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Anti-synchronization of the rigid body exhibiting chaotic dynamics

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Abstract

Based on a method derived from nonlinear control theory, we present a novel technical approach for synchronizing the dynamics of a rigid body exhibiting chaotic motion. In this framework, the active control technique is modified and employed to design control functions based on Lyapunov stability theory and Routh-Hurwitz criteria, so that a drive-response system of a rigid body achieves anti-synchronism in the chaotic state. Global asymptotic stability and convergence of the sum of the dynamical variables representing the Eulerian state space of the two rigid bodies was verified by numerical simulations.

1.0 Introduction

For over two centuries, the dynamics of rigid body motion has been a problem of interest to scientists and mathematicians in particular. The rigid body has many practical engineering applications such as gyroscopes, satellites, spacecraft and rockets. However, analytic solution to the general problem of a rigid body under the influence of arbitrary external torques is far from being complete. In fact, most existing analytic theories were applied to highly idealized cases, and as torque-free or symmetric bodies. Solutions have been obtained for these and several other special cases by Euler, Jacobi, Poinsot and other researchers; and have been reported by Leimanis (1965). Unfortunately, these solutions are hardly of practical importance to the complex problems encountered in spacecraft dynamics and control. In 1981, Liepnik and Newton (1981), found strange attractors in rigid body motion. Due to this discovery, studies on the chaotic dynamics in the rigid body motion have been a subject of intense focus by many researchers from different perspectives (Ge et al., 1996; Ge and Chen, 1996; Tong and Mrad, 2001; Chen, 2002; Chen and Lee, 2004)

Among these reports, the recent works of Chen and Lee (2004) on anti-control of chaos in rigid body motion is of significant interest in the present study. "Anti-control of chaos" or "*chaotification*" is a mechanism of making a non-chaotic dynamical system chaotic or retaining (or enhancing) the chaos of a chaotic system (Chen and Lai, 1998). Anti-control of chaos is important when chaotic behaviour and chaos synchronization are beneficial. For instance, chaos is important in secure communication, information processing, liquid mixing as well as biological systems and cognitive processes (Kapitaniak, 1992; Kapitaniak, 1995; Stefanski and Kapitaniak, 2003).

Since Pecora and Carroll (1990) first introduced the intriguing concept of the synchronization of chaotic systems in 1990, the phenomenon has been widely explored in a variety of fields. Advances in the study of the synchronization have led to the identification of

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various types of synchronization phenomena. These includes complete synchronization (CS) (Pecora and Carroll, 1990; Vincent et al., 2005a; Vincent et al., 2005b), phase synchronization (PS) (Vincent et al., 2004; Vincent et al., 2006a), anticipated synchronization (ACS) (Voss, 2000; Masoller, 2001; Kostur et al., 2005); measure synchronization (MS) (Hampton and Zanette, 1999; Wang et al., 2002; Wang and Zhang, 2003; Vincent, 2005a;Vincent et al., 2005c; Vincent et al., 2006b), generalized synchronization (GS) (Rulkov et al., 1995; Kocarev and Parlitz, 1996), lag synchronization (Rosenblum et al., 1997; Boccaletti et al., 2000) and anti-synchronization (AS) (Kim et al., 2003; Zhang and Sun, 2004, Li and Liao, 2006). Anti-synchronization (AS), which we consider in this paper for the rigid body motion is a situation in which the state variables of the synchronized systems have the same absolute values but opposite signs. AS is said to be achieved when the $\lim_{t \to \infty} \| x_1 + x_2 \| \to 0$; where x_1 and x_2 are the state variable of the two synchronizing systems. This synchronization phenomenon has been known for a long time. Indeed, the first observation of synchronization of two oscillators by Huygens in the seventeenth century was AS between two pendulum clocks (Kim et al, 2003). The emergency of the theoretical studies of AS phenomenon in chaotic oscillators has been motivated by the report of Liu et al. (2000) on AS of coupled map lattices. Some few studies have been carried out subsequently by Kim et al.,(2003), Zhang and Sun (2004), Emadzadeh and Haeri, (2005), Li and Liao (2006) and Idowu et al., (2007). It is worth noting that experimental observation of AS phenomenon has been reported earlier in the context of self-synchronization in salt-water oscillators (Nakata et al., 1998).

