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Streaming instability in a velocity–sheared dusty plasma

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A two-stream instability, obtained from kinetic theory, of strongly velocity-sheared inhomogeneous streaming electron in a magnetized plasma in the presence of negatively charged dust is discussed. Various cold plasma approximations were considered and it is shown that when the diamagnetic effect of ion can be ignored, the excited mode could be dust lower hybrid-like. On other hand, if the dust is treated as immobile background, the excited wave is ion lower hybrid-like. In both cases, the growth rate is reduced due to the presence of the dust particles and the velocity shear scale length, L_E , is on the order of k^{-1} (where k is wave vector) for the most unstable mode. An example is given from the plume.

Keywords*:* Ionosphere, Space, Dusty plasma.

1.0 Introduction

Dusty plasmas are a main constituent of many space and astrophysical environments such as interstellar clouds, circum stellar clouds, asteroid zones, earth's atmosphere, planetary rings, interplanetary dust, nebulae, comet tail, etc. The charged dust particles play a significant role in the dynamics and wave behaviour of many natural systems. Several analyses, treating the dust as a charged particle species of uniform mass and charge, have shown that the presence of charged dust component leads to the appearance of new plasma modes arising from the grain dynamics. de Angelis *et al*. [1] studied the linear properties of ion acoustic waves in the presence of massive, immobile charged dust grains in an unmagnetized plasma. They applied their result in interpreting the low frequency electrostatic noise enhancement associated with Halley's Comet. Rao *et al*. [2] considered the dynamics of a tenous dust fluid and assumed Boltzmann distributions for electrons and ions fluids. They predicted the existence of dust acoustic wave in dusty unmagnetized plasmas. D'Angelo [3] investigated low – frequency electrostatic waves in dusty magnetoplasma, and studied the ion acoustic and cyclotron modes. In a study of dust drift waves, it was found that the dust could modify the usual drift waves and also lead to the appearance of a dust drift wave arising from the dynamics of the dust grains [4]

Charged particle streams flowing in magnetized plasma are a common occurrence in terrestrial and astrophysical plasmas and this, two-stream instability, has also received some attention. The presence of charged dust in plasma can also affect the behaviour of plasma instabilities. Havnes [5] studied streaming instability between solar wind and cometary dust particles using a kinetic model in which the dust particles were drifting relative to a hot background of hot ions and electrons. The effect of dust particle dynamics on ion-ion two stream instability and two stream instability generated by drifting dust beams have been investigated Bharuthram *et al*. [6]. The existence of dust lower hybrid modes arising due to finite Larmor radius effect was demonstrated by Salimullah [7] while Rosenberg *et. al*. [8] showed how the mode can be driven by negatively charged dust and went on to apply the result to radar backscatter from Space Shuttle exhaust in the ionosphere.

A velocity-sheared dust induced instability was proposed in [9] as a possible means of interpreting the some observations in helical structures and streamer splitting in cometary tails. The authors, using fluid model, determined the critical dust shear for instability to occur. Scales *et al.* [1997] used simulation to study expansion of irregularities caused by velocity sheared electron flow in dusty plasmas. The mechanism for the production of irregularities in the boundary of the expanding dust cloud, found by the the authors, is the Electron-Ion Hybrid (EIH) shear driven instability.

In this letter we use the kinetic theory approach which allows one to take Larmor radius into account. However, unlike Scales et al, we simplify our study by ignoring the effect of collisions and dust charging. Our approximation can be justified in the regime where [ ]. An immediate leap from this approximation is to include an imaginary term analogous to the one derived by Goetz et al for dust fluctuations to the growth rate obtained here. A more complete theory, however, which we are looking at for future work may be the kinetic theory of dusty plasma suggested in Tsytovich where... Notwithstanding, the results presented here can give a qualitative description of what to expect in system under study when relatively more complete is applied.

