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Dynamic Analysis of a non-linear vibrating circular cylindrical shell using the regular perturbation technique

[1]M. Jiya, and [2]Y. M. Aiyesimi
Department of Mathematics and Computer Science
Federal University of Technology, Minna
e-mail: jiyason@yahoo.com. 08029303652

Abstract

We investigated in this paper the effect of non-linear vibration of a circular cylindrical shell subject to axially symmetric loading. We consider the approximation of the equation using the regular perturbation technique and thereby solving the resulting linear equation analytically. The result indicates an exponential decay away from the edge of the shell, which is one of the unique characteristics of a shell. From the numerically simulated results it was observed that increase in the excitation amplitude produces a wrinkling effect on the shell.

1.0 Introduction

The word shell is an old one and is commonly used to describe the hard covering of eggs, crustacean, tortoises etc. The dictionary says that the word shell is derived from the Latin scalus, as in fish scale. But to us now there is a clear difference between the tough but flexible scaly covering of a fish and the tough but rigid shell of, say a turtle. In this paper we will be concerned with man made shell structures as used in branches of science and Engineering. Vol'mir et. al. [7] studied nonlinear oscillations of simply supported, circular cylindrical panels and plates subject to an initial deviation from the equilibrium position (response of the panel to initial conditions) by using Donnell's nonlinear shallow theory. Results were calculated by numerical integration of the equations of motion obtained by Gerlerkin projection, retaining three or five modes in the expansion. Mikhlin,[3] studied vibrations of circular cylindrical shells under a radial excitation and an axial static load, using Donnell's nonlinear shallow-shell theory with Gerlerkin projection and two different mode expansions. Amabili et. al. [1] experimentally studied large amplitude vibrations of a stainless-steel circular cylindrical panel supported at four edges. The nonlinear response to harmonic excitation of different magnitudes in the neighborhood of three resonances was investigated. Experiments showed that the curved panel exhibited a relatively strong geometric nonlinearly of softening type. Nayfeh A. H. [5] used a perturbation technique to reduce the eight-order vibration problem for presstred, clamped cylindrical shells to an equivalent sixth-order membrane problem. In the transformation to a membrane problem composite expansion are utilized, uniformly over the length of the shell, to form modified boundary conditions that account for the effects of bending near the shells. In most of these works one form of loading is considered. We demonstrate in this paper the approximation of the equation using the regular perturbation technique and thereby solving the resulting linear equation analytically. The forms of loading considered are the axially symmetric loading and the axial-inplane stress function as that of an equivalent lateral distributed load.

2.0 Shell model equation

The Donnell's non – linear shallow shell theory, gives the equation for transverse vibration of a very, thin, circular cylindrical shell as

$$D\nabla^4 w + ch\dot{w} + \rho h\ddot{w} = f + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \left(\frac{\partial^2 F}{R^2\partial\theta^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{R\partial x\partial\theta}\frac{\partial^2 w}{R\partial x\partial\theta} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{R^2\partial\theta^2}\right) \qquad (2.1)$$

where $D = -\frac{\text{Eh}^3}{12(\text{I} - 2)}$ is the flexural rigidity, E is the Young's Modulus, ν is the Poisson's ratio, L the shell thickness, R the mean shell radius, ρ the mass density of the shell, c the damping coefficient and f the radial pressures applied to the surface of the shell as a consequences of external forces. The radial deflection w is positive inward $\dot{w} = \frac{\partial w}{\partial t}$, $\ddot{w} = \frac{\partial^2 w}{\partial t^2}$, and F is the in plane stress function; F is given by

$$\frac{1}{Eh}\nabla^4 F = \frac{1}{R}\frac{\partial^2 w}{\partial x^2}\left[\left(\frac{\partial^2 w}{R\partial x\partial\theta}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{R^2\partial\theta^2}\right] \qquad (2.2)$$

where ∇^4 is the bi-harmonic operator

Using Donnell's non linear shallow shell theory, the middle surface strain – displacement relationships are obtained.

$$E_X = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad E_\theta = \frac{\partial v}{R\partial\theta} - \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{R\partial\theta}\right)^2, Y_{x\theta} = \frac{\partial u}{R\partial\theta} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{R\partial\theta} \qquad (2.3)$$

were u, v, w are displacement at the middle surface of the shell.

