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Excitation of low-frequency electrostatic instability on the auroral field lines due to precipitation electron beam

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Abstract

Low-Frequency Electrostatic Instability That Is Observed By Both Ground Facilities And Satellites Have Been Studied In The Auroral Acceleration Region Consisting Of Hot Precipitating Electron Beam From The Magnetosphere, Cold Background Electron And Ion Beam Moving Upward Away From The Earth Along The Auroral Field Lines. The Model Distribution For Both The Electron And Ion Are Taken, As Drifting Maxwellians While The Cool Background Electron Is Maxwellian. The Excited Mode And Growth Rate For The Resonant Instability Driven By The Precipitating Electron Beams Are Derived. We Also Discuss The Growth Rate And The Real Frequency Of The Resonant Instability. It Is Also Shown That, The Precipitating Electrons Can Generate Low-Frequency Electric Field Fluctuations (Lefs) Within The Frequency Range 55.4Hz To About 174.7Hz.

1.0 Introduction

The auroral region of the earth's magnetosphere (altitude of 6000-12000Km) is a fascinating part of space, and also a part that can be investigated in some detail with sounding rockets, satellites and ground based instrument which indicates presence of upward and downward propagating charged particle beams, ion conics, low frequency electrostatic wave fluctuations (LEFs), including other phenomena such as shocks and double layers (Temerin, 1978; Hultqvist et al, 1988;Kaufmam, 1984; Mozer et al, 1997). Evidence (Sharp et al, 1980) of the presence of both electronic streams parallel and anti – parallel to the local magnetic field have been observed. In this report we investigate the LEFs (i.e. n = 0 Bernstein models) by precipitating beams. We are motivated by the fact that very little work has been done in this area even with recent observation of occurrences of LEF with field aligned electron beams (Miyake et. al 2001.). It appears likely that the LEF could be playing an important role in the auroral acceleration processes.

LEFs span the frequency range from essentially zero to the tens of KHz (Lakhina, 1993). A good correlation has been found between the occurrence of LEFs and ion conics and field aligned electron beams (Lundin et at, 1990; Miyake et al, 2001). This paper is organized as follows: in section 2, we analysis the excitation of low-frequency electrostatic instability by precipitating electrons having drifted Maxwellian distribution function. In section 3, the result are compared and discussed while section 4 is the concluding remarks.

2.0 Theoretical Considerations

2.1 Distribution function

We consider our models for the auroral acceleration region to consist of a three component plasma: an ion beam moving upward away from the earth along the auroral field lines with drift velocity V_{hi}, temperature T_i and density N_i, cold background electrons with temperature T_c and density N_c and a hot precipitating electron component which streams downwards the earth in a direction opposite to the ions with drift velocity V_{he}, temperature T_h and density N_h. The external equilibrium geomagnetic field is pointing upwards along the Z-direction, i.e. $\mathbf{B_o} = B_o\,\mathbf{Z}$. The velocity distribution function of the cool electrons is taken to be Maxwellian while that of the hot electrons and ion the appropriate drifting Maxwellians.

2.2 Excitation of low frequency electrostatic wave instability

For low frequency electrostatic waves the relevant dispersion relation can be written in analytic form in which $\left|w - K_\Pi V_{hj}\right|^2 \prec\prec \Omega_j^2,\ K_\Pi \prec\prec K_\perp \Omega_j = \left(e_j B_0 / m_j c\right)$ is the gyro frequency of the jth species , where $j = h$ for hot electrons, c for the cold background electrons and i for the ions , and the subscripts Π and $\perp$ stand for the parallel and perpendicular component of the wave vector ***K***. The condition $\mu_{nj} \equiv \left(w - K_\Pi V_{hj} - n\Omega_j\right)/K_\Pi C_{ij} \succ\succ 1$ is satisfied for all species for $n \geq 1$, where $C_{tj} = (2T_j/m_j)^{1/2}$ is the thermal speed of the jth species. The dispersion relation for the low–frequency electrostatic waves become

$$K^2\lambda^2_{dj} = H_1 + H_2 + H_3 \qquad (2.1)$$

where

$H_1 = \Sigma W([w - n\,\Sigma_c\,]/K_{II}C_{te}\,)\Lambda_{nc}(\beta_c)$

$H_2 = \Sigma W([w - K_{II}V_{he} - n\,\Sigma_h]/K_{II}\,C_{th}\,)\Lambda_{nh}(\beta_h)$

$H_3 = \Sigma W([w - K_{II}V_{hi} - n\Sigma_i]/K_{II}\,C_{ti})\Lambda_{ni}(\beta_i)$

$\Lambda_{nj}(\beta_h) = I_n(\beta_j)e^{-\beta}{}_j$ with $\beta_j = K_\perp{}^2\,C_{ti}{}^2/\Sigma^2{}_j$, $\lambda_{dj} = (T_j/4\,Bn_{jo}\,e^2)^{1/2}$

is associated Debye length and I_n is the modified Bessel function of order n.

