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Magnetic and velocity fields MHD flow of a stretched vertical permeable surface with buoyancy in the presence of heat generation and a first order chemical reaction

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Abstract

Analytical solutions for heat and mass transfer by laminar flow of Newtonian, viscous, electrically conducting and heat generation/absorbing fluid on a continuously moving vertical permeable surface with buoyancy in the presence of a magnetic field and a first order chemical reaction are reported. The solutions for magnetic and velocity fields are obtained for various thermal grashof number, mass grashof number, Hartmann number, and magnetic buoyancy. The effect of various parameters on skin friction coefficient C_f were also examined and reported. Graphical illustration features prominently in this work.

***Keywords*:** Permeable surface, Buoyancy, source term, thermal expansion, suction, ambient, magnetic induction, asymptotic variable.

Classification: 76W05

1.0 Introduction

A study of boundary–layer behaviour on continuously moving solid surfaces has attracted the attention of several researchers. The analysis of magneto – hydrodynamics (MHD) flow of electrically conducting fluid finds application in different areas, such as the aerodynamic extrusion of plastic sheets, and the boundary–layer along a liquid film in condensation processes, (Chankha 2003) [2].

In order to understand the basic features of such a process, we consider a continuous flat plate which issues from a slot and moves with a constant velocity into a fluid which is at rest. As a result, the fluid adjacent to the plate moves and the region of penetration of the fluid motion into the ambient fluid depends on the Reynolds number of the flow. For large Reynolds number the region of penetration

Nomenclatures

c: concentration
c_p: specific heat
C: fluid concentration
C_w: wall concentration
α_D: mass diffusion-coefficient
D: chemical reaction parameter
Q_0 : heat generation coefficient
Nu : Nuselt number
$\Pr$: Prandtl number
Sc : Shmidt number
E: activation energy
g : gravitational pull
B_0 : magnetic induction
T : fluid temperature
n: reaction order
R: universal gas constant
T_w : wall temperature
v : fluid transverse velocity
v_W : suction velocity
x : axial or vertical coordinate
y : transverse coordinate

Greek Symbols

γ : chemical reaction parameter
ρ : fluid density

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Greek Symbols
γ : chemical reaction parameter
ρ : fluid density
θ : dimensionless temperature
R_i : Richardson number
σ : electric conductivity
β_τ : coefficient of thermal expansion
β_c : coefficient of mass expansion

Subscripts
w : condition on the wall
∞ : ambient condition

increases down stream of the slot, with the momentum and thermal boundary–layers originating from the slot and growing in the direction of the motion of the plane. Vajravelu and Hadjinicolaou (1990) [7], reported on convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. Other example of studies dealing with hydro - magnetic flows can be found in the papers by Gray (1979) [3], Michiyoshi et al. (1976) [5] and Funmizawa (1980) [4].

A.J. Chamkha (2003) [2] examined the boundary – layer of an MHD flow when heat generation is linear in temperature. Ayeni et al (2004) [1] extended the problem posed to heat generation that is quadratic in temperature. Okedoye A. M. et al. (2007) [6] examined and report Temperature and concentration fields and Local maximum in temperature field of MHD flow with buoyancy in the presence of heat generation. Other related papers on this work could be found in the literature of Vajravelu and Hadjinicolaou (1990) [7] and Chamkha, (2003) [2].

In this paper, we examined the velocity and magnetic fields of MHD flow with buoyancy in the presence of heat generation which has not been given an attention the way we considered it here.

2.0 Governing equations

The flow is steady and two-dimensional and the surface is maintained at a uniform temperature and concentration species, and is assumed to be infinitely long, i.e the dependent variables are not dependent on the vertical or axial coordinate. The physical coordinates (x, y) are chosen such that the x – axis lies in the plane of the plate. It is also assumed that the applied transverse magnetic field is non - uniform and that the induced magnetic field is not negligible. In addition, there is no applied electric field and all of the Hall Effect, viscous dissipation and Joule heating are neglected, thermo - physical properties are assumed constant. The governing equations for the problem are the equations for mass and energy conservation, momentum and scalar transport equations and magnetic transport equation. The fliud properties are also assumed constant, and the Boussinesq approximation is used to restrict the effect of variation of density with temperature exclusively to the body fierce term. With these assumptions, the equations that describe the physical situation are given by