2.0 Dispersion Relations

We consider a three-component plasma which consists of singly charged ions, electrons and negatively charged dust particles. The mass of the dust particle is assumed constant. We also neglect dust charging and collisions, and consider rarefied dust so that the kinetic analysis is justifiable [Tsytovich et al, 2002]. The Maxwellian distribution for the dust particles is acceptable in a frequency regime that is not too large compared to the dust plasma frequency [see eg. Rosenberg and Shukla, 2003 and]. There is an external magnetic field, **B**, in the same z-direction as the relative velocity, u to the dust, of the plasma components. If we also have a density gradient oriented along the perpendicular axis, then the dispersion relation is [Hirose and Alexeff , 1975]

$$k^2 + \sum_{s=e,i,d} k_D^2 \left(1 + \frac{\omega}{k_\parallel v} \sum_n e^{-\lambda} I_n(\lambda) Z(\xi_n) \right)$$

$$- \sum_s \omega_s \left(\frac{k_\perp K_n}{\omega_{cs}} - \frac{\partial u/\partial x k_\perp k_\parallel}{\omega_{cs}} \frac{\partial}{\partial \Omega} \right) (k_\parallel v)^{-1} \sum_n e^{-\lambda} I_n(\lambda) Z(\xi_n) \;=\; 0 \qquad (2.1)$$

Here, e, i and d stand for electron, ion and dust, respectively, k is wave vector, $k_\perp^2 = k_x^2 + k_y^2$, k = k_z and k_D is Debye wave vector ; $v_s = (2T_s/m_s)^{1/2}$ is the specie thermal velocity, and T is the electron temperature. The arguments are defined by $\xi_n = (\omega - k_\parallel u + n\omega_c)/k_\parallel v_s$, where u(x) is the drift velocity, $\lambda = k_\perp^2 r_L^2$, $r_L = v/\omega_c$ and r_L is Larmor radius. Z(ξ) is the plasma dispersion function, $I_n(\lambda_n)$ is the modified Bessel function, $K_n = \frac{\partial \ln n}{\partial x}$ and $\Omega = \omega - k_\parallel u_s$.

The plasma and cyclotron frequencies are $\omega_{s=i,d,e} = 4\pi ne^2/m_s$, and $\omega_{cs} = qB/m_s$,; $-eZ_d$, e, -e and m_d, m_i, m_e are the charges and masses of the charged dusts, ions and the electrons, respectively. We shall assume that $\omega_{cd}^2 \ll |\omega|^2 \ll \omega_{ci}^2 \ll \omega_{ce}^2$, i.e, the electrons and the ions are magnetized while the dusts particles are unmagnetized but $\lambda_i \ll 1$. Dust diamagnetic effects are therefore neglected. We shall also assume that the inhomogeneities only show in the electrons and that only the electrons are hot while both the ions and the dusts particles are cold. Further assumptions are as follows: $\xi_e = (\omega - k_z u)/k_z v_e \gg 1$, $\xi_i = (\omega - k_y u)/kv_i \gg 1$, $\xi_d = \omega/kv_d \gg 1$ and $k_z \ll k_y$, $\omega \ll \omega_{ce}$, $k_z v \ll \omega_{ce}$. With the last two conditions only the term n = 0 appears to be significant and all other harmonics can be neglected. Given the quasi-neutrality condition for the system, to be $n_d + n_e = n_i = 0$ where $Z_d = 1$, the dispersion relation is written in terms of the summation of the responses of the dust, χ_d, ions, χ_i, and electrons, χ_e, as

$$D(\omega,k) = 1 - \frac{\omega_d^2}{\omega^2} - \frac{\omega_i^2}{(\omega - k_z u)^2} + \frac{\omega_i^2}{\omega_{ci}^2} + \frac{\omega_e^2}{\omega_{ce}^2} - \frac{\omega_e^2}{kL_n \omega_{ce}(\omega - k_z u\ \ )} - \frac{k_z}{k_y}\left(\frac{k_z}{k_y} + \frac{u'_e}{\omega_{ce}} \right) \frac{\omega_e^2}{(\omega - k_z u)^2}$$

$$+ \frac{i2\sqrt{\pi}\omega_e^2 e^{-\xi_e^2}}{k^2 v_e^2} \left\{ \frac{k_y v_e}{2k_z L_n \omega_{ce}} + \xi_e \left(1 + \frac{k_y u'_e}{k_z \omega_{ce}} \right) \right\} = 0 \qquad (2..2)$$

