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Constrained relative controllability of semilinear dynamical neutral systems with multiple delays in state and control.

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Abstract

In this work, we study the semi linear dimensional abstract control neutral system with multiple constant delays in the state and control given by

$$\frac{d}{dt}\left[x(t-h)\right] = \sum_{i=0}^{M} A_i x(t-h_i) + F(x(t)) + \sum_{i=0}^{M} B_i u(t-h_i)$$

with zero initial conditions $x(0) = 0,\ u(t) = 0 \quad for \quad t \in [-h, 0]$ ***where the state x(t) takes values in a real Banach space X and the control u(t) is in another real Banach space U.***

1.0 Introduction

Optimally, a dynamical system was understood as an isolated mechanical system which motion is described by the Newtonian differential equations and which is characterized by a finite set of generalized coordinates and velocities. In this work we associated any time dependent process with the motion of a dynamical system. This is a qualitative property of dynamical control systems and is of particular importance in control theory. In recent years, various control problems for different types of nonlinear dynamical systems have been considered in many publications and monographs. However it should be stressed that the most literature in this direction has been mainly concerned with controllability problems for finite dimensional nonlinear dynamical systems with unconstrained controls and without delays, [5] or for linear infinite dimensional dynamical systems with constrained controls and without delays [2, 4, 5]. In this paper, we shall consider constrained relative controllability problems for finite dimensional stationary semi-linear dynamical neutral systems with multiple point delays in the state and control described by ordinary differential state equations. Let us recall that semi-linear dynamical control systems contain linear and pure nonlinear parts in the differential state equation. More precisely we shall formulate and prove sufficient conditions for constrained 1n relative controllability in a prescribed time interval for semi-linear dynamical neutral systems with multiple point delays in the state and control which nonlinear terms is continuously differentiable near the origin.

It will be proved that, under suitable assumptions, constrained local relative controllability of linear associated, approximated dynamical systems implies local relative controllability near the origin of the original semi-linear abstract dynamical system. Klamka [5] and Onwuatu [10] have previously investigated the relative controllability of ordinary differential systems with delays in the control. Chukwu [1] considered linear delay systems but without delays in the control variable. He established his results using limited controls. Most recently, Iheagwm and Nse in [3] tackled the question of the controllability of the third world economies. Their model is a neutral dynamical system which provides broad-based policy guidelines for the revamping of the world economies.

Owing to the obvious difficulties in the handling systems which consist of linear and strictly nonlinear parts with delays in both the state and control variables, this work seeks to resolve these difficulties.

2.0 System description

In this paper, we study the semi-linear stationary finite dimensional dynamical with multiple delays in the state and control defined by the following ordinary differential state equation.

$$\frac{d}{dt}x(t-h) = \sum_{i=0}^{M} A_i x(t-h_i) + F(x(t)) + \sum_{i=0}^{M} B_i u(t-h_i) \qquad (2.1)$$

for $t \in [0, T]$, $T > h$ with zero initial conditions $x(0) = 0,\ u(t) = 0 \quad for \quad t \in [-h,\ 0)$ (2.2)

where the state $x(t) \in R^n = X$ and the control $u(t) \in R^m = U$. $A_i i = 0,\ 1\ldots, M$ are $n \times n$ dimensional constant matrices. $B_i,\ i = 0,\ 1\ldots, M$ are $n \times m$ dimensional constant matrices.

Moreover, let us assume that the nonlinear mapping $F{:}X \rightarrow X$ is continuously differentiable near the origin and such that $F(0) = 0$. We assume also that the set of values of controls $U_o \subset U$ is a given closed and convex cone with non-empty interior and vertex at zero. Then the set of admissible controls for the dynamical control system (2.1) has the following form: $U_{ad} = L_\infty([0,T], U_o)$

Thus for a given admissible control $u(t)$, there exists a unique solution $x(t,u)$ *for* $t \in [0, T]$ of the state equation (2.1) with zero initial conditions (2.2) described by the integral formula

$$x(t,u) = X(t,\phi,0)\phi(t_o) + \int_0^t \left[\sum_{i=0}^{M} S_i(t-s)\big(F(x(s,u))\big) + \sum_{i=0}^{M} B_{i1} u(t-h_i) \right] ds \qquad (2.3)$$

where the semi group $S_i(t) = \exp(A_i t)$ are nxn transition matrices for the linear part of the semi-linear control system (2.1).