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\upsilon\partial^2 u}{\partial y^2} + g\beta\left(T - T_\infty\right) - \frac{\sigma}{\rho}B^2 u \tag{2.2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{\alpha}{\rho\ cp}\frac{\partial^2 T}{\partial y^2} + \frac{ku_W\theta_0}{\rho\ cpd}\left(c - c_\infty\right) \tag{2.3}$$

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial y} = \frac{\alpha_D}{\rho}\frac{\partial^2 c}{\partial y^2} + \frac{\gamma}{\rho}\left(c - c_\infty\right) \tag{2.4}$$

$$\frac{\partial B}{\partial t} = \eta\frac{\partial^2 B}{\partial y^2} - v\frac{\partial B}{\partial y} - B\frac{\partial u}{\partial y} \tag{2.5}$$

$$p = Q\rho R(T - T_\infty). \tag{2.6}$$

In the above equations, y is the horizontal or transverse coordinate, u is the axial velocity, v is the transverse velocity, T is the fluid temperature, C is the species concentration and ρ, g, β, $\upsilon, \sigma, B_0, Q, \alpha_D$ and γ are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, dynamic viscosity, fluid electrical conductivity, magnetic induction, heat generation absorption coefficient, mass diffusion coefficient, the chemical reaction parameter and real number, respectively.

The physical boundary conditions for the problem are:

$$u(o,t) = u_W,\ v(0) = -v_W,\ T(0,t) = T_W,\ \ c(0,t) = c_W, B(0,t) = B_W \tag{2.7}$$

$$as\ \ y \to \infty;\ \ u(y,t) \to 0,\ T(y,t) \to T_\infty,\ \ c(y,t) \to c_\infty, B(y,t) \to B_\infty \tag{2.8}$$

$$u(y,0) \to u_W,\ T(y,0) \to T_W,\ \ c(y,0) \to c_W, B(y,0) \to B_W \tag{2.9}$$

where u_w (a parameter dependent on time), $v_w > 0$, T_w and c_w are the surface velocity, suction velocity, surface temperature and concentration respectively, q''' is the volumetric power.

The magnitude of local heat absorption/generation (or the strength of the energy source term) is a function of the local scalar concentration. This replicates the experimental situation in which the presence of acid lowers the local electrical resistance and increases the Ohmic heating rate. Further, heating is confined to the fluid in the channel which corresponds to a non – zero scalar concentration, i.e., selective heating of the channel fluid occurs. The presence of $g\beta\Delta T$ term in the momentum equation suggests a plume – like behavior. A plume experiences buoyancy addition only at its source, and not off – source as in the present study.

There is no difference in the pre-Heat Injected Zone (HIZ) region, and a jet-like behavior applies in this region. Moreover, the temperature can increase with the axial coordinate in the HIZ, unlike for a plume which experiences an axial decay in temperature at the centerline. It is however not immediately obvious whether the behavior will be closer to a jet or a plume at the exit of the HIZ, although it is known that a jet with buoyancy addition will eventually evolve into a plume [20]. Advection of the scalar depends on the velocity components. As seen above, the velocities are dependent on the local temperature in the HIZ (and post-HIZ); the temperature rise in turn depends on the scalar concentration in the HIZ. Therefore, there is a three-way coupling between temperature, velocities, and scalar concentration in the channel.

Heat addition to MHD flows can lead to turbulence flow when the magnetic induction is zero (or very small), that is a viscous dominated flow. The buoyant flow can transit into turbulence state by the buoyancy alone, the initial momentum will be lost after a certain length scales and the buoyancy will dominate the flow behaviour.