On the other hand, different types of synchronization methods such as APD method of Pecora and Carroll (1990), adaptive control scheme, backstepping design, sliding-mode control, linear and nonlinear feedbacks, linear matrix inequality, invariant manifold method, impulsive control method and active control have been proposed and applied successfully to achieve different synchronization goals. Among these methods, the active control scheme, which was originally proposed by Bai and Lonngren (1997, 2000) has received increasing interest in the recent times and have been widely employed by many other researchers. Some of the recent applications of the active control technique includes the following dynamical systems: chaotic ratchets (Vincent and Laoye, 2007), RCL-shunted Josephson junctions (Ucar et al., 2007), the unified chaotic system (Ucar et al., 2006), Chua's circuit (Tang and Wang, 2006), Rikitake two-disc dynamo (Vincent, 2005b), nonlinear equations of acoustic gravity waves (Vincent, 2006), Qi system (Vincent, 2006; Lei et al., 2007); Van-der Pol-Duffing oscillator (Njah and Vincent, 2006), nuclear magnetic resonance (NMR) modeled by the nonlinear Bloch equations (Ucar et al., 2003), parametrically excited oscillators (Lei et al., 2006) and permanent magnet reluctance machine (Vincent and Ucar, 2007.

In addition, Chen (2005a) also examined the synchronization of non-identical systems consisting of the rigid body dynamics and each of Lorenz, Chen and L$\ddot{u}$ dynamical systems using the method of active control. In addition, a nonlinear control approach for synchronizing the rigid body motion was considered by Chen (2005b). To the best of our knowledge, the anti-synchronization dynamics of the rigid body has not been investigated. The goal of this paper is to modify the active control technique so as to achieve anti-synchronization for two identical rigid bodies evolving from different initial states and exhibiting chaos. In our design technique, we employ the Lyapunov stability theory and Routh-Hurwitz criteria to investigate the stability of the synchronized state. The rest of the paper is organized as follows: In section 2, we describe the rigid body and discuss its dynamical properties; while in section 3, active control is formulated for anti-synchronization and numerical simulations are also presented. Section 4 concludes the paper.

2.0 The Rigid body dynamics

Here, we review the basic dynamical properties of the rigid body as presented by Chen and Lee (2004). Let us consider the Euler equations for the motion of a rigid body with principal axes at the center of mass:

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 + M_1 \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 + M_2 \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 + M_3 \end{aligned} \tag{2.1}$$

where $I_i\,(i=1,2,3)$ are the principal moments of inertias, $\omega_i\,(i=1,2,3)$ are the angular velocities about the principal axes fixed at the center of mass and $M_i\,(i=1,2,3)$ are applied moments. If the applied moments are considered to be linear feedback, then: $M = A\omega$, where

$$A = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}, \tag{2.2}$$

then the equations are represented as

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 + a_{11}\omega_1 \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 + a_{22}\omega_2 \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 + a_{33}\omega_3 \end{aligned} \tag{2.3}$$

Denoting $\omega_1 = x$, $\omega_2 = y$, $\omega_3 = z$, $a_{11}/I_1 = a$, $a_{22}/I_2 = b$, $a_{33}/I_3 = c$, we can re-write equation (2.3) in the form:

$$\begin{aligned} \dot{x} &= \frac{I_2 - I_3}{I_1}yz + ax \\ \dot{y} &= \frac{I_3 - I_1}{I_2}xz + by \\ \dot{z} &= \frac{I_1 - I_2}{I_3}xy + cz \end{aligned} \tag{2.4}$$

For system (2.4) to exhibit chaos, the equilibrium must be unstable. According to the results of Liu and Chen (2003), the system parameters *a, b* and *c* must satisfy the following necessary conditions:

$$a > 0,\ b < 0,\ c > 0 \ \text{ and } \ 0 < a < -(b + c) \tag{2.5}$$

which is just one of the three possible cases. In addition, the parameters $I_i\,(i=1,2,3)$ need to satisfy

$$\frac{I_2 - I_3}{I_1} < 0,\ \frac{I_3 - I_1}{I_2} > 0,\ \frac{I_1 - I_2}{I_3} > 0;\ \text{i.e. } I_3 > I_1 > I_2 \tag{2.6}$$

