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# The effects of permeability and radiation on the stability of plane Couette-Poiseuille flow in a porous medium

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***Abstract***

***A study on the effects of permeability and radiation on Couette-Poiseuille flow stability was carried out. Solutions to the governing hydrodynamic equations was developed using the method of undetermined coefficients. On the basis of linear theory using analysis of normal modes, it was observed that both parameters, independently affect the stability of Couette-Poiseuille flow but that of radiation is prominent at high wave numbers and Reynolds number regime.***

## 1.0 Introduction

Couette flow results when two plates moving relative to each other cause a flow of fluid between them whereas Poiseuille flow is observed as the steady laminar flow of an incompressible fluid between two parallel plates. Stability of Couette flow problems is widely accepted to have started with Rayleigh [11] and ever since the trend has been that of steady increase in different methods and configurations [6, 14, 15 ]. If we consider literatures of poiseuille flow [5, 9, 10, 13], the story is not different perhaps due to the very similarity of the two types of flows. A study of the combination of the two flows also commenced. Corenflos et al [4] carried out an experimental and numerical study of a plane couette- poiseuille flow as a test case for turbulence modeling from experimental data available for both the developing and developed flows. Spurk [12] also studied Couette-Poiseuille flow by developing general solution wherein he distinguished between them by setting $k_p$ =0 for Couette flow, U=0, $k_p \neq 0$ for poiseuille flow and U$\neq 0$, $k_p \neq 0$ for Couette- Poiseuille flow.

Recently, an excellent investigation on the Anchoring distortions coupled with plane Couette –Poiseuille flows of nematic polymers in viscous solvents: Morphology in molecular orientation, stress and flow was examined by Zhou and Forest [16] to model and simulate processing-induced heterogeneity in rigid, rod –like nematic polymers in viscous solvents .In the study so far highlighted, it is apparent that the combined effect of radiation and permeability as often been neglected whereas its importance in structural Engineering, Geology, and Geophysics cannot be over-emphasized. Hence, in this paper, our goal is to investigate the stability or otherwise of Couette-Poiseuille flow in the presence of radiation and permeability at varying Reynolds number regime.

## 2.0 Mathematical formulation

The study under consideration simplify the hydrodynamic equations of continuity, Navier-Stokes and energy respectively as

$$\frac{\partial \rho}{\partial t} = -\nabla . \rho V \qquad (2.1)$$

$$\rho \frac{\partial V}{\partial t} = -\nabla P + \mu \nabla^2 V + \rho g \qquad (2.2)$$

$$\frac{dT}{dt} = a^2 \nabla^2 T \qquad (2.3)$$

where P is pressure of fluid, $\rho$ is fluid density, g is acceleration due to gravity, $\mu$ is absolute viscosity and $t$ is temperature. Others are $a^2 \left( \frac{k}{\rho C_V} \right)$, thermal diffusivity and $\nabla$ is a del operator. Invoking the radiative [3] and permeability [8] terms, transform (3.2) and (2.3) respectively.

$$\rho \frac{\partial V}{\partial t} = -\nabla P + \mu \nabla^2 V + \rho g - \frac{\upsilon}{K} V \qquad (2.4)$$

$$\frac{dT}{dt} = a^2 \nabla^2 T + \delta^2 (T - T_0) \qquad (2.5)$$

where $\upsilon$ is kinematic viscosity, $K$ is permeability of the medium under consideration, and $\delta^2$ is defined as $\delta^2 = 4\int_0^\infty \left( \alpha_{K*} \frac{\partial B}{\partial T} \right) dK*,$ B is Planck's function, $\alpha_{K*}$ is absorption coefficient and $k*$ is frequency of radiation.

## 3.0 Perturbation

Denoting the disturbance in the velocity, temperature and pressure field by

$$V' = V - V_e, T' = T - T_e, and, P' = P - p_e \qquad (3.1)$$

where subscript e denotes equilibrium position. If we put (3.1) in (2.1), (2.4) and (2.5) and neglect all terms greater than unity while also considering the Z-axis as the direction of flow, we obtain the following linearized equations

$$\frac{\partial V'}{\partial Z'} = 0 \qquad (3.2)$$

$$\rho \frac{\partial V'}{\partial t} = -\frac{\partial P'}{\partial Z'} + \mu \frac{\partial^2 V'}{\partial Z'^2} + \rho g - \frac{\upsilon}{K} V' \qquad (3.3)$$

$$\frac{dT'}{dt} = a^2 \frac{\partial^2 T'}{\partial Z'^2} + \delta^2 (T' - T'_0) \qquad (3.4)$$