3.0 The non–dimensional governing equations

The channel scales are employed for non – dimensionalisation. Therefore, all distances are normalized by the channel width, d , velocities by the velocity at the wall, u_w , concentration by the concentration at the wall, c_w . Time is non–dimensionalised as

$$t^1 = \frac{u_W}{d} t \tag{3.1}$$

Pressure is non–dimensional as $p' = \dfrac{p}{\rho u_W^2}$. Integrating equation (2.1) subject to $v(0) = -vw$, we have the solution

$$v(y) = -v_w \tag{3.2}$$

Using equation (3.2), the momentum, energy, magnetic and species equations (2.2 – 2.6) can be non–dimensionalised using the following non – dimensional variables.

$$y^1 = y\frac{v_W}{v},\ u^1 = \frac{u}{u_W},\ \theta = \frac{(T - T_\infty)}{(T_W - T_\infty)},\ c^1 = \frac{c - c_\infty}{c_W - c_\infty}, \tag{3.3}$$

Dropping the primes for convenience, the steady governing equations is written as follows.

$$\frac{d^2u}{dy^2} + \frac{du}{dy} + \frac{\varepsilon G_r}{\mathrm{Re}}\theta + \frac{\varepsilon^2\phi_1}{\mathrm{Re}}\frac{d\theta}{dy} - \varepsilon M^2 B^2 u = 0 \tag{3.4}$$

$$\frac{d^2\theta}{dy^2} + \Pr\frac{d\theta}{dy} + \Pr Ri\,\phi c = 0 \tag{3.5}$$

$$\frac{d^2c}{dy^2} + S_c\frac{dc}{dy} + KS_c\,c = 0 \tag{3.6}$$

$$\frac{d^2B}{dy^2} + \alpha_B\frac{dB}{dy} - \mu_B B\frac{du}{dy} = 0 \tag{3.7}$$

The boundary conditions and the initial conditions becomes $u(0) = \theta(0) = c(0) = B(0) = 1$,

$$u \to 0,\ \theta \to 0,\ C \to 0,\ B \to 0 \quad as\ y \to \infty \tag{3.8}$$

where $Gr = \frac{g\beta_\tau \varepsilon T_\infty v^2}{\upsilon v_W^2 u_W}$, $\varepsilon M^2 = \frac{\sigma B_0^2}{\rho\upsilon}$, $\Pr = \frac{\mu c_p}{k}$, $\mathrm{Re} = \frac{u_W d}{\upsilon}$, $a_0 = \frac{u_\infty}{\rho\upsilon}$

$$\phi = \frac{Qv}{\mu c_p v_W^2},\ \varepsilon = \frac{RT_\infty}{E},\ Ri = \frac{g\beta}{\rho c_p}\frac{Q}{du_W^3},\ Sc = \frac{\upsilon}{\alpha_D},\ K = \frac{\gamma v}{v_W},$$

4.0 Method of solution

The analytical solutions for temperature and concentration fields were given as:

$$c(y) = e^{-ny}, \tag{4.1}$$

where,

$$n = \frac{1}{2}\left[Sc + \sqrt{Sc^2 + 4KSc}\right] \tag{4.2}$$

and

$$\theta(y) = a_1 e^{-\Pr y} + a_2 e^{-ny} \tag{4.3}$$

where

$$a_1 = 1 - a_2,\ a_2 = -\frac{\Pr Ri\phi}{n(n - \Pr)} \tag{4.4}$$

Using (4.1) – (4.2), the system of equations reduces to

$$\frac{d^2u}{dy^2} + \frac{du}{dy} - \varepsilon M^2 B^2 u = \frac{\varepsilon^2\phi_1}{\mathrm{Re}}\left(a_1 \Pr e^{-\Pr y} + a_2 n e^{-ny}\right) - \frac{\varepsilon G_r}{\mathrm{Re}}\left(a_1 e^{-\Pr y} + a_2 e^{-ny}\right) \tag{4.5}$$

$$\frac{d^2B}{dy^2} + \alpha_B\frac{dB}{dy} - \mu_B B\frac{du}{dy} = 0 \tag{4.6}$$

$$u(0) = B(0) = 1 \quad u(\infty) = B(\infty) = 0 \tag{4.7}$$

Now using the asymptotic variables,

$$u = u_0 + \varepsilon u_1 + + \varepsilon^2 u_2 + h.o.t$$

$$B = B_0 + \varepsilon B_1 + + \varepsilon^2 B_2 + h.o.t$$
$$\mu_B = \varepsilon \mu_0 + \varepsilon^2 \mu_1$$

