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Numerical solution of heat and mass transfer in MHD flow in the presence of chemical reaction and Arrhenius heat generation of a stretched vertical permeable membrane

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Abstract

We present a magnetohydrodynamic flow of a uniformly stretched vertical permeable surface undergoing Arrhenius heat reaction. It is shown that the temperature, concentration and the velocity fields depend on the chemical reaction parameter. The values of temperature field increase as the order of the reaction increases, while that of velocity field decreases as the order reaction increases. Moreover, the reactant field decreases faster as we move away form the wall as we increase the reaction parameter. This paper also shows that the temperature field and reacting layers get thinner as the heat deposit Q per unit mass increases while the velocity field and the boundary layer get thinner as thermal Grashof number increases. We also show that magnetic induction and cooling of the plate (thermal Grashof number Grt>0) lowers the velocity field.

***Keywords*:** Permeable surface, source term, thermal expansion, suction, ambient, magnetic induction.

Classification: 76W05

1.0 Introduction

Magneto - hydrodynamic (MHD) flow of electrically conducting fluids in the presence of magnetic field is encountered in many problems in geophysics and astrophysics, Ayeni R. O. Okedoye A. M. Balogun F. O. and Ayodele T. O. (2004) [1], Chamkha (2003) [2] and the literature cited therein. Also many industrial processes involve fluid flow, heat and mass transfer in the boundary layers induced by a surface moving with a uniform velocity.

Nomenclatures

c: concentration
c_p: specific heat
C: fluid concentration
C_w: wall concentration
α_D: mass diffusion-coefficient
D: chemical reaction parameter
Q_0 : heat generation coefficient
Nu : Nuselt number
$\Pr$: Prandtl number
Sc : Shmidt number
E: activation energy
g : gravitational pull
B_0 : magnetic induction
T : fluid temperature
n: reaction order
R: universal gas constant
T_w : wall temperature
v : fluid transverse velocity
v_W : suction velocity
x : axial or vertical coordinate
y : transverse coordinate

Greek Symbols

γ : chemical reaction parameter
ρ : fluid density

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Greek Symbols
γ : chemical reaction parameter
ρ : fluid density
θ : dimensionless temperature
R_i : Richardson number
σ : electric conductivity
β_τ : coefficient of thermal expansion
β_c : coefficient of mass expansion

Subscripts
w : condition on the wall
∞ : ambient condition

Chamkha (2003) [2] examined the boundary layer of an MHD flow when the heat generation is linear in temperature. Ayeni et al (2004) [1] extended the problem posed in Chamkha (2003) [2] to heat generation that is quadratic in temperature. Sakiads, (1996) studied the boundary – adjacent to a continuous moving surface. He obtained solutions by approximate and exact methods of momentum boundary – layer equations, with no heat transfer on flat and cylindrical surfaces. The corresponding heat transfer problems were considered experimentally by Griffin and Throne (1967). Vajravelu and Hadjinicolaou (1990) [5], reported on convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. Other examples of studies dealing with hydro – magnetic flows can be found in the papers by Gray (1980). Michiyoshi et al. (1976) and Funmizawa (1979) [4].

Okedoye and Ayeni (2007) [6] examined the problem of MHD and, mass and heat transfer when the chemical reaction is of order zero. Since most chemical reaction in conducting fluid is of order > 0, the present work is more appropriate. In this paper we extend the problem posed by Chamkha to Arrhenius heat generation and chemical reaction of order n, so that previous cases of heat generation become special cases of the present paper.

2.0 Mathematical formulation

Consider coupled heat and mass transfer by hydro – magnetic flow of a continuously moving vertical permeable surface in the presence of suction, heat generation/absorption effects, transverse magnetic field effect and Arrhenius reactions. The flow is assumed steady, laminar and two – dimensional and the surface is maintained at a uniform temperature and the concentration species, and is assumed to be infinitely long. It is also assumed that the applied transverse magnetic Reynolds number is small so that the induced magnetic field is neglected. In addition, there is no applied electric field and all of the Hall effect, viscous dissipation and Joule heating are neglected, thermo - physical properties are assumed constant except the density in the buoyancy terms of the momentum equation which is approximated according to the Boussinesq approximation.