## 4.0 Non-dimensional analysis

Applying the following dimensionless quantities:

$$Z = \frac{Z'}{d}, P = \frac{P'}{\rho V^2}, \alpha = \frac{\delta^2 d^2}{\upsilon},$$

$$K_0 = \frac{\upsilon \mu d^2}{K \rho}, V = \frac{V'}{U}, \beta = \frac{a^2 \rho t}{T_\infty},$$

$$g = \frac{gd}{V^2}, \quad \theta = \frac{T' - T_0}{T - T_Z},$$

$$\mathrm{Re}^{-1} = \frac{\mu}{Vd}, t = \frac{t'}{t}$$

to (3.2), (3.3) and (3.4) results in

$$\frac{\partial V}{\partial Z} = 0 \tag{4.1}$$

$$\rho \frac{\partial V}{\partial t} = -\frac{\partial P}{\partial Z} + \mathrm{Re}^{-1} \frac{\partial^2 V}{\partial Z^2} + \rho g - K_0 V \tag{4.2}$$

$$\frac{d\theta}{dt} = \beta \frac{\partial^2 \theta}{\partial Z^2} + \alpha\theta \quad (4.3$$

## 5.0 Analytical solution

If we assume a solution of the form

$$\theta = A \exp \lambda Z \tag{5.1}$$

then, we arrive at the feasible solution after the boundary condition $\theta(0 = 0)$ has been imposed, as

$$\theta(Z) = C \sin\left(\frac{\alpha}{\beta}\right)^{0.5} Z \tag{5.2}$$

where $C$ is a constant. Employing the boussinesg approximation

$$\Delta\rho = -\rho_0 E(T - T_0) \tag{5.3}$$

where, $\Delta\rho$ is change in fluid density, $\rho_0$ is fluid density at some properly chosen $T_0, T_0$ is temperature at which $\rho = \rho_0$ and E is coefficient of volume expansion. Dimensionless form of (5.3) results in

$$\Delta\rho = -\rho_0 E\theta \tag{5.4}$$

where $\rho$ is the perturbed fluid density. Substituting (5.4) in (5.2) and the overall in (4.2) results in

$$-\frac{\partial P}{\partial Z} = \mathrm{Re}^{-1} \frac{\partial^2 V}{\partial Z^2} - g\rho_0 C \sin\left(\frac{\alpha}{\beta}\right)^{0.5} Z - K_0 V \tag{5.5}$$

If we take $\rho_0 EgC = H$ and assume $\frac{\partial P}{\partial Z} = -K_p$ (constant), then (5.5) takes the form

$$\frac{\partial^2 V}{\partial Z^2} - \mathrm{Re}\, K_0 V = \mathrm{Re}\, H \sin\left(\frac{\alpha}{\beta}\right)^{0.5} Z - \mathrm{Re}\, K_p \tag{5.6}$$

The solution of (5.6) is

$$V(Z) = C_1 \cosh(\mathrm{Re}\, K_0)^{0.5} Z + C_2 \sinh(\mathrm{Re}\, K_o)^{0.5} Z + \left(\frac{\mathrm{Re}\, H}{-\frac{\alpha}{\beta} + \mathrm{Re}\, K_o}\right) \sin\left(\frac{\alpha}{\beta}\right)^{0.5} Z + \frac{K_p}{K_o} \tag{5.7}$$

Assuming that the fluid velocity at the wall of the plates is equal to the wall velocity,

then the boundary condition is satisfied by

$$V(0) = 0,\ V(d) = V \tag{5.8}$$

Applying the boundary conditions (5.8) gives the solution as

$$V(Z) = -\frac{K_p}{K_0} \cosh(\mathrm{Re}\ K_0)^{0.5} Z + \left(V - \frac{\mathrm{Re}\ H}{-\frac{\alpha}{\beta} + \mathrm{Re}\ K_0} - \frac{K_p}{K_0}\right) \sinh(\mathrm{Re}\ K_0)^{0..5} Z$$
$$+ \left(\frac{\mathrm{Re}\ H}{-\frac{\alpha}{\beta} + \mathrm{Re}\ K_0}\right) \sin\left(\frac{\alpha}{\beta}\right)^{0.5} Z + \frac{K_p}{K_0} \tag{5.9}$$