and equating the terms in power of ε, we have the systems

$$\frac{d^2u_0}{dy^2} + \frac{du_0}{dy} = 0 \tag{4.5.1}$$

$$\frac{d^2B_0}{dy^2} + \alpha_B \frac{dB_0}{dy} = 0 \tag{4.6.1}$$

$$u_0(0) = B_0(0) = 1,\ u_0(\infty) = B_0(\infty) = 0 \tag{4.7.1}$$

$$\frac{d^2u_1}{dy^2} + \frac{du_1}{dy} - M^2 B_0^2 u_0 = -\frac{Gr}{\mathrm{Re}}\left(a_1 e^{-\Pr y} + a_2 e^{-ny}\right) \tag{4.5.2}$$

$$\frac{d^2B_1}{dy^2} + \alpha_B \frac{dB_1}{dy} = \mu_0 B_0 \frac{du_0}{dy} \tag{4.6.2}$$

$$u_1(0) = B_1(0) = 0,\ u_1(\infty) = B_1(\infty) = 0 \tag{4.7.2}$$

$$\frac{d^2u_2}{dy^2} + \frac{du_2}{dy} - M^2\left(B_0^2 u_1 + 2B_1 B_0 u_0\right) = \frac{\phi_1}{\mathrm{Re}}\left(a_1 \Pr e^{-\Pr y} + a_2 n e^{-ny}\right) \tag{4.5.3}$$

$$\frac{d^2B_2}{dy^2} + \alpha_B \frac{dB_2}{dy} = \mu_0\left(B_0 \frac{du_1}{dy} + B_1 \frac{du_0}{dy}\right) + \mu_1 B_0 \frac{du_0}{dy} \tag{4.6.3}$$

$$u_2(0) = B_2(0) = 0,\ u_2(\infty) = B_2(\infty) = 0 \tag{4.7.3}$$

The solutions to equations (4.5.1) and (4.6.1) subject to (4.7.1) respectively are

$$\left.\begin{aligned} u_0(y) &= e^{-y} \\ B_0(y) &= e^{-\alpha_B y} \end{aligned}\right\} \tag{4.8}$$

Substitute (4.5.1) in to (4.5.2), we obtain

$$\frac{d^2u_1}{dy^2} + \frac{du_1}{dy} = M^2 e^{-(1+2\alpha_B)y} - \frac{Gr}{\mathrm{Re}}\left(a_1 e^{-\Pr y} + a_2 e^{-ny}\right)$$

The particular function $u_{1p}(y)$ is taken to be $a_3 e^{-\Pr y} + a_4 e^{-ny} + a_5 e^{-(1+2\alpha_B)y}$, the homo-genous part gives $a_6 e^{-y}$. This gives

$$u_1(y) = a_3 e^{-\Pr y} + a_4 e^{-ny} + a_5 e^{-(1+2\alpha_B)y} + a_6 e^{-y} \tag{4.9}$$

where $a_3 = -\dfrac{Gra_1}{\mathrm{Re}\Pr(\Pr-1)}$, $a_4 = -\dfrac{Gra_2}{\mathrm{Re}\, n(n-1)}$, $a_5 = \dfrac{M^2}{2\alpha_B(1+2\alpha_B)}$, $a_6 = -\left(a_3 + a_4 + a_5\right)$

$\dfrac{d^2B_1}{dy^2} + \alpha_B \dfrac{dB_1}{dy} = -\mu_0 e^{-(1+\alpha_B)}$. $B_{1P}(y) = a_7 e^{-(1+\alpha_B)y}$. Let $B_{1c}(y) = a_8 e^{-\alpha_B y} + a_9$,

thus $B_1(y) = a_7 e^{-(1+\alpha_B)y} + a_8 e^{-\alpha_B y} + a_9$, $a_7 = -\dfrac{\mu_0}{(1+\alpha_B)}$ $a_8 = -a_7$ $a_9 = 0$ (4.10)

And from equation (4.5.3) $\dfrac{d^2u_2}{dy^2} + \dfrac{du_2}{dy} - M^2\left(B_0^2 u_1 + 2B_1 B_0 u_0\right) = \dfrac{\phi_1}{\mathrm{Re}}\left(a_1 \Pr e^{-\Pr y} + a_2 n e^{-ny}\right)$.