For simplicity, assume that $I_3 = 3I_0, I_1 = 2I_0, I_2 = I_0; (I_3 > I_1 > I_2)$, then the system (2.4) can be re-written as

$$\begin{aligned} \dot{x} &= -yz + ax \\ \dot{y} &= xz + by \\ \dot{z} &= \left(1/3\right)xy + cz \end{aligned} \tag{2.7}$$

System (2.7) is invariant under the transformation $(x,y,z) \to (x,-y,-z)$, $(x,y,z) \to (-x,y,-z)$ and $(x,y,z) \to (-x,-y,z)$; implying that system (2.7) is symmetrical about the three coordinates x, y, z, respectively. The symmetries persist for all values of the system parameters; a property that makes it robust to various small perturbation. It is also important to state that the system is dissipative and therefore all orbits ultimately are confined to a specific subset of zero volume, and the asymptotic motion settles onto an attractor

By making appropriate choices of the parameters *a, b, c*, the system (2.7) could exhibit strange chaotic attractors and limit cycles. For instance, for $a = 5$, $b = -10$ and $c = -3.8$, the motion is clearly chaotic and symmetrical about an axis as illustrated in Figure 1.

3.0 Anti-synchronization via active control

Chen (2005a) and Chen (2005b) presented two different schemes for synchronizing the rigid body chaotic motions. In this section, we present a modification to the work reported by Chen (2005a). The method is aimed at achieving global anti-synchronization.

3.1 Design of Active Control

Consider a drive-response system of a rigid body given by

$$\begin{aligned} \dot{x}_1 &= -y_1 z_1 + a x_1, \\ \dot{y}_1 &= x_1 z_1 + b y_1, \\ \dot{z}_1 &= \left(1/3\right) x_1 y_1 + c z_1, \end{aligned} \tag{3.1}$$

for the drive system and

$$\begin{aligned} \dot{x}_2 &= -y_2 z_2 + a x_2 + u_1, \\ \dot{y}_2 &= x_2 z_2 + b y_2 + u_2, \\ \dot{z}_2 &= \left(1/3\right) x_2 y_2 + c z_2' + u_3, \end{aligned} \tag{3.2}$$

for the response system, where $u_i (i = 1,2,3)$ are active control functions that are to be determined. In the synchronization scheme, as presented in Chen (2005a) and Chen (2005b) for instance, the synchronization error is defined as the difference between two dynamical variables from the drive-response system. In the anti-synchronization scheme, the anti-synchronization error is defined as the sum of the relevant dynamical variables. Thus, let the anti-synchronization error be defined as follows:

$$e_x = x_2 + x_1 \; ; \; e_y = y_2 + y_1 \; ; \; e_z = z_2 + z_1 . \tag{3.3}$$

By adding equations (3.1) and (3.2) and using the definition (3.3), we obtain the following error dynamics systems:

$$\begin{aligned} \dot{e}_x &= a e_x - y_2 z_2 - y_1 z_1 + u_1, \\ \dot{e}_y &= b e_y + x_2 z_2 + x_1 z_1 + u_2, \\ \dot{e}_z &= c e_z + \left(1/3\right)\left(x_2 y_2 + x_1 y_1\right) + u_3 . \end{aligned} \tag{3.4}$$

According to the active control method, the control functions should be re-defined so that the error dynamics system (3.4) is expressed in terms of the synchronization errors $e_i (i = x, y, z)$ only. Let

$$\begin{aligned} u_1 &= y_2 z_2 + y_1 z_1 + V_1, \\ u_2 &= -x_2 z_2 - x_1 z_1 + V_2, \\ u_3 &= -\left(1/3\right)\left(x_2 y_2 + x_1 y_1\right) + V_3, \end{aligned} \tag{3.5}$$

so that the error system (3.4) becomes

$$\begin{aligned} \dot{e}_x &= a e_x + V_1, \\ \dot{e}_y &= b e_y + V_2, \\ \dot{e}_z &= c e_z + V_3, \end{aligned} \tag{3.6}$$

where $V_i (i = 1,2,3)$ are new control functions that are to be determined. There are many possible choices for the control $V_i (i = 1,2,3)$ that could lead to synchronized dynamics. Let $V_i (i = 1,2,3)$ be defined as