Making use of the well –known plasma dispersion function {Fried and Conte, 1961} as:

$$W\{X\} = -\{1+XZ\{X\}\} \qquad (2.2)$$

in the dispersion relation Equation (2.1) it reduces to

$$1 + \sum\chi_j = 0 \qquad (2.3)$$

where

$$\chi_h = \frac{1}{K^2\lambda_{dj}^2}\left[1 + \frac{w - K_\Pi V_{he}}{K_\Pi C_{th}}\Lambda_{nh} Z\left(\left[\frac{w - K_\Pi V_{he}}{K_\Pi C_{th}}\right]\right)\right]$$

$$\chi_c = \frac{1}{K^2\lambda_{dc}^2}\left[1 + \frac{w}{K_\Pi C_{tc}}\Lambda_{nh} Z\left(\left[\frac{w}{K_\Pi C_{tc}}\right]\right)\right] \qquad (2.4)$$

$$\chi_i = \frac{1}{K^2\lambda_{di}^2}\left[1 + \frac{w - K_\Pi V_{hi}}{K_\Pi C_{ti}} Z\left(\left[\frac{w - K_\Pi V_{hi}}{K_\Pi C_{ti}}\right]\right)\right]$$

To solve Equation (2.3), we let $w = w_r + i\gamma$ and assume $\left|\gamma^2\right| \prec\prec \left|w_r^2\right|$, where $\gamma = 0$ denotes instability. Considering the limit $\mu_\alpha^2 \succ\succ 1,\ \mu_{0i}^2 \succ\succ 1$ and $\mu^2_{oh} \leq 1$ in Equation (2.4), the dispersion relation now becomes

$$D_R(w,k) + iD_I(w,k) = 0 \qquad (2.5)$$

where

$$D_R(w,k) = \Delta - \frac{K_\Pi w_{pc}^2}{K^2 w^2} - \frac{K_\Pi w_{pi}^2 \Lambda_{ni}(\beta_j)}{K^2\left(w^2 - K_\Pi V_{hi}\right)^2} = 0 \qquad (2.6)$$

and

$$\Delta = 1 + -\frac{w_{pc}^2}{\Omega_c^2} + \frac{Mw_{pi}^2}{\Omega_i^2} + \frac{w_{ph}^2}{K^2 V_{th}^2} = 0 \qquad (2.7)$$

with $M = [1 - \Lambda_{ni}(\beta_i)]/\beta_i$.

$$D_1(w,k) = i\pi^{\frac{1}{2}} - \frac{w_{ph}^2}{K^2 V_{th}^2}\frac{\left(w + K_\Pi V_{he}\right)\Lambda_{nh}(\beta_h)}{K_\Pi V_{thi}} \bullet \exp\left[\frac{-\left(w + K_\Pi V_{he}\right)^2}{2K_\Pi C_{th}^2}\right] \qquad (2.8)$$

The real frequency of the resonant instability driven by the precipitating electron beams becomes (Bhatia and Lakhina, 1980; Lakhina, 1993):

$$w_r = -K_{II}\alpha\Sigma_i/K, \qquad (2.9)$$

where

$$\alpha = \frac{w_{pc}\left[1 + \Lambda_{ni}(\beta_i) N_j m_e / N_c m_i\right]^{\frac{1}{2}}}{\Omega_1 \Delta^{\frac{1}{2}}}$$

$$\approx \{N_c\,m_i/M\,N_i\,m_e\}^{1/2}$$

Then using the formular:

$$\gamma = -D_1 \frac{\left(w_r, k\right)}{\frac{\partial}{\partial w_r}} D_R\left(w_r, k\right) \tag{2.10}$$

the expression for growth rate is found to be γ

$$\gamma = \frac{\pi^{\frac{1}{2}} N_h K_\Pi \alpha^3 \Omega_i^3 \Lambda_{ni}\left(\beta_j\right)\left(V_{he} - V_{he}^*\right)}{2N_c K^3 C_{th}^3} \exp \frac{\left|-\left(V_{he} - V_{he}^*\right)^2\right|}{2C_{th}^2} \tag{2.11}$$

V^*_{he} is the critical drift speed above which instability develops, it is given as

$$V_{he}^* = \frac{\alpha \Omega_i}{K} \approx \left\{ \frac{N_c T}{M N_i T_h \beta_i} \right\}^{\frac{1}{2}} C_{th}^2 - \tag{2.12}$$

3.0 Results and discussion

As wide variety of modes can be excited in plasma under different conditions; depending on the parameters of the auroral plasma, several mechanisms (as discussed early) may operate at the sane time. The present study deals with the generation of low- frequency electrostatic modes by electron beams on the auroral field lines at altitudes of 1 to 2 R_E. Since the excited LEFs are electrostatic waves, they will resonate with the electrons thus transferring energy and momentum from the beam – electrons.

