac{\mu N_b L^4}{EI}\frac{L^4}{\pi^4}\dot{q}(t) + 2\frac{M_b w_b}{EI}\frac{L^4}{\pi^4}\dot{q}(t) + \frac{K_1 L^4}{EI\pi^4}q(t)$$
$$= \frac{2\rho_o L^4}{EI\pi^5}[1-\cos]\cos\ \Omega t \tag{3.6}$$

take $\lambda_a = \frac{K_a L^2}{EI}$ *and* $\lambda = \frac{KL^4}{EI}$, we have

$$\ddot{q}(t) + 2w_b\dot{q}(t) + \frac{K_1\pi^4}{M_b\lambda}\left[1 + \frac{\lambda_o}{\pi^4}\right]q(t) = \frac{2P_o}{\mu\pi}[1-\cos\pi]\cos\Omega t \tag{3.7}$$

Again if $C_o = \frac{K_1\pi^4}{M_b\pi}\left[1 + \frac{\lambda_o}{\pi} + \frac{\lambda}{\pi^4}\right]$

$$d = \frac{2P_o}{M_b\pi}[1-\cos\pi] \tag{3.8}$$

We have $\ddot{q}(t) + 2w_b\dot{q}(t) + c_o q(t) = d\cos\eta t$

We shall now proceed to apply the Laplace transform to obtain

$$S^2 q(s) = 2w_b S + co\ q(s) = \frac{ds}{S^2 + \Omega^2} \tag{3.9}$$

Hence
$$q(s) = d\cdot\frac{S}{S^2 + \Omega^2}\frac{1}{S^2 + 2w_b + co} \tag{3.10}$$

So that the inverse Laplace transform gives

$$q(t) = \frac{d}{co - \Omega^2 + 4wb^2\Omega^2}\left[\frac{co - \Omega^2}{\Omega^2}\sin\ \Omega^2 t - \frac{co - 2w_b^2 - \Omega^2}{\sqrt{co - w_b^2}}\ell^{w_b t}w_b t\right.$$
$$\left. -2w_b\left[\cos w_b t - \ell^{w_b t}\cos\sqrt{co - w_b^2 t}\right]\right] \tag{3.11}$$

$$\therefore \quad W_1(x,t) = q(t)\sin\frac{\pi x}{L} \tag{3.12}$$

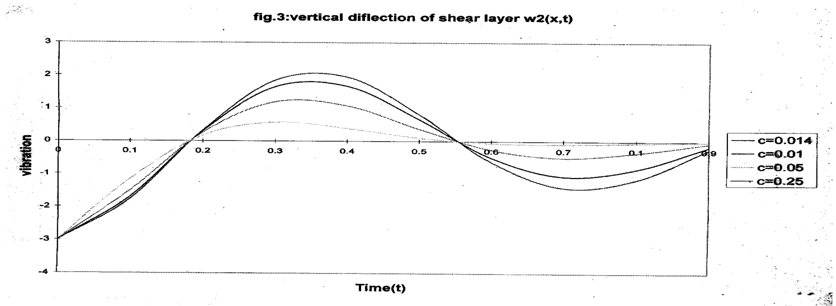
Also following a similar procedure

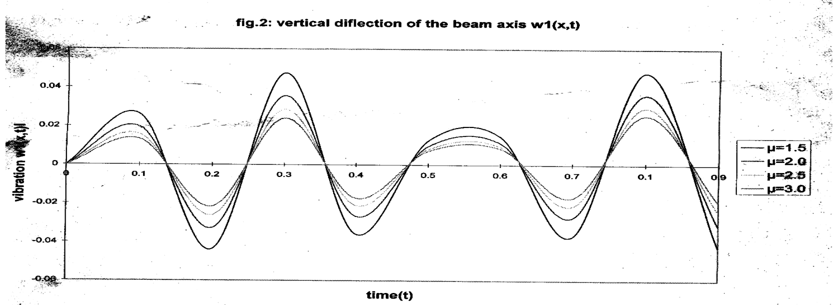
$$W_2(x,t) = \ell\frac{-M_b w_b t}{mf}\left[\frac{V_o M_b w_b}{mf\lambda n}\sin\quad \lambda\Omega t + V_o\cos\ \lambda\Omega t\right]\sin\frac{\pi x}{L} \tag{3.13}$$

where $$\lambda n = \frac{\sqrt{\frac{4M_b^{\,2} w_b^{\,2}}{mf} - 4\left(\frac{K_G \pi^2}{L^2 mf} + \frac{k}{mf}\right)}}{2}$$

4.0 Discussion of results

In figure 2 above we plot a graph of vibration against time for various values of the viscous damping $M_b w_b = \mu$ for $K_1 = 0.1$, $\lambda = 0.5$, $P_i = 180°$, $p_0 = 1.5$, $\Omega = 10$, $X = 0.1$. It can be shown that the viscous damping term reduces the amplitude of vibration of beam increases in the viscous term leads to decrease in the mode of vibration. While in figure 3, we plot the vertical deflection of the shear layer for values of the damping coefficient it is also observed that the damping term reduces the amplitude of vibration.





5.0 Conclusion

A solution has been presented for the effect of viscous damping for the vibration of a finite beam on a tensionless Pasternak foundation subjected to a harmonic load. The solution we obtained using the fundamental mode of vibration using the Galerkin weighted residual method and the resulting initial value problem has been solved using the Laplace transform technique result were also presented for various values of the damping coefficient. Result obtained shows that increase in damping reduces the amplitude of vibrations. It is also observed that the results obtained shows lower amplitude of vibration than that of Coskun (2003) [1] as a result of the viscous damping. The flexural layer has a negative phase shift and it is also observed that there is a phase differences in the mode of vibration of about ¾ of a cycle between the flexural layer and shear layer.

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