With these assumptions, the steady equations that describe the physical situation are given as

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial^2 y} + g\beta_\tau\left(T-T_\infty\right) + g\beta_C\left(c-c_\infty\right) - \frac{\sigma B_0^2 u}{\sigma} \tag{2.2}$$

$$pc_p\, v\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + Q\left(c-c_\infty\right)^n e^{-E/RT} \tag{2.3}$$

$$v\frac{\partial u}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \gamma\left(c-c_\infty\right)^n e^{-E/RT} \tag{2.4}$$

where y is the horizontal or transverse coordinate, it is the axial velocity, *v* is the transverse velocity, *T* is the fluid temperature, *c* is the concentration, $T\infty$ is the ambient temperature $c\infty$ is the ambient concentration, and ρ, g, B_T, β_c v σ, B_c, Q, D, γ and n are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, kinematics viscosity, fluid electrical conductivity, magnetic induction, heat generation/coefficient and the chemical reaction parameter and real number respectively.

With the physical boundary conditions

$$\begin{aligned} &u(0)=u_w, v(0)=-v_w, T(0)=T_w, C(0)=C_w \\ &as\ y \to \infty, u \to \infty, T \to \infty, C \to \infty, \end{aligned} \tag{2.5}$$

where u_w, $v_w > 0$, T_w and c_w are surface velocity, suction velocity, surface temperature and concentration respectively.

3.0 Method of solution

We use the non – dimensional variables

$$y^1 = yv_w / \upsilon,\ u^1 = \frac{u}{u_w},\ \theta = (\mathrm{T}-\mathrm{T}_\infty)\frac{E}{RT_\infty^2},\ c^1 = \frac{c-c_\infty}{c_w-c_\infty} \tag{3.1}$$

From equation (2.1) i.e $\frac{dv}{dy}=0$ on integrating we have v = constant

but $v(0)=-v_w$ = constant, $\Rightarrow v(y)=-v_w$ (3.2)

Substituting equations (3.1) and (3.2) and dropping (‘), we have

$$\frac{d^2u}{dy^2}+\frac{du}{dy}+G_{r\mathrm{T}}\,\theta+G_{rc}\,c-M^2u=0 \tag{3.3}$$

$$\frac{d^2\theta}{dy^2}+p_r\frac{d\theta}{dy}+b\Pr\phi C^n e^{\frac{\theta}{1+\varepsilon\theta}}=0 \tag{3.4}$$

$$\frac{d^2c}{dy^2}+s_c\frac{dc}{dy}-KScbC^n e^{\frac{\theta}{1+\varepsilon\theta}}=0 \tag{3.5}$$

The dimensionless boundary conditions becomes

$$\begin{aligned} &u(0) = 1,\ \theta(0) = 1,\ c(0) = 1 \\ &y \to \infty\ u \to 0,\ \theta \to 0,\ c \to 0 \end{aligned} \tag{3.6}$$

where

$$G_{r\mathrm{T}}=\frac{B_\mathrm{T}\,v_g\,RT_o^2}{U_w\,EV_w^2},\ G_{rc}=g\frac{B_c\,v(C_w-C_\infty)}{U_w V_w^2},\ P_r=\frac{\mu cp}{k},\ S_c=\frac{V_w}{D},\ M^2=\frac{\sigma\beta_o v}{\rho v_w^2},$$

$$\frac{vQ}{\mu cpv_w^2}=\frac{\upsilon Q}{\rho c_p v_w{}^2}\cdot\frac{\upsilon\rho c_p}{k}=\phi\Pr,\ \frac{vQ}{\mu cpv_w^2}\frac{RT_0^2}{E}=\varepsilon\phi\ \Pr,\ b=(C_w-C_\infty)^n\frac{E}{RT_o^2}e^{-\frac{1}{\varepsilon}},$$

We now re – write the system of equations (3.3) – (3.5) as a system of 1st – order equations.