For the semi-linear dynamical system with multiple point delays in the state and control, it is possible to define many concepts of control. In the sequel, we shall focus our attention on the so-called constrained global relative controllability in the time interval $[0, T)$. In order to do this, we first introduce the notion of the attainable set at time $T > 0$ from zero initial conditions (2.2) denoted by $K_T(U_c)$ and defined as

$$K_T(U_c) = \{x \in X : x = x(T,u), u(t) \in U_o \quad for \quad a.e.\ t \in [0,T]\} \qquad (2.4)$$

where $x(t,u)$, $t > 0$ is the unique solution of equation (2.1) with initial conditions (2.2). Now using the concept of the attainable set, let us recall the well-known definitions of constrained relative controllability in $[0, T]$ for the dynamical system (2.1).

3.0 Preliminaries

***Definition* 3.1**

The dynamical system (2.1) *is said to be U_c-exactly locally relatively controllable in* [0, T] *if the attainable set $K_T(U_c)$ contains neighbourhood of zero in the space X.*

***Definition* 3.2**:

T*he dynamical system* (2.1) *is said to be U_c-exactly globally relatively controllable in* [0, T] *if the attainable set $K_T(U_c)$ contains neighborhoods of zero in the space X.*

***Lemma* 3.1**:

Let X, U, U_c, and Ω be as described above. Let $g{:}\Omega \rightarrow X$ be a nonlinear mapping and suppose that on Ω, the nonlinear mapping has derivative Dg which is continuous at zero. Moreover, suppose that $g(0) = 0$ and assume that linear map $Dg(0)$ maps U_c onto the whole space. Then there exists neighborhoods $N_0 \subset X$ about 0 εX and $M_0 \subset \Omega$ about 0 εU such that the nonlinear equation $x = g(u)$ has for each $x \varepsilon N_0$, at least one solution $u \varepsilon M_o \cap U_c$, where $M_o \cap U_c$ is a set called conical neighborhood of zero in the space U.

Lemma **3.2**

Let $D_u x$ denotes derivative of x with respect to u. Moreover, if x(t;u) is continuously differentiable with respect to its u argument, we have for u $\in$*:* $L_\infty([0,T], U) D_u x(t;u)(v) = z(t,u,$ $v)$ *where the mapping* $t \to z(t,u,v)$ *is the solution of the linear ordinary equation*

$$\frac{d}{dt}z(t-h) = \sum_{i=0}^{M} A_i z(t-h_i) + D_x(F(x;u))\,z(t) + \sum_{i=0}^{M} B_1 u(t-h_i) \qquad (3.1)$$

with zero initial conditions $z(0,u,v) = 0$ and $u(t) = 0$ for t ε $[-h\ 0]$

Proof

Using formula (2.3) and the well-known differentiability result, we have

$$D_u x(t;u) = \int_0^t \left[D_u \sum_{i=0}^{M} S_i(t-s)\,F\big(x(t,u)\big) + \sum_{i=0}^{M} B_i u(t-h_i) \right] ds$$

$$= \int_0^t \left[\sum_{i=0}^{M} S_i(t-s)\,D_u F\big(x(t,u)\big) + \sum_{i=0}^{M} B_i u(t-h_i) \right] ds$$

$$= \int_0^t \sum_{i=0}^{M} S_i(t-s)\,D_X F\big(x(t,u)\big) D_n x(t,u)\,ds + \int_0^t \sum_{i=0}^{M} S_i(t-s) \sum_{i=0}^{M} B_i u(t-h_i)\ ds \qquad (3.2)$$

Differentiating equality (3.2) with respect to the time variable, t, we have

$$\left(\frac{d}{dt}\right) D_u x(t;u)v = D_X F\big(x(t,u)\big) D_u x(t;u)v + \sum_{i=0}^{M} B_i u(t-h_i)\,v$$

$$+ \int_0^t \left(\frac{d}{dt}\right) \sum_{i=0}^{M} S_i(t-s) * \sum_{i=0}^{M} B_i u(t-h_i)\,ds \cdot v + \int_0^t \left(\frac{d}{dt}\right) \sum_{i=0}^{M} S_i(t-s) D_u F(x(s;u)) D_u x(s;u)\,ds \cdot v \qquad (3.3)$$