$$[V_i (i = 1,2,3)]^T = A[e_i (i = x, y, z)]^T, \tag{3.7}$$

where $\boldsymbol{A}$ is a 3×3 constant matrix and $A[e_i (i = x, y, z)]^T$ is the feedback matrix. In the usual approach of active control the matrix $\boldsymbol{A}$ is determined from the eigenvalues equation of the error system (3.6). In this paper, we provide an alternative method for determining the matrix ***A*** that is base on Lyapunov stability theory and Routh-Hurwitz criteria. Suppose ***A*** is of the form

$$A = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix}, \tag{3.8}$$

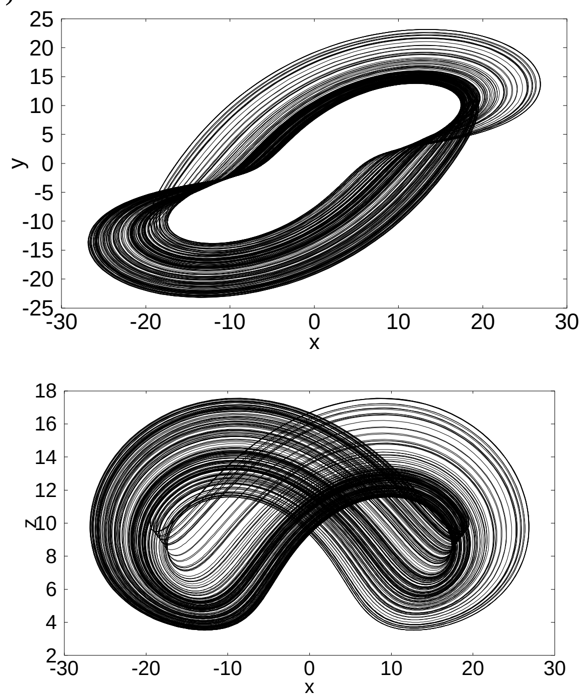
where the $k_i\,(i = 1,2,3)$ are the controller gains of the feedback matrix $A[e_i\,(i = x, y, z)]^T$, the value of which will modify the time at which synchronization will occur. With the matrix **A** defined in equation. (3.8), the error system (3.6) now becomes

$$\begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} = \begin{pmatrix} a + k_1 & 0 & 0 \\ 0 & b + k_2 & 0 \\ 0 & 0 & c + k_3 \end{pmatrix} \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} \tag{3.9}$$

According to Lyapunov stability theory and Routh-Hurwitz criteria, if

$$\begin{cases} -(a + k_1 + b + k_2) > 0 \\ (a + k_1)(b + k_2) < 0 \\ -(c + k_3) > 0 \end{cases} \tag{3.10}$$

then the error system (3.6) must have all of the eigenvalues with negative real parts. This implies that the system (3.9) would be stable and the two rigid bodies would achieve anti-synchronization. Since the system parameters *a, b,* and *c* are bounded, it is convenient to choose the feedback gains $k_i\,(i = 1,2,3)$ that satisfies the conditions (3.10).



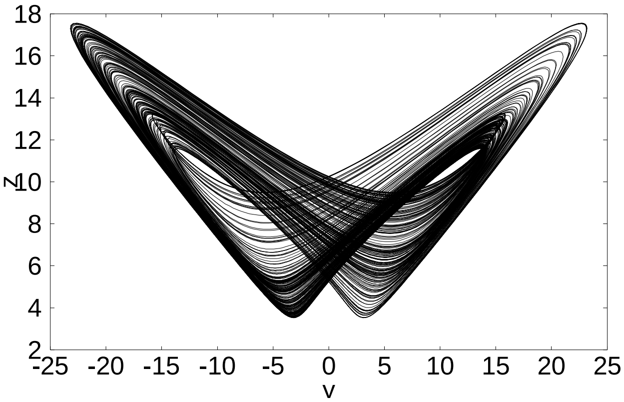


Figure 1: Phase space illustrating the chaotic attractors of the rigid body for $a = 5$, $b = -10$ and $c = -3.8$.