where $L_n = \left(\frac{\partial \ln n_e}{\partial x} \right)^{-1}$ is scale length of electron density inhomogeneity, $u' = \frac{\partial u}{\partial x}$ and $\xi_e = (\omega - k_z u)/kv_e$. If we set $\partial \ln n/\partial x = 0$ and $u = 0$ (since we are interested in instability caused by velocity shear, u'), $\omega_d = 0$ and assume $\omega = \omega_r + i\gamma$ with $\omega_r >> \gamma$ then the real part (wave mode) is

$$\omega_r \approx \omega_h \left[1 + \frac{m_i}{m_d}\frac{n_d}{n_i} + \left(1 - \frac{n_d}{n_i}\right)\frac{m_i}{m_e}\frac{k_z}{k_y}\left(\frac{k_z}{k_y} + \frac{u'}{\omega_{ce}}\right)\right]^{1/2} \tag{2.3}$$

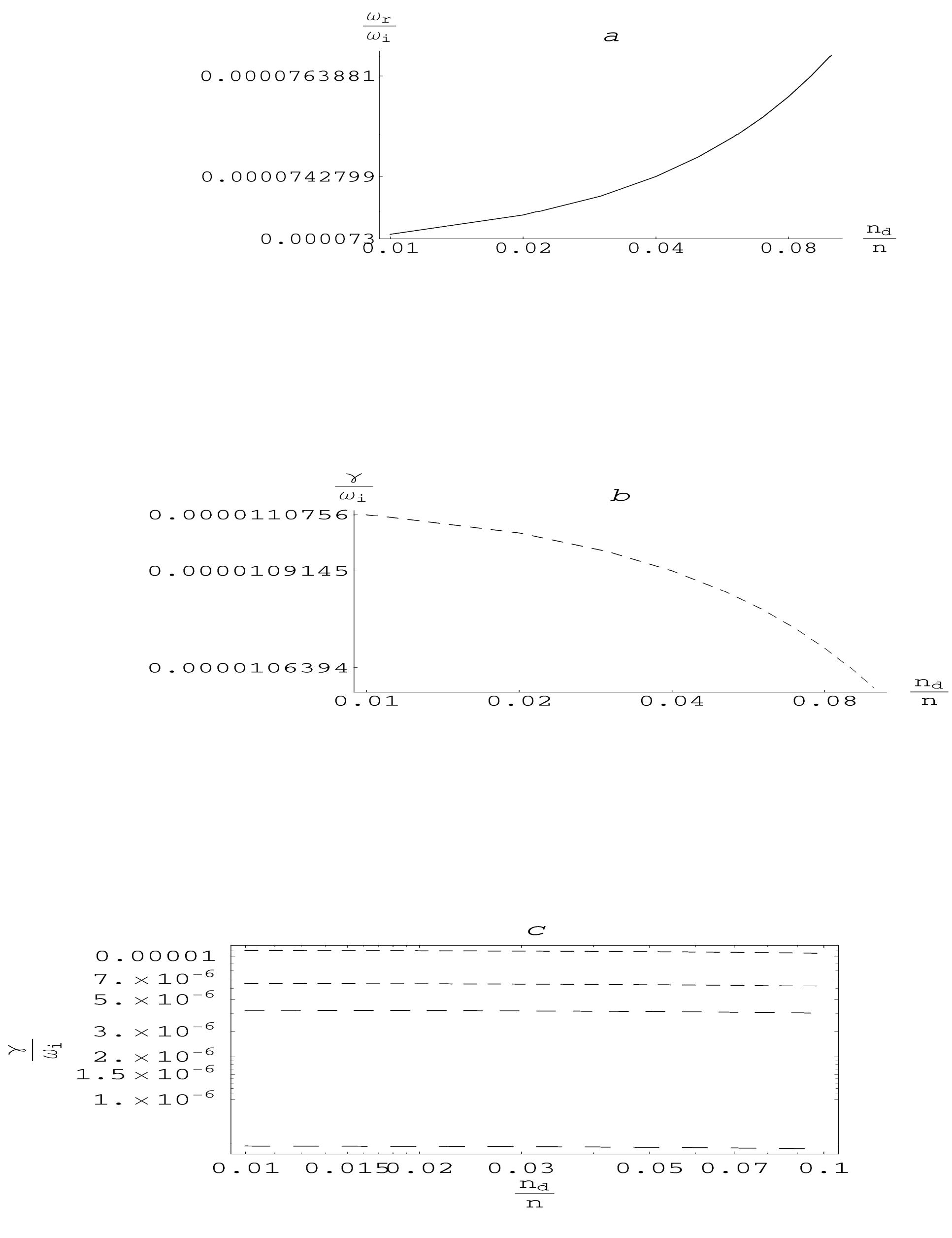

Fig.ure 1: Plots of (a) ω_r/ω_i, versus n_d/n_i for $u'/\omega_{ce} = -0.025$ (b) γ/ω_i, versus n_d/n_i for $u'/\omega_{ce} = -0.025$ and (c) γ/ω_i, versus n_d/n_i, for $u'/\omega_{ce} = -0.025, -0.015, -0.010, -0.002$, shown in that order from top to bottom. The parameters used are the following: $m_i/m_d = 10^{-5}$, $m_i/me = 2000$, $k_z/k_y = 10^{-3}$, $\omega_i/\omega_{ci} = 100$, $\omega_{cd} = .01 s^{-1}$, $v_e = 10^5$ $k = 1$ and $u = .005 v_e$.

where

$$\omega_h = \omega_i\left[1 + A^2\right]^{-1/2} \quad (2.4)$$

Damping or instability is given by the imaginary part

$$\gamma = -\frac{\sqrt{\pi}}{(1+A^2)}\frac{2\omega_e^2}{k^2 v_e^2}\left(1-\frac{n_d}{n_i}\right)\left(1+\frac{k_y u'}{k_z \omega_{ce}}\right)\xi_e \exp(-\xi_e^2)\omega_r \quad (2.5)$$

where $A^2 = \frac{\omega_e^2}{\omega_{ce}} + \frac{\omega_i^2}{\omega_{ci}}$. The condition for instability is

