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Anti-synchronization of two new different chaotic systems via active control

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Abstract

This paper investigates the anti-synchronization of chaos between two new different chaotic systems by using active control. Numerical simulations are used to show the robustness of the active control scheme in anti-synchronizing the two different coupled systems.

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***Keywords*: Chaos, Anti-Synchronization; Active Control**

1.0 Introduction

Synchronization of chaotic attractors are of fundamental importance in the study of nonlinear dynamics; and have been extensively investigated both theoretically, numerically and experimentally in many chaotic systems [1,2]. The application of synchronization of chaotic systems in chemical reactors, secure communications, laser physics, ecological systems and so on have been explored [1]. This has made it one of the striking discoveries in the study of Chaos [2].

The most familiar synchronization phenomenon is that the difference of states of synchronized systems converges to zero, and is called complete synchronization (CS) as first reported by Pecora and Carroll [3]. Other synchronization phenomena reported for coupled chaotic oscillators are phase synchronization, lag synchronization, anti-synchronization, generalized synchronization, anticipated synchronization and measure synchronization.

Anti-synchronization (AS) is a phenomena observed in periodic oscillators that has been known for quite a long time [4]. It is well known that the first observation of synchronization of two oscillators by Huygens in the seventeeth century was, infact, AS between two pendulum clocks. Blekham [5] shows that either synchronization or AS can appear depending on the initial conditions of the coupled pendula. AS have been observed experimentally in the context of self-synchronization e.g. in salt water oscillators [6], and some biological systems where a nonchaotic signal is generated.

Over the last decade, a large variety of approaches have been proposed for chaos synchronization such as the master-slave method[3], backstepping design method[7], active control[8,9], invariant manifold method[10], adaptive method[11], feedback approach[11], etc. Most of the methods mentioned synchronize two identical chaotic systems. However, the method of the synchronization of two different chaotic systems is far from being straightforward, notwithstanding, the applications of chaos synchronization in secure communications makes it much more important to synchronize two different chaotic systems[12].

The application of the active control technique to nonidentical systems have demonstrated its advantage over other schemes [13].

AS have been applied to identical systems mainly in the past. In Zhang [14], the AS for chaotic systems (original Chua's circuit) was investigated and some simple but generic criteria for AS was derived, along with a simple configuration by the corresponding suitable separation. They show that there is no need to calculate the Lyapunov exponents and eigenvalues of the Jacobian matrix A, hence it is simple and convenient. Kim et al observed AS phenomena in coupled identical chaotic oscillators, which is different from complete synchronization phenomenon. They qualitatively analyse its base mechanism by using the dynamics of the difference and the sum of the relevant variables, since AS can be characterised by the vanishing of the sum of relevant variables. Only recently did Emadzadeh [15] presents AS of chaos between two different chaotic systems, where Rossler system is controlled to be AS with Lu system.

In practice, anti-synchronization is a phenomenon wherein the state vectors of synchronized systems have the same absolute values but opposite signs[15], it is an example of Phase Shifted Synchronization (PSS). Thus anti-synchronization of two systems S_1 and S_2 is achieved if $\lim_{t\to\infty}\|x_1(t)+x_2(t)\|=0$, where $x_1(t)$, $x_2(t)$ are the state vectors of the systems S_1, S_2.

In this paper we study ***anti-synchronization between two chaotic systems recently introduced by Lu et. al.* [16] *and studied by Sun* [17] *using the active control technique***.

2.0 Model and Active Control

Active control technique gives the flexibility to construct a control law so that it can be used widely to control various nonlinear systems including chaotic systems [18]. The pioneering work [19,20] was on identical systems (Lorenz system), however, the method has been generalized to nonidentical systems, thus, breaking the limit of synchronization of identical systems and demonstrating further the advantage of the active control technique over other schemes [18].

To clearly state the problem, consider a chaotic dynamical system, a master (or drive), together with another system (the slave) coupled together via active controllers. The aim is to synchronize the response of the slave system to the master system by driving the slave system with the control signals derived from the master, that is, the designed controller with the state variable of the master will make the trajectories of the state variables of the slave system to follow the trajectories of the state variables of the driver system [21].