3.0 Solution technique

The governing equation in cylindrical coordinate is given as

$$\left[\frac{\partial^4 w}{\partial x^4} + \frac{2}{R^2}\frac{\partial^4 w}{\partial x^2\partial\theta^2} + \frac{1}{R^4}\frac{\partial^4 w}{\partial\theta^4}\right] + \text{ch}\dot{w} + \rho\text{h}\ddot{w} = f + \frac{1}{\text{R}}\frac{\partial^2 F}{\partial x^2}$$
$$+\frac{1}{R}\left[\frac{\partial^2 F}{R^2\partial\theta^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{R\partial x\partial\theta}\frac{\partial^2 w}{R\partial x\partial\theta} + \frac{\partial^2 F}{\partial x\partial\theta}\frac{\partial^2 w}{R^2\partial x\partial\theta}\right] \qquad (3.1)$$

Assuming a solution of the form

$$\left.\begin{aligned} w &= W_n(x,t)Sinn\theta \\ F &= F_n(x)Sinn\theta \end{aligned}\right\} \qquad (3.2)$$

and dividing through by $DSinn\theta$ gives

$$W_n^{(iv)} - \frac{2n^2}{R^2}W_n'' + \frac{n_4}{R^4}W_n + \frac{ch}{D}\dot{W}_n + \frac{\rho h}{D}\ddot{W}_n = \frac{f}{DSinn\theta} - \frac{1}{DR}F_n'' + \frac{1}{DR^2} \qquad (3.3)$$
$$\left[-n^2F_nW_n''Sinn\theta\right] - 2n^2F_n'W_n'\frac{Cos^2 n\theta}{Sinn\theta} - n^2F_n''W_nSinn\theta$$

Neglecting the radial pressure i.e. $f = 0$ results in the equation

$$W_n^{iv} - \frac{2n^2}{R^2}W_n'' + \frac{n^4}{R^4}W_n + \frac{ch}{D}\dot{W}_n + \frac{eh}{D}\ddot{W}_n = \frac{1}{DR}F_n''$$

$$+\frac{1}{DR^2}\left[-n^2F_nW_n''\,Sinn\theta-2n^2F_n'W_n'\frac{Cos^2n\theta}{Sinn\theta}-n^2F_n''\,W_nSinn\theta\right] \qquad (3.4)$$

We introduce the following non dimensional quantities: N_0 – prestress, L – unit length, D – bending rigidity and T_0 – Time. Hence

$$W=\frac{1}{N_oL^2}W^*,\;\; F=\frac{N_o}{D}F^* \;\text{ and }\; T_o=\left(\frac{ehL_0^2}{DN_o}\right)^{1/2} \qquad (3.5)$$

We take $\varepsilon=\frac{1}{DR^2}$ to be the perturbed parameter (normalized bending rigidity) where DR^2 >>1 equation (3.4) now becomes

$$W_n^{iv}-2n^2D.\varepsilon.W_n''+\frac{n^4}{R^4}W_n+\frac{ch}{D}\dot{W}_n+\frac{\rho h}{D}\ddot{W}_0=\varepsilon.RF_n''+\varepsilon \qquad (3.6)$$
$$\left[-n^2F_nW_n''\;Sinn\;\theta-2n^2\frac{Cos^2n\theta}{Sinn\;\theta}F_n'W_n'-n^2Sinn\;\theta F_n''\;W_n\right]$$

also substituting eqn. (3.2) into eqn. (2.2) and dividing through by Sinnθ gives

$$F_n^{iv}-\frac{2n^2}{R^2}F_n''+\frac{n^4}{R^4}F_n=-\frac{Eh}{R}W_n''+\frac{Eh}{R^2}\left[\frac{n^2Cos^2n\theta}{Sinn\;\theta}(W_0')^2+n^2Sinn\;\theta W_n''\;W_n\right] \qquad (3.7)$$