The particular solution suggested by equation (4.5.3) could be written as
$u_{2c}(y)=\ a_9 e^{-(\Pr+2\alpha_B)y}+a_{10}e^{-(n+2\alpha_B)y}+a_{11}e^{-(1+4\alpha_B)y}+a_{12}e^{-2(1+\alpha_B)y}+a_{13}e^{-(1+2\alpha_B)y}\ +a_{14}e^{-\Pr y}+a_{15}e^{-ny}$ Let $u_{2c}(y)=a_{16}e^{-y}+a_{17}$. Thus

$$u_2(y)=\ a_9 e^{-(\Pr+2\alpha_B)y}+a_{10}e^{-(n+2\alpha_B)y}+a_{11}e^{-(1+4\alpha_B)y}+a_{12}e^{-2(1+\alpha_B)y}+a_{13}e^{-(1+2\alpha_B)y}+a_{14}e^{-\Pr y}+a_{15}e^{-ny}+a_{16}e^{-y}+a_{17}$$

Now using $u_2(0)=u_2(\infty)=0$, we have

$$u_2(y)=\ a_9 e^{-(\Pr+2\alpha_B)y}+a_{10}e^{-(n+2\alpha_B)y}+a_{11}e^{-(1+4\alpha_B)y}+a_{12}e^{-2(1+\alpha_B)y}+a_{13}e^{-(1+2\alpha_B)y}+a_{14}e^{-\Pr y}+a_{15}e^{-ny}+a_{16}e^{-y} \qquad (4.11)$$

where $a_9=\dfrac{M^2a_3}{(\Pr+2\alpha_B)(\Pr+2\alpha_B-1)}$, $a_{10}=\dfrac{M^2a_4}{(n+2\alpha_B)(n+2\alpha_B-1)}$, $a_{11}=\dfrac{M^2a_5}{4\alpha_B(1+4\alpha_B)}$,
$a_{12}=\dfrac{M^2a_7}{(1+\alpha_B)(1+2\alpha_B)}$, $a_{13}=\dfrac{M^2(a_6+2a_8)}{2\alpha_B(1+2\alpha_B)}$, $a_{14}=\dfrac{\Pr\phi_1 a_1}{\mathrm{Re}\Pr(\Pr-1)}$, $a_{15}=\dfrac{n\phi_0 a_2}{\mathrm{Re}\, n(n-1)}$
$a_{16}=-(a_9+a_{10}+a_{11}+a_{12}+a_{13}+a_{14}+a_{15})$
Now, from (4.6.3)

$$\frac{d^2B_2}{dy^2}+\alpha_B\frac{dB_2}{dy}=-\mu_0\left[a_3\Pr e^{-(\Pr+\alpha_B)y}+na_4e^{-(n+\alpha_B)y}+a_5(1+2\alpha_B)e^{-(1+3\alpha_B)y}+a_7e^{-(2+\alpha_B)y}+(a_8+a_6)e^{-(1+\alpha_B)y}\right]-\mu_1e^{-(1+\alpha_B)y}$$

then $B_{2p}(y)=a_{17}e^{-(\Pr+\alpha_B)y}+a_{18}e^{-(n+\alpha_B)y}+a_{19}e^{-(1+3\alpha_B)y}+a_{20}e^{-(2+\alpha_B)y}+a_{21}e^{-(1+\alpha_B)y}$
and $B_{2c}(y)=a_{22}e^{-\alpha_B y}+a_{23}$.
Combining $B_{2p}(y)$ and $B_{2c}(y)$, and applying the boundary conditions we have

$$B_2(y)=a_{17}e^{-(\Pr+\alpha_B)y}+a_{18}e^{-(n+\alpha_B)y}+a_{19}e^{-(1+3\alpha_B)y}+a_{20}e^{-(2+\alpha_B)y}+a_{21}e^{-(1+\alpha_B)y}+a_{22}e^{-\alpha_B y} \qquad (4.12)$$