$L\ddot{u}$ et. al. [16] introduced the follwoing chaotic system of 3-D quadratic autonomous ordinary differential equations, which can display two 1-scroll chaotic attractors simultaneously with only three equilibra and two 2-scroll chaotic attractors simultaneously with five equilibra:

$$\begin{aligned}\dot{x} &= -kx - yz + c\\ \dot{y} &= ay + xz\\ \dot{z} &= bz + xy\end{aligned} \tag{2.1}$$

where $k=\frac{ab}{a+b}$, a, b and c are real constants and x, y and z are state variables. The system is chaotic for the parameters a = -10, b = -4 and || c || < 19.2, it displays the chaotic attractor shown in Figure 1.

The second system which can display two 2-scroll chaotic attractors and two 4-scroll chaotic attractors can be described as follows [22]:

$$\begin{aligned}\dot{x} &= px - yz\\ \dot{y} &= -vy + xz\\ \dot{z} &= -uz + xy\end{aligned} \tag{2.2}$$

where p, v and u are positive control parameters. This system exhibits a chaotic attractor at the parameter values p = 0.4, v = 12 and u = 5, the chaotic attractor is shown in Fig. 2. Recently, Sun [17] showed that systems (2.1) and (2.2) can be synchronized via active control. Here we follow the method of active control and show that system (2.1) and system (2.2) can be anti-synchronised as well. To achieve this, we make system (2.1) the drive, system (2.2) the response and then introduce three control signals into system (2.2) to obtain the following drive-response system:

$$\begin{aligned} \dot{x}_1 &= -kx_1 - y_1 z_1 + c \\ \dot{y}_1 &= ay_1 + x_1 z_1 \\ \dot{z} &= bz_1 + x_1 y_1 \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} \dot{x}_2 &= px_2 - y_2 z_2 + u_1(t) \\ \dot{y}_2 &= -vy_2 + x_2 z_2 + u_2(t) \\ \dot{z}_2 &= -uz_2 + x_2 y_2 + u_3(t) \end{aligned} \tag{2.4}$$

Let

$$\begin{aligned} s_1 &= x_1 + x_2 \\ s_2 &= y_1 + y_2 \\ s_3 &= z_1 + z_2 \end{aligned} \tag{2.5}$$

be the anti-synchronization errors of the state variables. By adding (2.3) and (2.4) and using the above notations, we get

$$\begin{aligned} \dot{s}_1 &= -kx_1 + y_1 z_1 + c + ps_1 - px_1 - y_2 z_2 + u_1(t) \\ \dot{s}_2 &= ay_1 + x_1 z_1 - vs_2 + vy_1 + x_2 z_2 + u_2(t) \\ \dot{s}_3 &= bz_1 + x_1 y_1 - us_3 + uz_1 + x_2 y_2 + u_3(t) \end{aligned} \tag{2.6}$$

The active control inputs can be defined as follows:

$$\begin{aligned} u_1(t) &= kx_1 + y_1 z_1 - c + px_1 + y_2 z_2 + v_1(t) \\ u_2(t) &= -ay_1 - x_1 z_1 - vy_1 - x_2 z_2 + v_2(t) \\ u_3(t) &= -bz_1 - x_1 y_1 - uz_1 - x_2 y_2 + v_3(t) \end{aligned} \tag{2.7}$$

where $v_1(t)$, $v_2(t)$, $v_3(t)$$ are new control inputs with equation (2.7), equation (2.6) becomes

$$\begin{aligned} \dot{s}_1 &= ps_1 + v_1(t) \\ \dot{s}_2 &= -vs_2 + v_2(t) \\ \dot{s}_3 &= -us_3 + v_3(t) \end{aligned} \tag{2.8}$$

The AS error system in (2.8) is a linear system with control inputs $v_1(t)$, $v_2(t)$ and $v_3(t)$. Design of an appropriate feedback control stabilizes the system so that s_1, s_2 and s_3 converge to zero as time t tends to infinity. This implies that the two new different systems are anti-synchronized with feedback control.