Therefore, since by assumption, $S(t)$ a differentiable semi group, then $(d/dt)\ S(t\text{-}s) - AS(t\text{-}s)$ and we have

$$z'(t) = D_X F(x(t;u)) \sum_{i=0}^{M} z(t-h_i) + \left[\int_0^t \sum_{i=0}^{M} A_i\ S(t-s) \sum_{i=0}^{M} B_i u(t-h_i)\,ds \right]$$

$$+ \left[\int_0^t \sum_{i=0}^{M} A_i\ S(t-s) D_X F(x(s;u)) z(s)\,ds \right] \qquad (3.4)$$

On the other hand, solution of the equation (3.1) has the following integral form

$$z(t) = \int_0^t\ S(t-s) \left[D_X F(x(s;u))\,z(s) + \sum_{i=0}^{M} B_i\ u(t-h_i) \right] ds \qquad (3.5)$$

Therefore differential equation (3.4) can be expressed as follows

$$z'(t) = \sum_{i=0}^{M} A_i z(t) + D_x F(x(t;u))\ z(t) + \sum_{i=0}^{M} B_i\ u(t-h_i)$$

Hence lemma 3.2 follows. ■

4.0 Controllability conditions

In this section we shall study constrained global relative controllability in $[0, T]$ for semi-linear dynamical system (2.1) using the associated linear dynamical system with multiple point delays in the state and control given by

$$\frac{d}{dt}z(t-h) = C_z(t-h) + \sum_{i=0}^{M}\ B_1\,u(t-h_i) \qquad (4.1)$$

for $t \in [0, T]\}$ with zero initial conditions $z(0) = 0,\ u(t) = 0\ \ for\ \ t \in [-h, 0)$ where

$$C = \sum_{i=0}^{M} A_i + D_x F(0) \tag{4.2}$$

The main result is the following sufficient condition for constrained local relative controllability of the semi-linear dynamical system (2.1).

***Theorem* 4.1**

Suppose that

(*i*) $F(0) = 0$

(*ii*) $U_c \subset U$ *is a closed and convex cone with vertex at zero*

(*iii*) *The associated linear control system with multiple point delays in the state and control is* U_c *– exactly locally relatively controllable in* $[0, T]$. *Then, the semi-linear dynamical control system with multiple point delays in the state and control* U_c *-0- exactly locally relatively controllable in* $[0, T]$.

Proof

Let us define for the nonlinear dynamical system (3.4) a nonlinear map

$$g : L_\infty([0,T], U_c) \to X$$

by $g(u) = x(T, u)$. Similarly for the associated linear dynamical system (4.1), we define a linear map

$$H : L_\infty([0,T], U_c \to X$$

by $Hv = x(T, v)$. ■

By the assumption (*iii*) the linear dynamical system (1) is U_c – globally relatively controllable in $[0, T]$. Therefore, by definition 3.2, the linear operator H is subjective, that is, it maps cones of admissible controls U_{ad} unto the whole space X. Furthermore, by lemma 3.2, we have that $Dg(0) = H$. Since U_c is a closed and convex cone, then the cone in the function space $L_\infty([0,T], U_c)$. Therefore, the nonlinear map g satisfies all the assumption of the generalized open mapping theorem stated in lemma 3.1. Therefore, the nonlinear map g transforms a conical neighborhood of zero in the cone of admissible controls U_{ad} onto some neighborhood of zero in the whole space X. This is by definition 3.1 is equivalent to the U_c-local controllability in {0, T] of the semi linear dynamical control system (2.1). Hence our theorem follows.

5.0 Concluding remarks

In the present paper, sufficient conditions for constrained relative controllability for semi linear finite dimensional stationary dynamical systems with multiple delays in the state and control, have been formulated and proved. In the proof of the main result, generalized open mapping theorem [5] has been extensively used. The relative controllability conditions given in this paper extends to the case of constrained relative controllability of finite dynamic semi linear stationary neutral control systems and also for unconstrained nonlinear stationary control systems.

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