

Horizontal and vertical projectile motion in a resistant medium under the influence of transverse magnetic field

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Abstract

Horizontal and vertical projectile motion in a resistant medium under the influence of magnetic field is carried out. Solutions to the governing equations is developed using integrating factor method. The results are in reasonable agreement with the findings of [2] and [3].

1.0 Introduction

Anyone who has observed a base ball in motion has observed projectile motion. In his paper Hayen [5] considered the projectile motion under the influence of a uniform gravitational field. In the study, he used drag force to act on the particle in the medium which he assumed is proportional to the square of the particle's velocity. Ayeni and Ayandokun [1] included electric field with a general power n in the earlier mentioned study of Hayen with the assumption that the initial motion is vertical and the resulting unsteady motion investigated. Recently, Fenuga and Ayeni [2] made some important and necessary remarks on the projectile motion of a particle in a resistant medium under the influence of a magnetic field and showed that both velocity and path angle increase as magnetic field increases and decrease as magnetic field decreases. They [3] also compared the impact of electric and magnetic fields in a resistant medium and showed that the magnetic field has much impact on the velocity than the electric field. In this paper, our aim is to critically examine the effect of magnetic field by considering explicitly the vertical and horizontal motions of the projectile where the resistant medium is air and the assumptions that the acceleration due to gravity is constant over the range of motion and directed downwards and the rotation of the earth does not affect the motion.

2.0 Model formulation

We consider the governing equations of two dimensional projectile motion as depicted by the physical model shown below

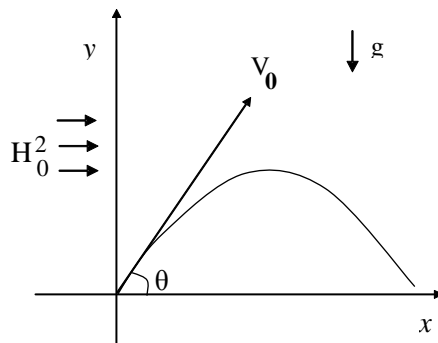


Figure 1: The Physical model and coordinate system of the problem.

as

$$\cos\theta \frac{dv_0}{dt} = -Kv_0^m + H_0^2 \quad (2.1)$$

$$\sin\theta \frac{dv_0}{dt} = -g - Kv_0^n + H_0^2 \quad (2.2)$$

with the initial condition $v_0(0) = 1$ (2.3)

Equations (2.1) and (2.2) are horizontal and vertical components of the acceleration vector, where

v_0 = velocity of projection

t = time

g = acceleration due to gravity

k = proportional constant

μ^2 = permeability

ρ = fluid density

θ = angle of projection

H_0^2 = imposed magnetic field

m, n = positive integers

3.0 Non-dimensional analysis

Applying the following dimensional quantities $v = \frac{v_0}{v}, t' = \frac{t}{t'}, G = \frac{gd}{v_0^2}, B^2 = \frac{\mu^2 \sigma_c H_0^2}{\rho v_0^2}$ to (2.1)

and (2.2) results in

$$\cos\theta \frac{dv}{dt'} = -kv^m + B^2 \quad (3.1)$$

$$\sin\theta \frac{dv}{dt'} = -G - kv^n + B^2 \quad (3.2)$$

4.0 Method of solution

Considering (3.1) and assuming that $m = 1$ [3] its solution after imposing the boundary condition $v(0) = 1$ can be written as

$$v(t) = \frac{B^2}{\cos\theta} + \left(1 - \frac{B^2}{\cos\theta}\right) e^{-\left(\frac{k}{\cos\theta}\right)t'} \quad (4.1)$$

Similarly, following the same procedure assumption and boundary condition, the solution of (3.2) is

$$v(t) = -\left(\frac{G - B^2}{\sin\theta}\right) + \left[1 + \left(\frac{G - B^2}{\sin\theta}\right)\right] e^{-\left(\frac{k}{\sin\theta}\right)t'} \quad (4.2)$$

5.0 Results

We assumed maximum value for $\sin\theta, \cos\theta$ $G = 9.8, B = 0.8, 4.8, k = 1$ in (4.1) and (4.2).