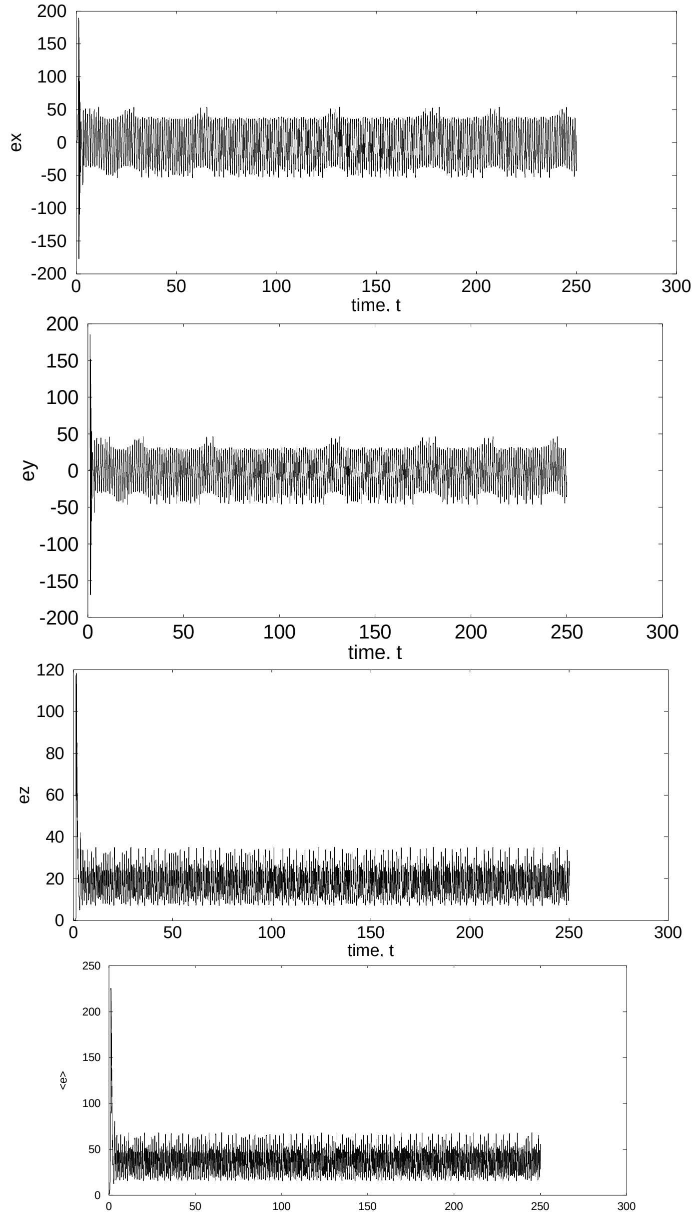


Figure 2: Error dynamics when controls are de-activated.