also introducing the perturbed parameter into eqn. (3.7) gives

$$F_n^{iv}-2n^2D.\varepsilon.F_n''+\frac{n^4}{R^4}F_n=-EhDR.\varepsilon.W_n''+DEh.\varepsilon.\left[\frac{n^2Cos^2n\theta}{Sinn\theta}(W_n')^2+n^2Sinn\theta W_n''W_n\right] \qquad (3.8)$$

By seeking an asymptotic expansion of the form

$$\left.\begin{aligned}W_n&=W_o+\varepsilon W_1+\varepsilon^2W_2+\cdots\\ F_n&=F_o+\varepsilon F_1+\varepsilon^2F_2+\cdots\end{aligned}\right\} \qquad (3.9)$$

We have from eqn.(3.6) and eqn.(3.8) the following system of equations:
order zero (ε^0)

$$W_o^{iv}+\frac{n^4}{R^4}W_o+\frac{ch}{D}\dot{W}_o+\frac{\rho h}{D}\ddot{W}_o=0 \qquad (3.10)$$

$$F_o^{iv}+\frac{n^4}{R^4}F_0=0 \qquad (3.11)$$

order one (ε^1)

$$W_o^{iv}-2n^2DW_0''+\frac{n^4}{R^4}W_1+\frac{ch}{D}\dot{W}_1+\frac{\rho h}{D}\ddot{W}_1=RF_0''-n^2Sinn\theta F_0W_0'' \qquad (3.12)$$
$$-\frac{2n^2Cos^2n\theta}{Sinn\theta}F_o^1W_o^1-n^2Sinn\theta F_o^{11}W_o$$

$$F_1^{iv}-2n^2DF_0''+\frac{n^4}{R^4}F_1=-EhDRW_0''+DEh\frac{n^2Cos^2n\theta}{Sinn\theta}W_0'+n^2Sinn\theta W_0W_0'' \qquad (3.13)$$

3.1 Solution of Order zero Radial Deflection $W^{(0)}(x,t)$

Equation (3.8) is the governing equations for the order zero for the Radial Displacement. This is the free vibration of a beam resting on an elastic foundation. Using the usual superposition method of separation of variables

$$W_0(x,t) = X(x)T(t) \tag{3.1.1}$$

Results in the following system of Ordinary differential equations:

$$\left.\begin{aligned} &\frac{d^4X}{dx^4} + p^4 X = 0 \\ \text{and}\quad &\alpha_1 \frac{d^2T}{dt^2} + \alpha_2 \frac{dT}{dt} + \alpha_3 T = 0 \end{aligned}\right\} \tag{3.1.2}$$

The first of these equation is the familiar steady state free vibration of a beam while thesecond equation represents a damped free oscillation, where $\alpha_1 = \frac{\rho h}{D}$, $\alpha_2 = \frac{ch}{D}$ and α_3 is an arbitrarily introduced constant. The spatial component is solved subject to the following boundary conditions:

$$\left.\begin{aligned} M_x = D\frac{\partial^2 w}{\partial x^2},\ \ Q_x = 0 = -D\frac{\partial^3 w}{\partial x^3}\ \ at\ \ x = 0, l \\ M_x = 0 = -D\frac{\partial^2 w}{\partial x^2},\ \ Q_x = -D\frac{\partial^3 w}{\partial x^3}\ \ at\ \ x = 0, l \end{aligned}\right\} \tag{3.1.3}$$

In addition to the above conditions we invoke a technique [2] for solving shell problem in which the boundary conditions involving w at the two ends are uncoupled from one another. Thus we have the following conditions satisfied by $X(x)$:

$$\left.\frac{\partial^2 X}{\partial x^2}\right|_{x=0} = \frac{-M_0}{D};\ \ \left.\frac{\partial^3 X}{\partial x^3}\right|_{x=0} = 0 \tag{3.1.4}$$