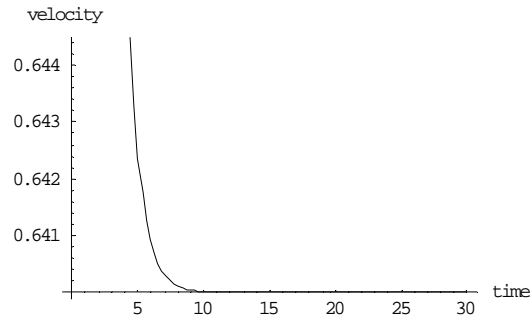


Figure 1: The graph of (4.1) with $B = 0.8$

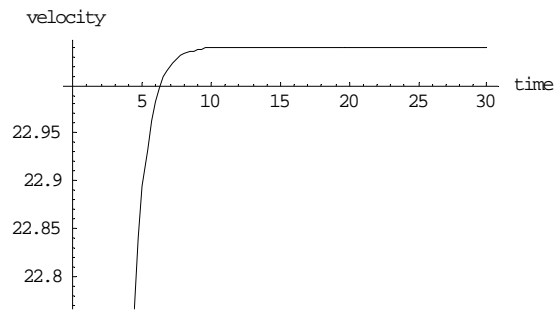


Figure 2: The graph of (4.1) with $B = 4.8$

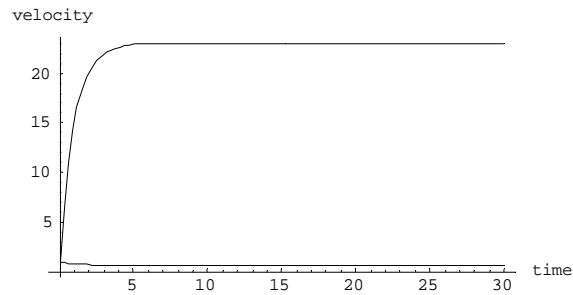


Figure 3: The combination of Figure 1 and Figure 2.

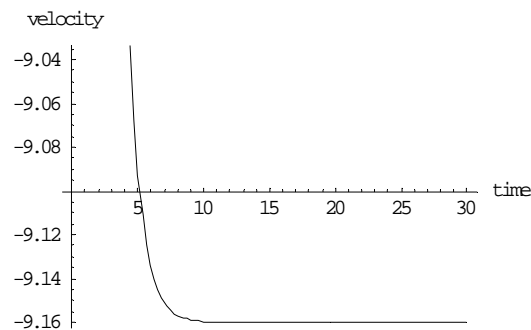


Figure 4: The graph of (4.2) with $B = 0.8$

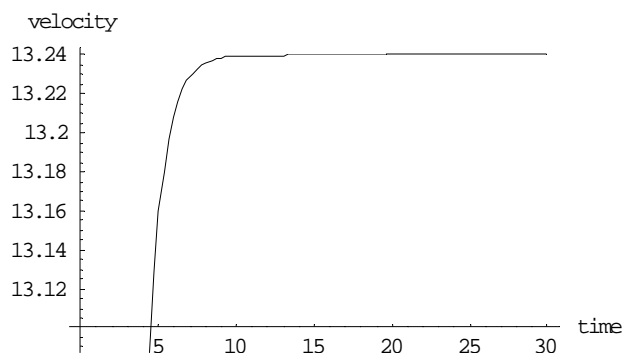


Figure 5: The graph of (4.2) with $B = 4.8$

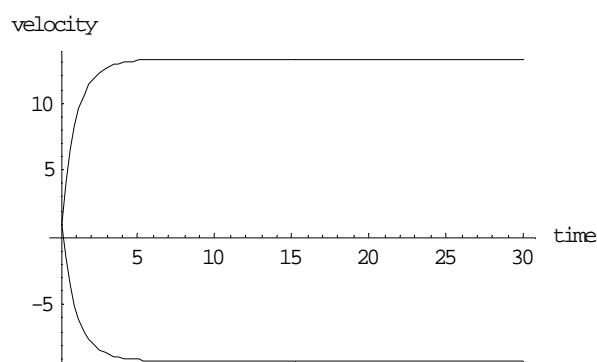


Figure 6: The combination of Figure 4 and Figure 5.

6.0 Discussion

Figures 1 and 2 show a steady increase in velocity when $B = 0.8$ and 4.8 respectively but assumes a constant state over time. This is further appreciated in Figure 3 which is a combination of Figure 1 and Figure 2 and this confirmed [2]. Similar situation is observed in Figure 4 and Figure 5 and its combination Figure 6 clearly shows the effect of the imposed magnetic field which is in accordance with [3].

7.0 Conclusion

Adjustment of magnetic field is essential to enhance flight of particles in the presence of magnetic field. Wrong projection of a given target may be experienced or hindrance to motions of particle if proper adjustment mechanism is not put in place.

References

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