Let, $y=X_1, C=X_2, \theta=X_3, u=X_4, C^1=X_5, \theta^1=X_6, u^1=X_7$

And in vector form we have

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \end{pmatrix}^1 = \begin{pmatrix} 1 \\ X_5 \\ X_6 \\ X_7 \\ Sc(KScX_2 - X_5)bX_2^n e^{\frac{X_3}{1+\varepsilon x_3}} \\ -\Pr(X_6 + b\phi X_2^n \frac{X_3}{1+\varepsilon x_3} \\ M^2 X_4 - aGrcX_2 - Gr_T X_3 - X_7 \end{pmatrix}$$

subject to

$$\begin{pmatrix} X_1(0) \\ X_2(0) \\ X_3(0) \\ X_4(0) \\ X_5(0) \\ X_6(0) \\ X_7(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$$, where $a_i, i=1,..3$ are guess values.

The essence of the above system is to transform the second order boundary value problem to a system of 1[st] order initial value problem. We continue to adjust the values of $a_i, i=1,..3$ until $u \to 0$, $\theta \to 0$, $c \to 0$, as $y \to \infty$ is satisfied. We then use Runge–Kutta of order four to solved the problem and the result for various choice of parameters were displayed graphically in figures 1 – 9.

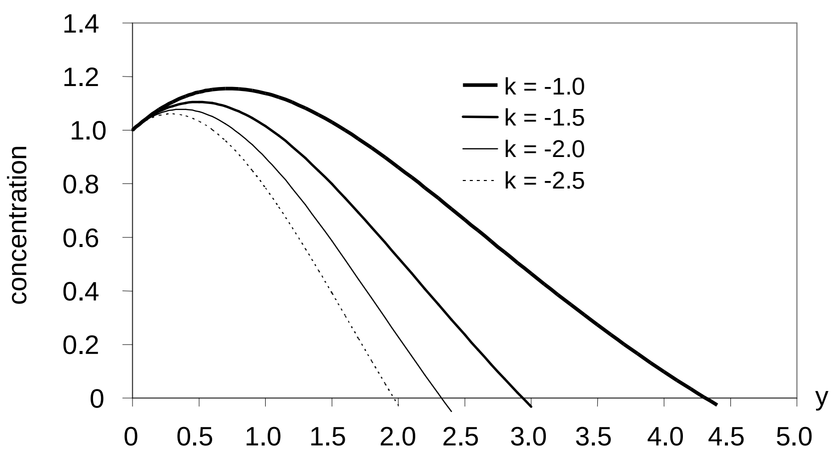

Figure 1: Concentration profile for various values of *k*

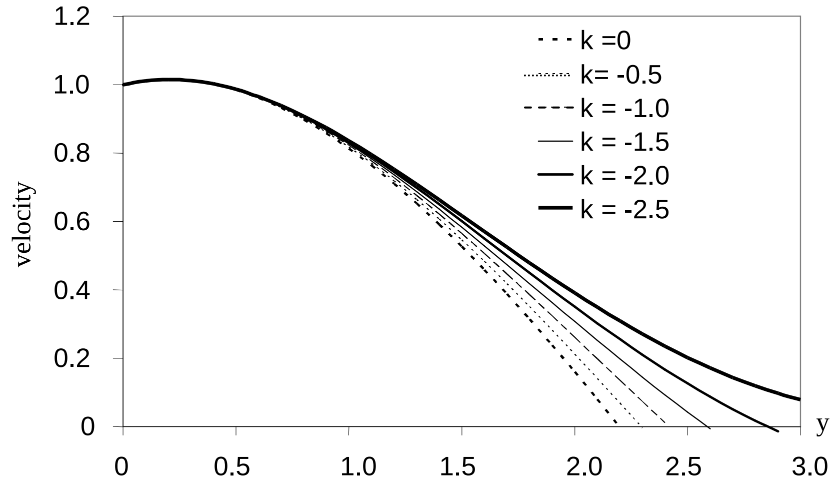

Figure 2: Velocity profile for various values of *k*

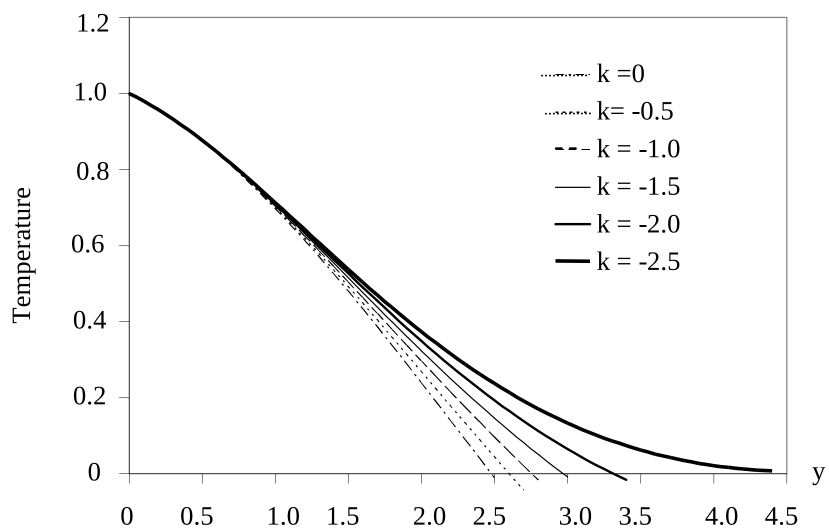

Figure 3: Temperature profile for various values of *k*