In addition we recall that the displacement $w(x,t)$ must be finite for all values of (x,t)

Hence

$$X(\mathrm{x}) = \frac{\mathrm{M}_0}{2p^2D} e^{-px}\,(Sinpx\ \text{-}\ Cospx)\,\frac{\text{-}Q_0}{2p^3D}\ \text{-}\ e^{-px}Cospx\ + \frac{M_l}{2p^2D}e^{-p(l-x)}\,Sinp(l\text{-}\ x)$$
$$+\ \frac{Q_l}{2p^3D}e^{-p(l-x)}Cosp(l-x) \tag{3.1.5}$$

On the other hand if we consider an over-damped system for the time dependent equation starting from a neutral position at a velocity u_0 we thus have;

$$W_0(x,t) = \begin{bmatrix} \frac{M_0}{2p^2D}e^{-px}(Sinpx - Cospx) - \frac{Q_0}{2p^3D}e^{-px}Cospx \\ + \frac{M_l}{2p^2D}e^{-p(l-x)}(Sinp(l-x) - Cosp(l-x)) + \frac{Q_l}{2p^3D}e^{-px}Cosp(l-x) \end{bmatrix} \frac{u_0}{r_2 - r_1}\left(e^{r_2 t} - e^{r_1 t}\right) \tag{3.1.6}$$

where

$$r_1 = \frac{-\alpha_2 + \sqrt{\alpha_2^2 - 4\alpha_1\lambda^2}}{2\alpha_1}, \qquad r_2 = \frac{-\alpha_2 - \sqrt{\alpha_2^2 - 4\alpha_1\lambda^2}}{2\alpha_1} \tag{3.1.7}$$

3.2 Order zero solution for the in-plane-stress function F

The steady state stress equation

$$F_0^{iv} + k^4 F_0 = 0 \tag{3.2.1}$$

satisfying the boundary conditions

$$F = 0 \quad and \quad \frac{d^2F}{dx^2} = \gamma_x \Big|_{x=0,\,L} \tag{3.2.2}$$

where γ_x is the intensity of the in-plane stress and $k^4 = \frac{n^4}{R^4}$ admits the solution;

$$F_0(x) = \frac{\gamma_l}{2k^2} e^{-k(l-x)} Sin(x-l) - \frac{\gamma_0}{2k^2} e^{-kx} Sinkx \tag{3.2.3}$$

3.3 Order one solution for radial deflection $W^{(1)}(x,t)$

The governing differential equation of the problem is given as

$$W_1^{iv} + \frac{n^4}{R^4} W_1 + \frac{ch}{D}\dot{W}_1 + \frac{\rho h}{D}\ddot{W}_1 = \frac{u_0}{(r_2 - r_1)}(e^{r_2 t} - e^{r_1 t})G(x) + H(x) \tag{3.3.1}$$

where

$$\begin{aligned} G(x) &= 2n^2R\left[-\left(M_0 + \frac{Q_0}{p}\right)e^{-px}Sinpx - M_0 e^{-px}Cospx - \left(M_l - \frac{Q_l}{P}\right)e^{-p(l-x)}Sinp(l-x) \right. \\ &\left. + M_L e^{-p(l-x)}Cosp(l-x)\right] - \frac{2n^2Cos^2n\theta}{pDSinn\theta}\left[\frac{r_0}{2k}e^{-kx}(Coskx - Sinkx) - \frac{r_l}{2k}e^{-k(l-x)}(Cosk(l-x) - Sink(l-x))\right] \\ &\left[\left(M_o + \frac{Q_0}{p}\right)e^{-px}Cospx + \frac{Q_0}{p}e^{-px}Sinpx - \left(M_L - \frac{Q_l}{p}\right)e^{-P(L-x)}Cosp(l-x) + \frac{Q_l}{p}Sinp(l-x)\right] \\ &- \frac{n^2Sinn\theta}{2pD}\left[r_o e^{-kx}Coskx + \gamma_l e^{-k(l-x)}Cosp(l-x)\right]\left[M_0 e^{-px}Sinpx - \left(M_o + \frac{Q_0}{p}\right)e^{-px}Cospx \right. \\ &\left. - \left(M_L - \frac{Q_l}{p}\right)e^{-p(l-x)}Cosp(l-x) + M_L e^{-P(L-x)}Sinp(l-x)\right] \end{aligned} \tag{3.3.2}$$