Using the active control method, we choose a constant matrix **A** which will control the error dynamics (2.8) such that

$$\begin{pmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{pmatrix} = A \begin{pmatrix} s_x(t) \\ s_y(t) \\ s_z(t) \end{pmatrix}$$

with

$$A = \begin{pmatrix} \lambda - p & 0 & 0 \\ 0 & \lambda + v & 0 \\ 0 & 0 & \lambda + u \end{pmatrix} \tag{2.9}$$

In (2.9), the three eigenvalues λ_1, λ_2 and λ_3 have been chosen as -1, -1, and -1 in order that a stable and anti-synchronized different systems are achieved. We can also make non-zero numbers less than -1. If the eigenvalues get smaller, the convergence will become smaller.

$$v_1(t) = (-1 - p)s_1$$
$$v_2(t) = (v - 1)s_2$$
$$v_2(t) = (u - 1)s_3 \qquad (2.10)$$

Finally, the active control inputs becomes,

$$u_1(t) = kx_1 + y_1z_1 + y_2z_2 - x_1 - x_2 - px_2 - c$$
$$u_2(t) = -ay_1 - x_1z_1 + x_2z_2 + vy_2 - y_2 - y_1 \qquad (2.11)$$
$$u_3(t) = -bz_1 - x_1y_1 + uz_2 - x_2y_2 - z_2 - z_1$$

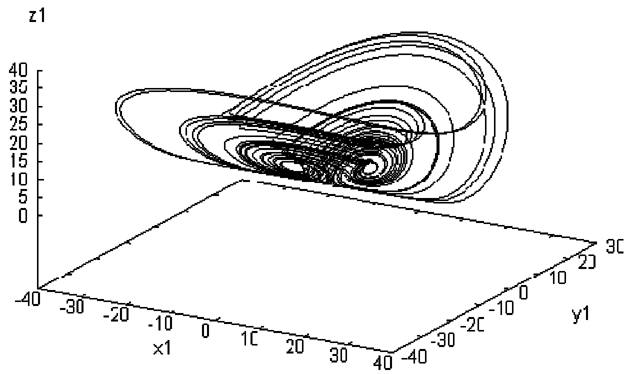

Figure 1: Chaotic attractor for the drive system.

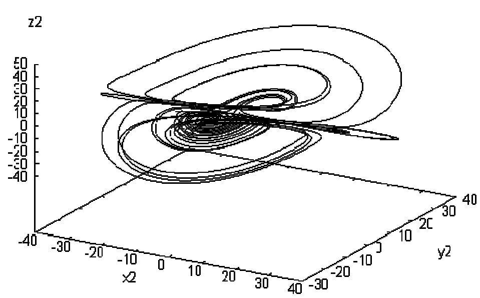

Figure 2: The two-scroll chaotic attractor for the slave system.

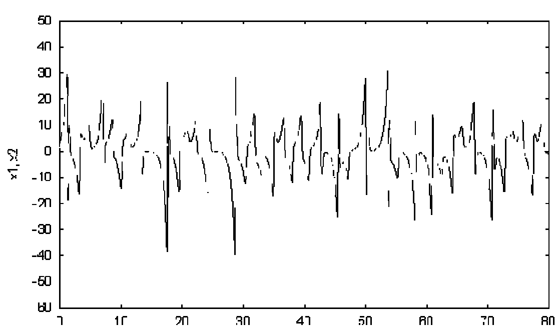

Figure 3: Time series for the state variables x_1, x_2 of the slave and drive system, when the control is deactivated.

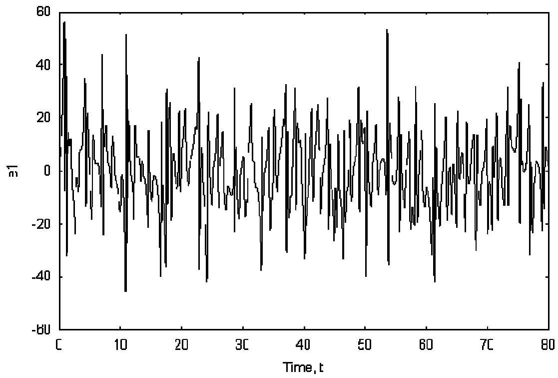

Figure 4: Time evolution of the error state, e_1, when the controllers are not activated.