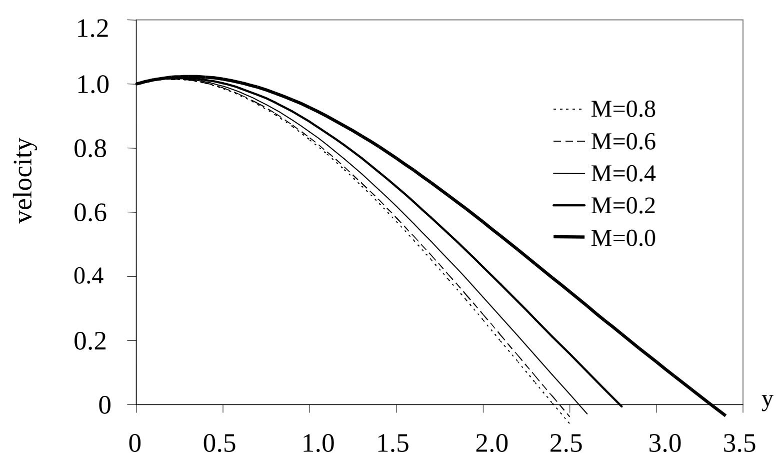

Figure 4: Velocity profile for various values of *M*

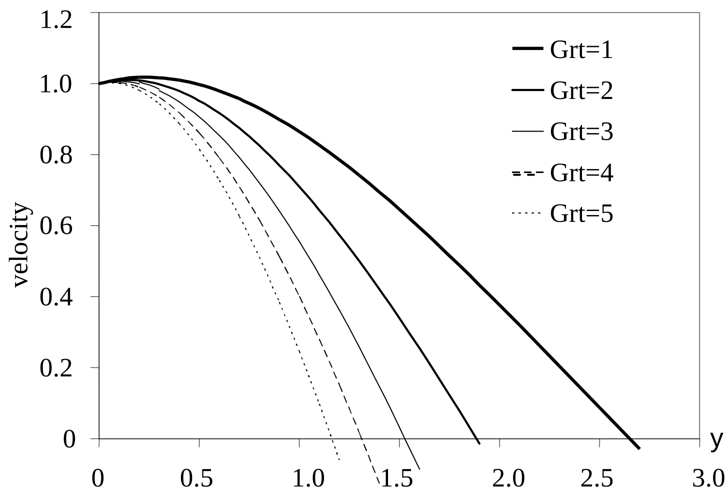

Figure 5: Velocity profile for various values of *Grt*

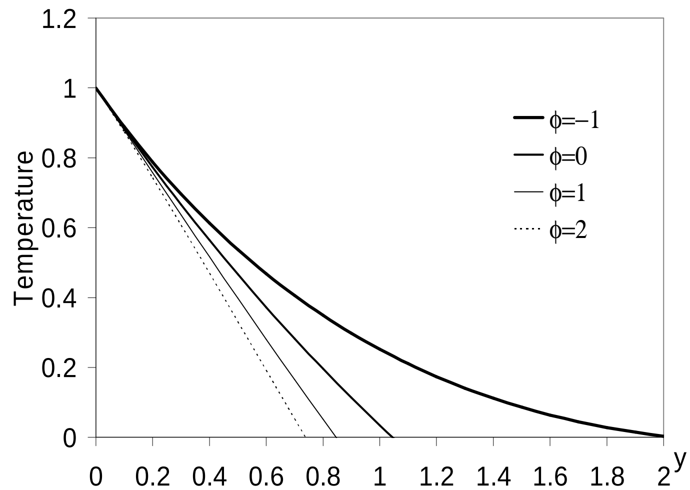

Figure 6: Temperature profile for various values of ϕ

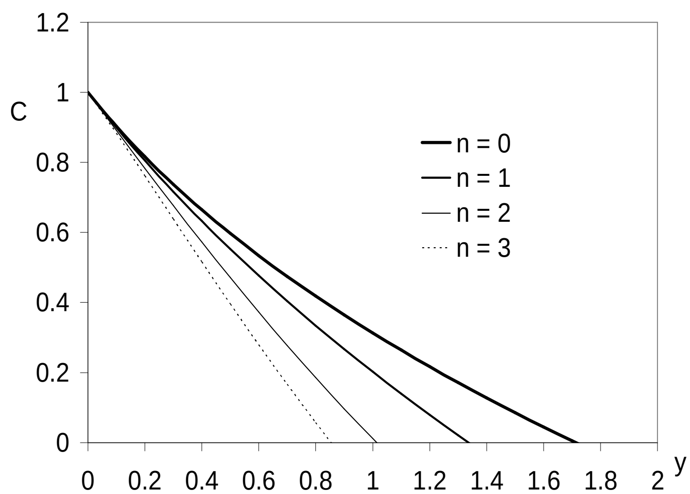

Figure 7: Concentration profile for various values of *n*

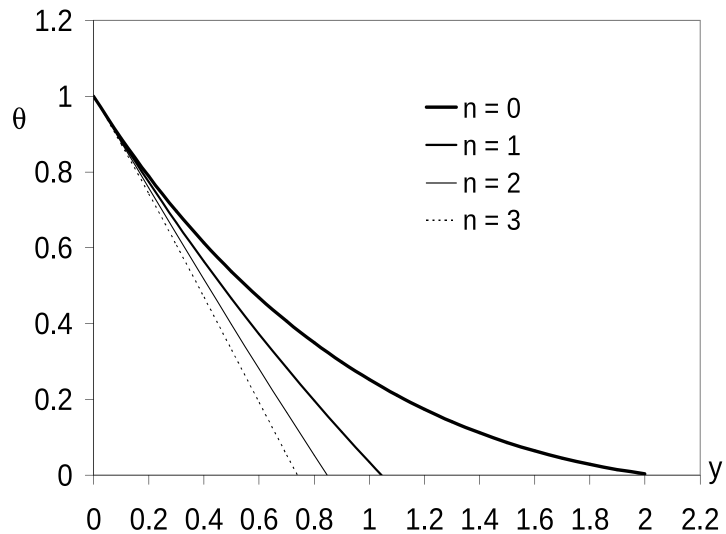

Figure 8: Temperature profile for various values of *n*

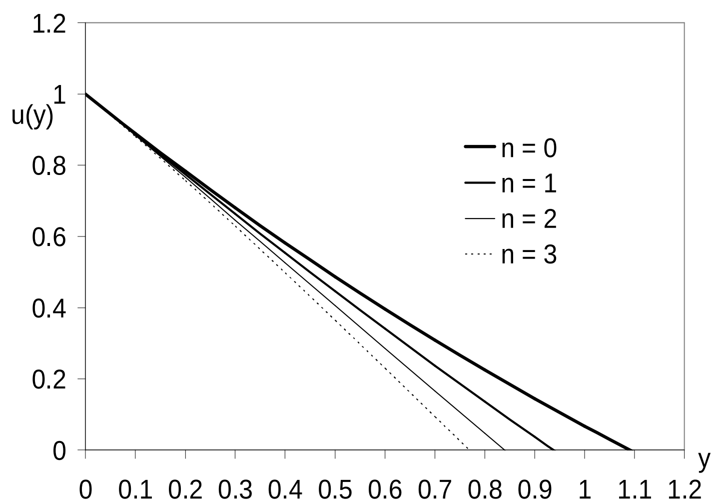

Figure 9: Velocity profile for various values of *n*

4.0 Discussion of results

In the analysis presented here, the parameters $\phi_0, K, Ri, \Pr$ *and* Sc are assigned values – Grt = 1, Grc=1, Pr=0.71, Sc=0.6, ϕ = -0.2, and M=0.5 except where stated. It should be noted that K > 0 indicate a destructive chemical reaction, while K< 0 correspond to a generative chemical reaction. Also, ϕ_0 < 0 indicate heat generation and ϕ_0 >0 imply heat absorption. And K=0 and ϕ_0 =0 indicate no chemical reaction and no heat generation/absorption effects respectively. Also $Grt > 0$ indicate cooling of the plate while $Grt < 0$ indicate heating of the plate.