and

$$H(x) = R\left[\gamma_0 e^{-kx}Coskx + \gamma_l e^{-k(l-x)}Cosk(l-x)\right] \tag{3.3.3}$$

To solve eqn (3.3.1) we assume that $W_1(x,t)$ can be expressed as a series of eigen functions

$$W_1(x,t) = \sum_{m=1}^{\infty} b_m(t)\phi_m(x) \tag{3.3.4}$$

where

$$\phi_m(x) = c_m v_m(x) \qquad (3.3.5)$$

$v_m(x)$ is then chosen to satisfy the boundary condition

$$v(x)\Big|_{x=0,1} = 0 \quad \text{and} \quad \left.\frac{d^2v}{dx^2}\right|_{x=0,1} = 0 \qquad (3.3.6)$$

$$v_m(x) = Sinp_m x + Sinp_m(l-x) \qquad (3.3.7)$$

Which satisfy the boundary condition when they are uncoupled from each end. The eigen function is now given by

$$\phi_m(x) = c_m\left[Sinp_m x + Sinp_m(l-x)\right] \qquad (3.3.8)$$

where c_m remain arbitrary constant chosen to normalize the eigen function

***Theorem* 3.1**

Let $\phi_n(x)$ *be a set of functions which are mutually orthonormal in* (*a*, *b*) *then* $\sum_{n=1}^{\infty} K_n\phi_n(x)$ *converges uniformly* to *f(x) in* (*a*, *b*)

Proof by Spiegel, [6]:

Given *f(x)* = $\sum_{n=1}^{\infty} K_n\phi_n(x)$ - - - - - (*i*)

Then $K_n = \int_a^b f(x)\phi_n(x)dx$. Multiply both sides of *f*(*x*) = $\sum_{n=1}^{\infty} K_n\phi_n(x)$ by $\phi_m(x)$ and integrate from *a* to *b* to get

$$\int_a^b f(x)\phi_n(x)dx = \sum_{n=1}^{\infty} K_n \int_a^b \phi_m(x)\phi_n(x)dx \qquad (ii)$$

When the interchange of integration and summation is justified by using the fact that the series converges uniformly to *f(x)*, and since the function $\{\phi_n(x)\}$ are mutually orthornomal in (a,b) we have $\int_a^b f(x)\phi_n(x)dx = \{^{om\neq n}_{1m=n}$ so that (ii) becomes $\int_a^b f(x)\phi_m(x)dx = k_n$ as required. ■

From equation (3.1.1), we can assume that

$$G(x) = \sum_{m=1}^{\infty} d_m\varphi_m(x) \qquad (3.3.9)$$

$$H(x) = \sum_{m=1}^{\infty} a_m\varphi_m(x) \qquad (3.3.10)$$

substituting equation (3.3.2) into (3.3.1) and since $\phi_m(x) \neq 0$, then

$$\alpha_1 b^{II}{}_n(t) + \alpha_2 b^{I}{}_m(t) + p^4 b_m(t) = \frac{u_0}{(r_2 - r_1)}(e^{r_2 t} - e^{r_1 t})d_m + a_m \qquad (3.3.11)$$

where the $\alpha_{i,s}$ are as defined earlier. Equation (3.3.11) is the forced motion of a particle where $d_m = \int_0^l G(x)\phi_m(x)dx$ and $a_m = \int_0^l H(x)\phi_m(x)dx$.