For Couette flow, $K_p = 0$ in (5.9). For Poiseuille flow, $V = 0$, $K_p \neq 0$ in (5.9). (5.9) can be written in parts as

$$V(Z) = -\frac{K_p}{K_0}\cosh(\operatorname{Re} K_0)^{0.5} nZ + \frac{K_p}{K_0}, \text{ for } n = 2, 4, 6,\ldots \tag{5.10}$$

and

$$V(Z) = +\left(V - \frac{\operatorname{Re} H}{-\frac{\alpha}{\beta} + \operatorname{Re} K_0} - \frac{K_p}{K_0}\right)\sinh(\operatorname{Re} K_0)^{0.5} nZ + \left(\frac{\operatorname{Re} H}{-\frac{\alpha}{\beta} + \operatorname{Re} K_0}\right)\sin\left(\frac{\alpha}{\beta}\right)^{0.5} nZ + \frac{K_p}{K_0},.$$
$$for, n = 1,3,5\cdots \tag{5.11}$$

## 6.0 Analysis into normal modes

Following Bestman [1] and Bestman and Opara [2], we examine the stability of these modes individually. The analysis can be made in terms of two dimensional periodic wave numbers. Thus we assign to all quantities describing the perturbation on *X, Y*, and *t* in the form

$$\exp\left[i\left(K_x X + K_y Y\right) + \tau t\right] \tag{6.1}$$

where $\tau$ is the time constant, and $\left(K^2{}_x + K_y^2\right)^{0.5}$ is given as $\kappa$ ,the resultant wave number of the disturbance. Employing the non-dimensional variables $a = \kappa$ d, $\sigma = \frac{\tau d^2}{\kappa}$, we write

$$\left.\begin{aligned}
&(V,\theta,P) = (V(Z),\theta(Z),P(Z))\exp\left[i\left(K_x X + K_y Y\right) + \tau t\right]\\
&\nabla^2 = \frac{\partial^2}{\partial Z^2} - \kappa^2\\
&-\kappa^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\\
&D = \frac{d}{dZ}
\end{aligned}\right\} \tag{6.2}$$

If we put (6.2) in (2.4) and (2.5) following Hocking [7] and eliminate pressure, we will obtain

$$\left[\sigma\frac{k}{\upsilon} - \left(D^2 - a^2 - K_0\right)\right]\left(D^2 - a^2\right)V = -Ra^2\theta \tag{6.3}$$

and

$$\left[\sigma - \left(D^2 - a^2 - \frac{\alpha}{\beta}\right)\right]\theta = V \tag{6.4}$$

Coupling (6.3) and (6.4) following Opara [2]

$$\left[\sigma\frac{\kappa}{\upsilon} - \left(D^2 - a^2 - K_0\right)\right]\left(D^2 - a^2\right)\left[\sigma - \left(D^2 - a^2 - \frac{\alpha}{\beta}\right)\right]\theta = -Ra^2\theta \tag{6.5}$$

where *a* is the resultant dimensionless wave number and $R = \frac{g\alpha E d^2}{k\upsilon}$ is the Rayleigh number. To find the critical value of R as a function of a we set $\sigma = 0$ and (6.5) reduces to

$$\left[-\left(D^2-a^2-K_0\right)\left(D^2-a^2\right)\left(D^2-a^2-\frac{\alpha}{\beta}\right)\right]\theta=-Ra^2\theta \qquad (6.6)$$

Following Hocking [7], the proper solution for $\theta$ appropriate for the lowest mode is

$$\theta = A\sin \pi Z \qquad (6.7)$$

If we put (6.7) in (6.8) and after simplification, results in

$$\left[-\left(\pi^2+a^2+K_0\right)\left(\pi^2+a^2\right)\left(\pi^2+a^2+\frac{\alpha}{\beta}\right)\right]=-Ra^2 \qquad (6.8)$$