where $a_{17}=-\dfrac{\mu_0\Pr a_3}{\Pr(\Pr+\alpha_B)}$ $a_{18}=-\dfrac{\mu_0 na_4}{n(n+\alpha_B)}$ $a_{19}=-\dfrac{\mu_0(1+2\alpha_B)a_5}{(1+3\alpha_B)(1+2\alpha_B)}$ $a_{20}=-\dfrac{\mu_0a_7}{2(2+\alpha_B)}$
$a_{21}=-\dfrac{\mu_0(a_8+a_6)-1}{\alpha_B(1+\alpha_B)}$ $a_{22}=-(a_{17}+a_{18}+a_{19}+a_{20}+a_{21})$. So that

$$\begin{aligned}B(y)&=B_0(y)+\varepsilon B_1(y)+\varepsilon^2B_2(y)\\ u(y)&=u_0(y)+\varepsilon u_1(y)+\varepsilon^2u_2(y)\end{aligned} \qquad (4.13)$$

5.0 Skin friction

The physical quantity of this problem is the shearing stress at a point on the plate. This is called skin friction coefficient C_f which is defined by

$$C_f=\frac{\tau_f}{\rho u_w v_w}=\frac{du}{dy}(0)\quad \tau_f=\mu\left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad (5.1)$$

According to the analytical solution reported earlier C_f, then take the form

$$
\begin{aligned}
C_f = & -1+\varepsilon\left(-a_3 \Pr - a_4 n + a_5\left(-1-2\alpha_B\right)-a_6\right)+\varepsilon^2\left(-a_9\left(\Pr+2\alpha_B\right)\right.\\
& -a_{10}\left(n+2\alpha_B\right)+a_{11}\left(-1-4\alpha_B\right)+a_{12}\left(-2-2\alpha_B\right)\\
& \left.+a_{13}\left(-1-2\alpha_B\right)-a_{14}\Pr - a_{15}n - a_{16}\right)
\end{aligned}
$$

6. 0 Result and discussion

The investigations on this problem is carried out using Pr = 0.71, Sc = 0.6, Grt = 5, Grc = 4, Ri = 0.3, ϕ = -1.2, α_B = 0.5, $\mathrm{Re}=-10$ and M = 0.5 except where stated otherwise. It should be noted that Grt > 0 and Grt < 0 represent heating of the plate and cooling of the plate respectively. Also, $\phi \succ 0$, ϕ = 0 and ϕ < 0 indicates heat absorption, no heat generation/absorption and heat generation respectively.

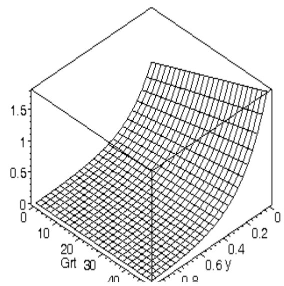

Figure 1: Magnetic field for several values of thermal Grashof number

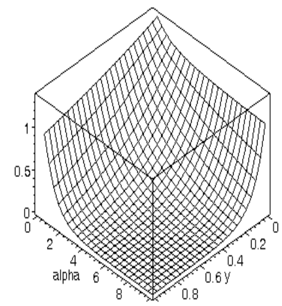

Figure 2: Magnetic field for several values of magnetic

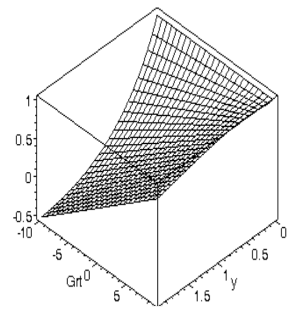

Figure 3: Velocity field for several Grashof number

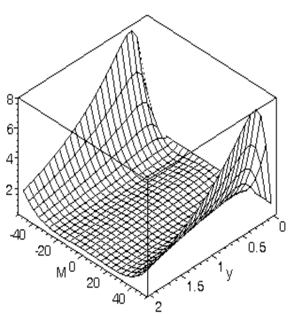

Figure 4: Velocity field for several values of thermal values of magnetic induction

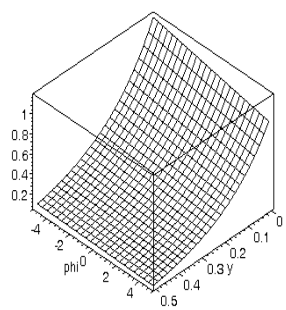

Figure 5: Magnetic field for several values of heat generation/absorption

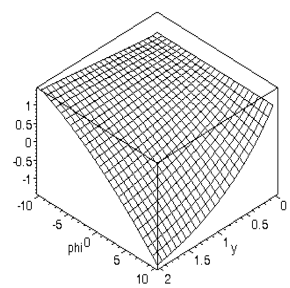

Figure 6: Velocity field for several values of heat generation/absorption.