Define
$$Z = \left(\frac{n^4}{R^4} + P_n^4\right) \tag{3.3.12}$$

then the reduced equation is given as
$$\alpha_1 b''_m(t) + \alpha_2 b'_m(t) + Z b_m(t) = 0 \tag{3.3.13}$$

$if \quad q_1 = \dfrac{-\alpha_2 + \sqrt{\alpha_1^2 - 4\alpha_1 Z}}{2\alpha_1} \; and \quad q_2 = \dfrac{-\alpha_2 - \sqrt{\alpha_2^2 - 4\alpha_1 Z}}{2\alpha_1}$, then

$$b_m(t) = u_0\left\{\frac{1}{(q_2 - q_1)}(e^{q_2 t} - e^{q_1 t}) + \frac{d_m}{(r_2 - r_1)}\left[\frac{e^{r_2 t}}{(\alpha_1 r_2^2 + \alpha_2 r_2 + Z)} - \frac{e^{r_1 t}}{(\alpha_1 r_1^2 + \alpha_1 r_1 + Z)}\right]\right\} + \frac{a_m R^2}{(n^4 - R^2 p_m^4)}$$ and,

$$W_1(x,t) = u_0 \sum_{m=1}^{\infty}\left[\frac{1}{(q_2 - q_1)}(e^{q_2 t} - e^{q_1 t}) + \frac{d_m}{(r_2 - r_1)}\left(\frac{e^{r_2 t}}{(\alpha_1 r_2^2 + \alpha_2 r_2 + Z)} - \frac{e^{r_1 t}}{(\alpha_1 r_1^2 + \alpha_1 r_1 + Z)}\right) + \frac{a_m R^2}{(n^4 - R^2 P_m^4)}\right]$$
$$[Sin p_m x + \sin p_m (l - x)] \tag{3.3.14}$$

The coefficients c_m is given as

$$c_m = \frac{2\sqrt{p_m}}{\left(4p_m l - 3Sin2p_m l - 4sSnp_m l + 4p_m l \cos p_m l\right)^{1/2}} \tag{3.3.15}$$

3.4 Solution of the order one in–plane stress function $F^{(1)}(x,t)$

The governing equation is
$$\frac{d^4 F^{(1)}}{dx^4} + \frac{n^4}{R^4}F^{(1)} = g\left(W^{(1)}, F^{(0)}\right) \tag{3.4.1}$$

where the r.h.s. of (3.4.1) is the consequence of (3.3.14).

Solving (3.1.4) via the Finite Fourier Sine transformation technique subject to the conditions

$$F(x)\big|_{x=0,l} = 0 \quad \text{and} \quad \left.\frac{d^2F}{dx^2}\right|_{x=0,l} = 0 \tag{3.4.2}$$

we have

$$F_1(X) = \frac{2}{l}\sum_{m=1}^{\infty}\frac{1}{\beta_1}\left\{2n^2DR[\gamma_0\beta_2 + \gamma_L\beta_l] + \frac{REh\tau_0}{r_2 - r_1}\left(e^{r_2t} - e^{r_1t}\right)\left[\left(M_0 + \frac{Q_0}{p}\right)\beta_4 + M_0\beta_5 + \left(M_L + \frac{Q_L}{p}\right)\beta_6 + M_L\beta_7\right]\right\}Sin\frac{m\pi}{l}x \text{ where}$$

$$\beta_1 = \left(\frac{m\pi}{l}\right)^4 + \frac{n^4}{R^4}$$

$$\beta_2 = \frac{1}{2\left(k^2 + \left(k + \frac{m\pi}{l}\right)^2\right)}\left[\left(k + \frac{m\pi}{l}\right)\left[1 - e^{-kl}\cos\left(kl + m\pi\right)\right] - ke^{-kl}Sin\left(kl + m\pi\right)\right]$$

$$- \frac{1}{2\left(k^2 + \left(k - \frac{m\pi}{l}\right)^2\right)}\left[\left(k + \frac{m\pi}{l}\right)\left[1 - e^{-kl}Cos\left(kl - m\pi\right)\right] - Ke^{-KL}\sin\left(kl - m\pi\right)\right]$$