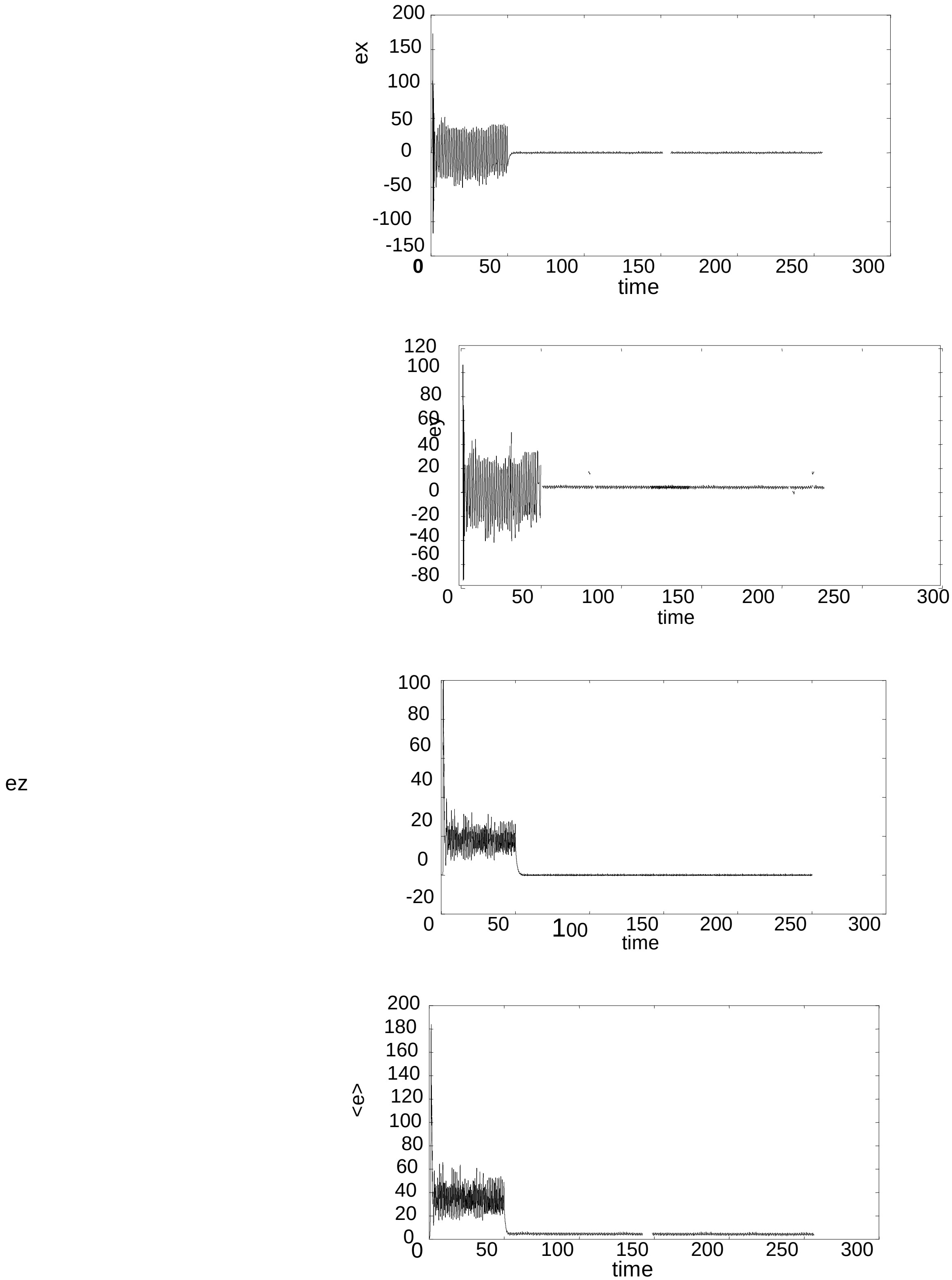


Figure 3: Error dynamics when controllers have been activated at t = 50

3.2 Numerical Results.

In the numerical simulations that follow, the Fourth-order Runge-Kutta algorithm has been employed with a fixed time-step of 0.001. To ensure chaotic behaviour in parameter space, the system parameters were set as in Figure1. That is a = 5, b = -10 and c = -3.8. However, the initial conditions for the drive-response system was set as follows:

$$x_1 = 0.2, y_1 = 0.2, z_1 = 0.2, x_2 = 0.2, y_2 = -0.25, z_2 = -0.25.$$

These corresponds to initial error states $e_x = 0, e_y = -0.05, e_z = -0.05$. For this choice of the system parameters, $k_1 = -6, k_2 = 10$ and $k_3 = 2.8$ satisfies the condition (3.10).

Figure 2 shows that in the absence of control, the error states grow chaotically with time. In Figure 3, the convergence of the error states when controls are activated at t = 50 is illustrated. The convergence is a clear indicator that anti-synchronization state has been reached. A confirmation of this result is shown by the plot of the average error, <e> defined by $< e >= \sqrt{e_x^2 + e_y^2 + e_z^2}$ (Baker et al., 1998; Baker et al., 1999). It should be noted that the controllers could be activated at any later time. In any case, anti-synchronization would be readily achieved as soon as the controls are activated

4.0 Conclusions

In conclusion, the anti-synchronization dynamics of a drive-response system of two rigid bodies derived from by the Euler's equations of motion has been studied based on a modified method of active control technique that employs Lyapunov stability theory and Routh-Hurwitz criteria. The performance of the synchronization scheme for the rigid body model has been tested using standard numerical simulations. Asymptotic convergence of the error dynamics defined for the anti-synchronization states confirm that the drive-response rigid body achieves stable anti-synchronized state. The method could be readily extended to other chaotic systems.

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