We display in figures 1 – 9, the temperature profile, velocity profile and concentration profile for various values of the parameters. Figure 1 show that the concentration layer gets thinner as the rate of reactant consumption increases. On the other hand, in figures 2 and 3, the velocity and temperature layer increases as the rate of reactant consumption per unit mass increases. Figures 4 and 5 show the effect of magnetic induction and thermal Grashof number the velocity field. In figure 4, we show that velocity layer decreases as magnetic induction M increases, this is due to the opposing force put up in the flow by

Lorenz force which always oppose the fluid velocity. On the other hand, figure 5 shows that the velocity layer decreases as thermal Grashof number Grt increases as a result of cooling of the channel plate. Figure 6 show that the temperature layer gets thinner as the heat deposit increases. Chemical reaction is being consumed more as the other of reaction increases. This is shown in figure 7, as we discovered that the concentration profile reduces as n increases. This effect of n on concentration field also affect the other fields – temperature and velocity fields. Figures 8 and 9 shows the effect of reaction order on velocity and temperature fields. We could see that increase in reaction order result in increase in temperature field, as more reaction is consumed the temperature of the system increases and vice – versa, while from figure 9, consumption of the chemical species reduces the strength of flow and thus increase in reaction order lowers the velocity field. The boundary layers of the three fields gets thinner as the reaction order increases.

5.0 Concluding remarks

Numerical solutions for mass and heat transfer by steady laminar flow of an electrically conducting and heat generating/absorbing fluid on a uniformly moving vertical permeable surface with buoyancy in the presence of Arrhenius heat generation/absorption and a first order chemical reaction were reported. Based on the obtained results, the following conclusions were deducted:

(1) Maximum velocity reduces in value as both Grt and K increases.

(2) A maximum concentration occurs during a generative chemical reaction.

(3) Increases in destructive chemical reaction produces lower species concentration and vice – versa.

(4) The fluid body temperature increases with increase in heat generation $(\phi_0 < 0)$, and a decreases a with a decrease in $(\phi_0 > 0)$.

(5) Velocity and temperature fields increase with increase in generative chemical reaction.

(6) Velocity field decreases with increase in both Hartmann number M and cooling of the plate (Grt > 0).

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