$$\beta_3 = \frac{1}{2\left(k^2 + \left(k + \frac{m\pi}{l}\right)^2\right)}\left[\left(k + \frac{m\pi}{l}\right)\left[1 - e^{-kl}Cos\left(pl + m\pi\right)\right] - pe^{-kl}Sin\left(kl + m\pi\right)\right]$$

$$- \frac{1}{2\left(k^2 + \left(k - \frac{m\pi}{l}\right)^2\right)}\left[\left(k - \frac{m\pi}{l}\right)\left[1 - e^{-kl}Cos\left(pl - m\pi\right)\right] - pe^{-kl}Sin\left(kl + m\pi\right)\right]$$

$$\beta_4 = \frac{1}{2\left(p^2 + \left(p + \frac{m\pi}{l}\right)^2\right)}\left[\left(p + \frac{m\pi}{l}\right)\left[e^{-pl}Sin(pl + S\pi)\right] + p(1 - e^{-pl}Cos(plPl + m\pi))\right]$$

$$- \frac{1}{2\left(p^2 + \left(p - \frac{m\pi}{l}\right)^2\right)}\left[\left(p - \frac{m\pi}{l}\right)\left[e^{-pL}Sin(pl - m\pi)\right] + p(1 - e^{-pl}Cos(pl - m\pi))\right]$$

$$\beta_5 = \frac{1}{2\left(p^2 + \left(p + \frac{m\pi}{l}\right)^2\right)}\left[\left(p + \frac{m\pi}{l}\right)\left[1 - e^{-pl}Cos\left(pl + m\pi\right)\right] - pe^{-pl}Sin\left(pl + m\pi\right)\right]$$

$$- \frac{1}{2\left(p^2 + \left(p - \frac{m\pi}{l}\right)^2\right)}\left[\left(p - \frac{m\pi}{l}\right)\left[1 - e^{-pl}Cos\left(pl - m\pi\right)\right] - pe^{-pl}Sin\left(pl - m\pi\right)\right]$$

$$\beta_6 = \frac{1}{2\left(p^2 + \left(p - \frac{m\pi}{l}\right)^2\right)}\left[p(Cosm\pi - e^{-pl}Cospl) - \left(p - \frac{m\pi}{l}\right)(Sinm\pi - e^{-pl}Sinpl)\right]$$

$$- \frac{1}{2\left(p^2 + \left(p + \frac{m\pi}{l}\right)^2\right)}\left[p(Cosm\pi - e^{-pl}Cospl) + \left(p + \frac{m\pi}{l}\right)(Sinm\pi + e^{-pl}Sinpl)\right]$$

$$\beta_7 = \frac{1}{2\left(p^2 + \left(p - \frac{m\pi}{l}\right)^2\right)}\left[\left(p - \frac{m\pi}{l}\right)\left[(Cosm\pi - e^{-pl}CosPl)\right] + p(Sinm\pi - e^{-pl}Sinpl)\right]$$

$$- \frac{1}{2\left(p^2 + \left(p + \frac{m\pi}{l}\right)^2\right)}\left[\left(p + \frac{m\pi}{l}\right)\left[(Cosm\pi - e^{-pl}Cospl)\right] - p(Sinm\pi + e^{-pl}Sinpl)\right]$$

4.0 Numerical Simulation

The displacement profiles of the shell are displayed graphically in what follows demonstrating the effect of bending moments, shear force and the damping parameter on the amplitude of vibration.