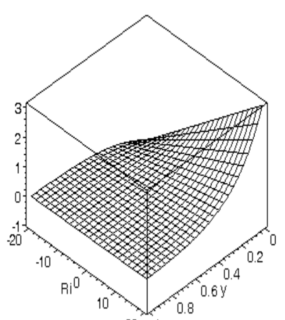

Figure 7: Magnetic field for several values of Richardson number

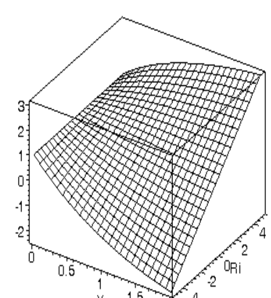

Figure 8: Velocity field for several values of Richardson number

The figures were displayed in 3 – dimensional form to give insight to what happens within the fluid in the channel as the chosen parameter increases or decreases. In figures 1 and 3, we display the magnetic and velocity profiles against heat generation/absorption (ϕ) and thermal Grashof number (Grt). It was discovered that increase in the thermal buoyancy increases both magnetic and velocity field. Figure 2 shows magnetic field profile as $\alpha_{\square}$ increases. It could be seen that as $\alpha_{\square}$ increases, the magnetic field reduces as flow progresses. While in figure 4, we show the effect of magnetic induction on the velocity field. It is obtained that maximum velocity exist at higher values of magnetic induction, this is likened to the push effect due to induction as an added factor to the flow velocity. For each value of M, the velocity reduces along the flow channel. Figures 5 and 6 shows the effect of heat generation/absorption on both magnetic and velocity fields. It could be seen that heat generation increases both the magnetic and velocity fields and vice versa. Whilein figures 7 and 8, the effect of Richardson number Ri. We observe that for each choice of Ri, the fields (velocity and magnetic) decreases along y – direction, and increase in Ri brings about increase in the magnetic and velocity fields.

Figures 9 - 14 present the effect of changing $\phi, Grt, M, \mathrm{Re}, Ri, \varepsilon$ on the values of the skin friction coefficient. It can be seen, in figure 9 that maximum skin friction coefficient is recorded when ϕ=-0.6975369003, we will obtain C_f = 0 when ϕ =-7.099538382 and 5.704464581. While figure 10 shows a linear relationship between the thermal Grashof number and the skin friction coefficient as a decreasing function Grt, which will be zero when Grt = 0. In figure 11 there are 3 – distinct extremes as a result of the reflexivity of the magnetic induction. This maxima occurs at M = 0, -3.22110442 and 3.22110442 with the maximum at M = 0 is small compare to that at the other points. This is clearly seen in figure 4. In figure 12, a rapid increase in skin friction is noticed between $0.1 < Ri < 1$, this established an ideal situation where $0 < Ri < 1$, after which the skin friction coefficient increases slowly. Figure 13 shows that skin friction coefficient increases as heat deposit Q per unit mass increases. We displayed in figure 14 the effect of ε on C_f. The maximum C_f =1.58 in this case occur at ε =0.4766108

7.0 Conclusion

The MHD of a vertical permeable surface with buoyancy is considered. The cases of heating and cooling of the channel are examined and the corresponding effects on the flow shows that velocity is enhanced in the heating of the plate thereby bring about increase in the velocity. Also, the magnetic induction is supported by the buoyant force which resultd into a higher velocity as the Hartmann number increases. The observation on the magnetic and velocity fields could be enumerated as below:

1. Increase in thermal Grashof number brings about increase both magnetic and velocity fields.
2. Increase in magnetic Prandtl number α_B, reduces the magnetic field.
3. Heat generation increases both magnetic and velocity fields
4. Maximum skin – friction coefficient occur when heat generation is -7.1

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