## 7.0 Results

$K_p = 2, K_0 = 1, X = 1-5, V = 1, H = 1$

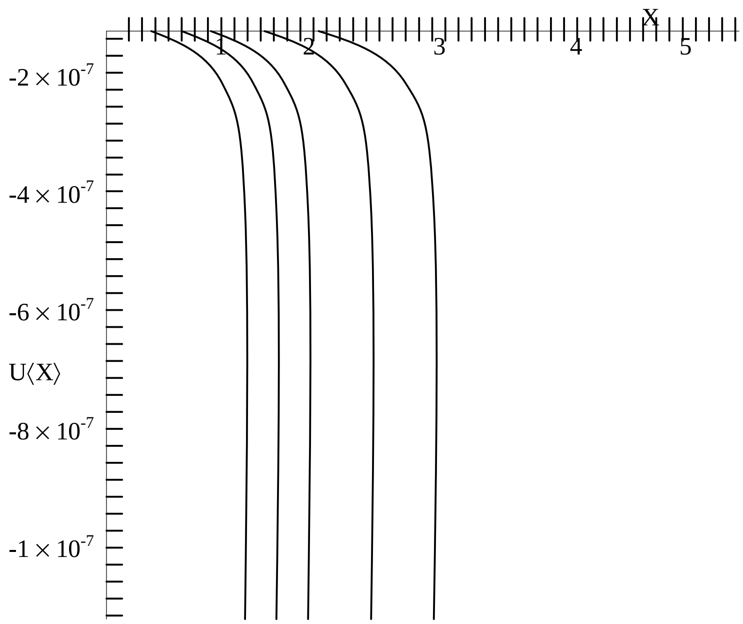


Figure 1: The dependence of the velocity on both the direction *X* and the Reynolds' number Re.

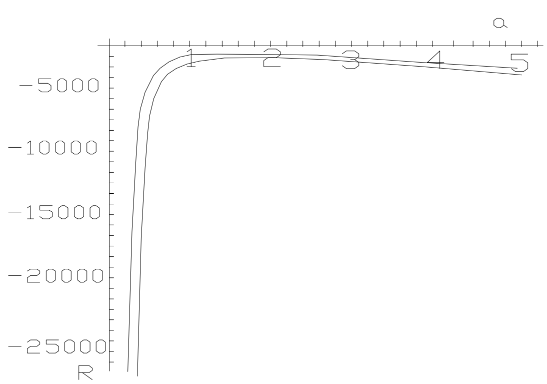


Figure 2: The dependence of the Rayleigh number on the wave number.

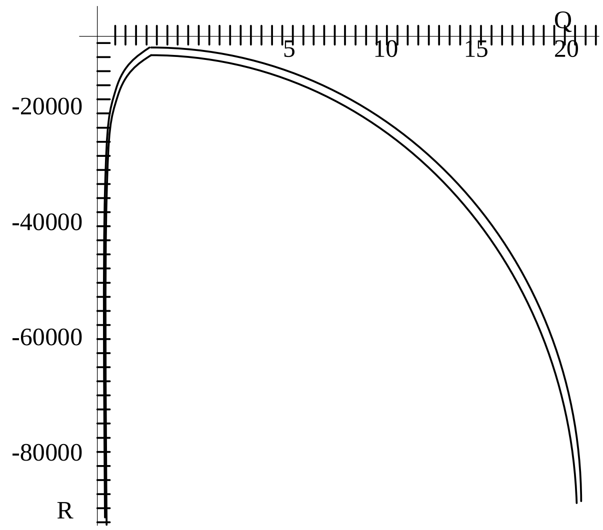


Figure 3: The dependence of the Rayleigh number on the wave number with $K_0$ kept constant and $\alpha$ varying.

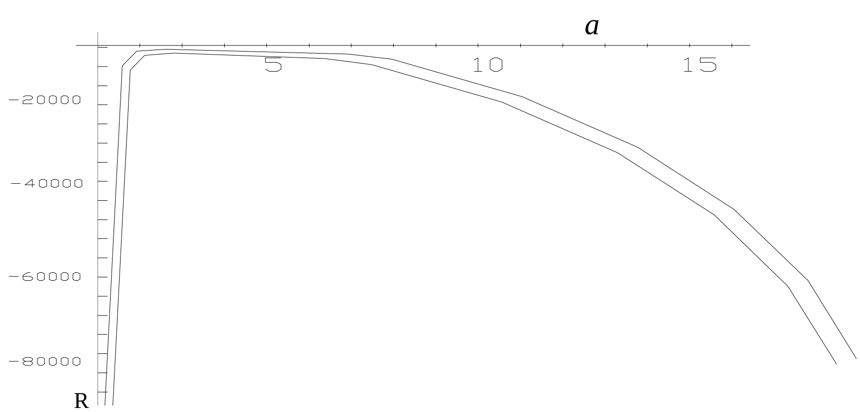


Figure4: The dependence of the Rayleigh number on the wave number with $K_0$ varying and $\alpha$ kept constant**.**