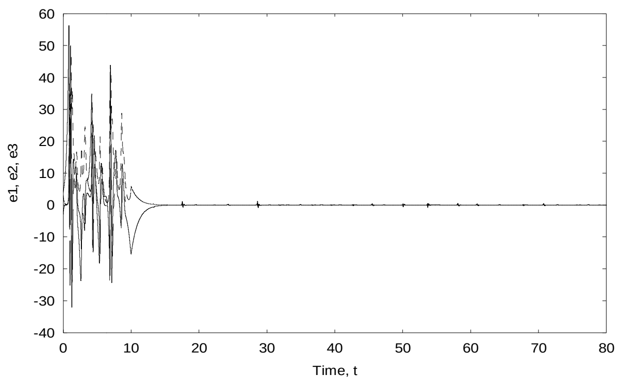

Figure 5: Time evolution of the error states when the controllers are activated-depicting anti-synchronization.

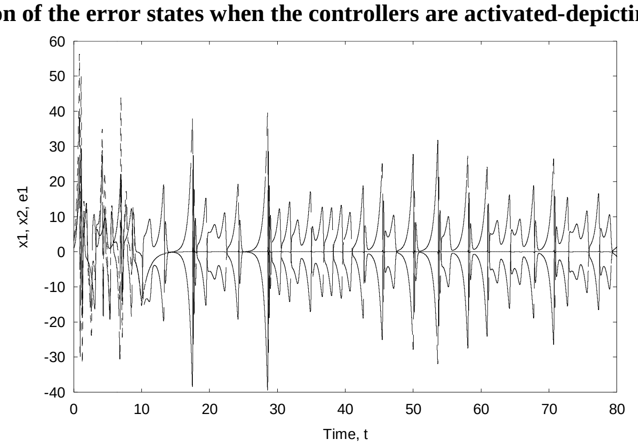

Figure 6:. Overlapping of the time series and the error state for the drive and slave system when the controllers are activated.

3.0 Simulation Results

In this section, numerical simulations are given to verify the active control method. In these simulations, the fourth-order Runge-Kutta method is used to solve the two systems (2.3) and (2.4), with time step 0.01. The parameters are chosen as a = -10, b = -4, c = 0, p = 4.5, v = 12, u = 5.

The initial values of the drive and response system is taken as $(x_1(0), y_1(0), z_1(0)) = (3,-4,2)$ and $(x_2(0), y_2(0), z_2(0)) = (1,1,1)$. In Figure 3, we display the time history for the state variables x_1, x_2 of the slave and drive system, when the control is deactivated. This clearly shows the non-periodic nature of the time series for the systems under consideration and Fig 4 displays the time evolution of the error states when the active control is also deactivated, indicating the chaotic form of the error states. Fig 5 shows the error dynamics when the active control is activated. Obviously, anti-synchronization has been achieved as soon as the controllers are activated, and satisfies $\lim_{t\to\infty}\|x_1(t)+x_2(t)\|=0$. The convergence of the error states e_1, e_2, and e_3 as the controllers are activated, indicates that anti-synchronization have been achieved by the proposed control technique. That is the controllers when activated controls the systems in such a way that the slave system tracks the trajectories of the master, thus, there seems to be no difference between their states.

In Figure 6, where the overlapping of the time series and the error series are displayed when the controllers are activated showed that the error line is the mean of the time series, when there is anti-synchronization. This visual display confirms the reliability of the designed active controller for the two systems. Normally, since the error states are the addition of the state variables, then the error states may not necessarily converge, but the activated controller acts in such a way as to make the state variables to be in anti-phase, thus, their addition tends to zero, which is the anti-synchronized state.

4.0 Conclusion

Conclusively, we have demonstrated in this Letter, a specific application of the active control for the anti-synchronization of the two different systems -which can display two 1-scroll chaotic attractors simultaneously with only three equilibra and two 2-scroll chaotic attractors simultaneously with five equilibra. The simulations confirm that AS of two systems operates satisfactorily in presence of the proposed control method, that is, the anti-synchronization error would converge to zero finally and two different systems from different initial values are indeed achieving chaos anti-synchronization. This technique can be extended to anti-synchronize higher dimensional systems.

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