In the auroral acceleration region (Lakhina, 1993), the density and energy of precipitating electrons are variable but in the range N_h ~ {0.1 to 5) Kev. For typical value of electron temperature as $T_{h\_=}$ 1Kev, this gives C_{th} = 1.32 x 10^4 Km s^{-1}. Outside the region of strong parallel potential drops, the typical values of background cold electrons parameters are N_c ~$10^{-3}$$cm^{-3}$ and T_c = 1 eV. But in the region of strong parallel electric fields the cold electrons are expelled resulting in N_c =10^{-3} cm^{-3} or smaller. The ion beam drift speed can be U_i = (5 -50) C_{ti} and C_{ti} = (9.8 – 30.9) km s^{-1} corresponding to T_i = 1 to 10 eV. The destiny of ion beam could be calculated from quasi-neutrality condition in the equilibrium state.

The result shown in Figures 1and 2 are calculated for plasma parameters corresponding to auroral acceleration region described above. In Fig. 1 the growth rate ((/ Σ_i) has been plotted against β_i. According to Fig.1, the growth rate of the resonant electron instability increases rapidly to its maximum value and then it states decreasing slowly as β_i increase. While in Fig 2 a plot of growth rate ((/ Σ_i) against the normalized frequency (w_r / Σ_i) presents slowly variations of the growth rate after it has attained its maximum value as the normalized frequency increases.

Substituting the value 101 x 10^{-3}<N_i<1001x10^{-3} cm $^{-3}$ for the auroral acceleration region, we find f_{pi} = w_i / 2 B is in the range 55.4 Hz < f_{pi} < 174.7 Hz suggesting that frequency range is in fairly good agreement with the observations from Viking (Hultqvist et al 1988) and from S3-3 (Temerin, 1978) representing low frequency electrostatic modes.

As the drift velocity of the hot precipitating electrons exceeds that of the critical velocity; the low frequency electrostatic instability sets in. It is possible that the excited low-frequency electrostatic mode could heat up the auroral plasma leading to acceleration mechanisms that result in the production of field aligned electron beams or upward flowing ions likewise electron or ion conics. The acceleration mechanism due to low- frequency electrostatic mode will be considered in a subsequent report.

3.0 Concluding remarks

Our theory is consistent with the data, and this suggests that the observed frequency prominences in the auroral acceleration region between 55.4 Hz and 174.7 Hz is due to low- frequency electrostatic instability which are generated by precipitating electron beam from the magnetosphere. It is our belief that, the unstable low-frequency electrostatic mode excited can resonate with the electrons and ions leading to the formation of ion conics, accelerated electrons and hot ions beams. This interesting subject will be pursued in a future paper.

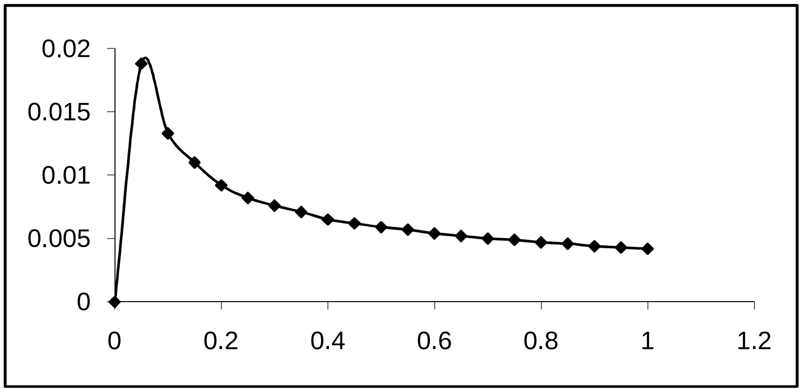

Figure 1: Variation of growth rate λ / Ω_i for the resonant hot electron instability versus β_i for K_Π / K =10m_e/m_i, Bo = 0.046G and V_{he}/C_{th} = 0.525.

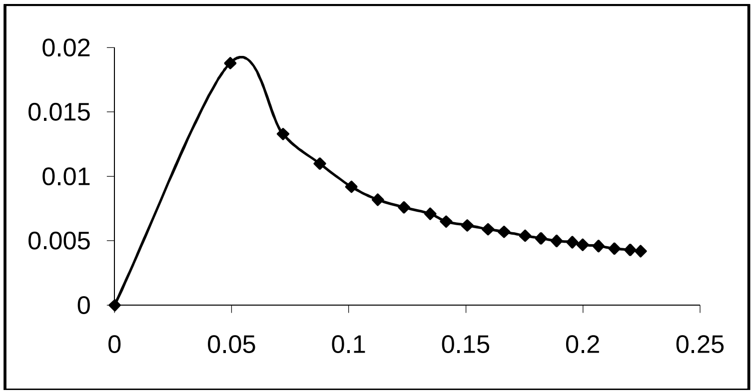

Figure 2: Variation of normalized frequency $\left(-w_r / \Omega_i\right)$ with growth rate $\left(\lambda / \Omega_i\right)$

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