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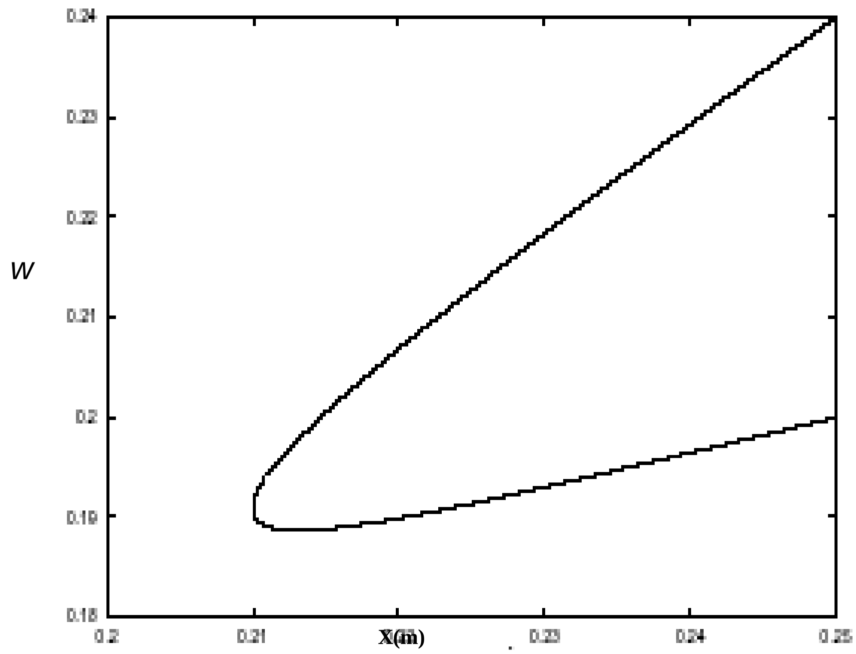

Figure 1: Displacement profile at a time t .

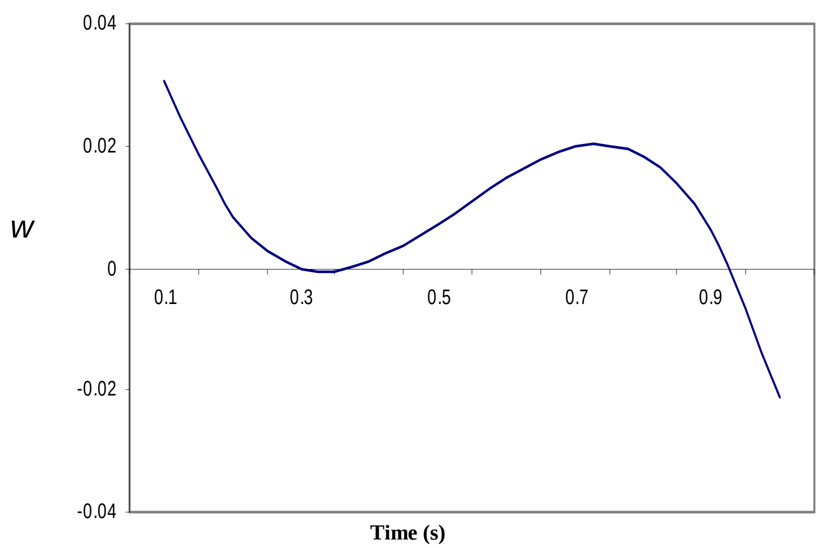

Figure 2: Displacement profile with time

5.0 Discussion of results and conclusion

The deflection profile is characterized by exponential decay away from the edge of the shell. This decay in the lateral deflection is due to edge stress couple or an edge transverse shear resultants. Since the shape bending number, and shear resultant away from the edge are all proportional to the derivatives of the lateral deflections, each of these also decays away from the edge where the edge load is acting (boundary layer) Jack R. V. [2]

From the graphs (figure 1 and figure 2) the nonlinearity is of the weak, softening type. An increase in the excitation amplitude produces wrinkling of the shell which my respond nonlinearly to disturbance with the amplitude of the breathing mode as well as the flexural mode increasing dramatically yielding a much larger response. This instability and the saturation phenomenon were first found analytically and verified numerically by Nayfeh et. al., [4] where consideration was given to circular rings with harmonic loading. However, in this paper we have a finite nonlinear circular cylindrical shell subjected to axially symmetric loading and the response obtained as shown in figure 1 and figure 2 physically depends on the initial condition with velocity u_0, whereas the long time response in [4] exhibit Hopf Bifurcation.

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