Figure 1. Shows that at any fixed Reynolds' number, presence of permeability and radiation, altered the mean curve of the theoretical description of Couette- Poiseuille flow. Figure 2 shows stability at small wave numbers regime (0.1≤a≤5) with $K_0 = 0.2$ and $\alpha = 0.4$, however the situation is reversed at high wave number (a≥10) with $K_0 = 0.2$ and radiation increased to 20.5. Also at high wave number with $\alpha = 0.4$ and permeability increased up to 10 as shown in Figures 3 and 4 respectively, are in agreement with the work of Hassard et al [6] and Takhar et al [14]. Analysis of Figures 3 and 4 also shows that the effect of radiation is more prominent than permeability at progressively high wave number and corresponding low Rayleigh number. It was also observed that at high Reynolds'number (Re>3000) and wave number (a$\geq 20$), instability sets in as the fluid flow progresses in the presence of permeability and radiative heat.

## 8.0 Conclusion

To demonstrate stable Couette-Poiseuille flow, through two parallel plates in a porous medium permeability and differential temperature must be put into consideration. Much of the interest in studying these flows has come from the discrepancies that exist between the critical Reynolds' number computed for linear instability, for fully non linear stability and those observed experimentally. Effort is also being made to consider the hydrodynamic equations in cylindrical coordinate system for the study where the geometry is cylindrical, which appears to be most suitable owing to the boundaries of the flow field in such configuration.

## References


[1] Bestman, A, R (1989). ICTP Internal Report IC/89/30

[2] Bestman, A, R; Opara, F, E (1990). J. Proc. Edward Bouchet Institute, Legion, Ghana. P.195.

[3] Cogley, A, C; Vincenti, W, G; Giles, E, S (1968). Differential approximation of a radiative heat transfer, ATAA Journal 6 551-4.

[4] Corenflos,K; Rida, S; Monnier, J, C; Dupont, P; Dangtran, K; Stanislas, M (!993). Experimental and numerical study of a Couette-Poiseuille flow as a test case for turbulence modeling. Proc. 5th int. symp. on refined flow modeling and turbulence measurements, Paris, pp. 499-508.

[5] Gupta, G, K (1999). Hydrodynamic stability of the plane poiseuille flow of an electrorheological fluid. Int.J. Non-Linear Mech. 34,589.

[6] Hassard, B, D; Chang, T, S; Ludford, G, S, S (1972). An exact solution in the stability of MHD Couette flow Proc. R. Soc. London. A 327, 269- 278.

[7] Hocking,L,M (1958). Theory of hydrodynamics stability. Univ. Coll. London. Ch.5,p. 437-469.

[8] Opara, F, E (1994).Thermosolutal instability of radiating partially-ionized Plasma in a porous medium. Astrophys. Space Sci.213, 197-204.

[9] Orzag, S,A and Kells, L,C (1980). Transition to turbulence in plane poiseuille and plane Couette flow. J. Fluid Mech. 96, 159.

[10] Porteous, K,C; Denn, M, M (1972). Linear stability of plane poiseuille flow of viscoelastic liquids. Trans. Soc. Rheol. 16,295.

[11] Rayleigh, L (1926). Phil. Mag. 32, 529.

[12] Spurk,J (1997). Fluid Mechanics Springer-Verlag. Berlin Helderberg New York. Ch,6 p. 176-177, p.213-216. Ch,12 p. 419-423.

[13] Tackels,G; Crochet, M, J (1973). Stability of a plane poiseuille flow of a finite linear viscoelastic fluid. Phys. Fluids 16, 790.

[14] Takhar, H, S; Ali, M, A; Soundalgekar, V, M (1989). Stability of MHD Couette flow in a narrow gap annulus Appl. Sc. Res. 461- 24.

[15] Waters, N, D; Keeley, A, M (1987). The stability of two stratified non-Newtonian liquids in Couette flow. J. Non-Newtonian fluid Mech. 24,161.

[16] Zhou, H; Forest, M, G (2006). Anchoring distortions coupled with plane Couette and Poiseuille flows of nematic polymers in viscous solvents :Morphology in molecular orientation, stress and flow. Descrete and continous dynamical systems-series B. vol.6, 2, 407-425