$$\frac{k_z}{k_y} + \frac{1}{\omega_{ce}}\frac{\partial u}{\partial x} < 0 \quad (2.6)$$

or

$$\frac{\partial u/\partial x}{\omega_{ce}} < -\frac{k_z}{k_y} \quad (2.7)$$

Here, k can take a positive or negative sign. Plots of equations (2.3) and (2.5) are given in figure 1. where data parameters for the plume given in Bernhardt et al [1995] were used.

3.0 Cold Plasma Limits

If all the particles are assumed cold and we have that $(k_\perp v_e/\omega_c)^2 << 1$, $k_z << k_\perp$, $\xi_e >>1$, $\xi_i >>1$ and $L_n \to \infty$ (i.e, very weak density gradient), equation (2.1) reduces

$$1 - \frac{\omega_i^2}{\omega^2} + \frac{\omega_i^2}{\omega_{ci}^2} + \frac{\omega_e^2}{\omega_{ce}^2} - \left(\frac{k_z^2}{k_y^2} + \frac{k_z}{k_y}\frac{u'_e}{\omega_{ce}}\right)\frac{\omega_e^2}{\omega^2} = 0 \quad (3.1)$$

Where we have also set $u = 0$. It has the criterion for instability given as

$$\frac{\partial u/\partial x}{\omega_{ce}} < -\left[\frac{k_z}{k_y} + \frac{k}{k_z}\left(\frac{m_e n_i}{m_i n_e} + \frac{m_e n_d}{m_d n_i}\right)\right] \quad (3.2)$$

On comparing equation (3.2) with equation (2.7) it shows that the presence of dust has the effect of reducing the instability as shown in Figures 1b and 1c.

When u and u' are not equal to zero and we consider that the instability is driven primarily by electrons we can ignore the ion term, (see Rosenberg and Krall, 1994), and write

$$1 + \frac{\omega_e^2}{\omega_{ce}^2} + \frac{\omega_i^2}{\omega_{ci}^2} = \frac{\hat{\omega}_e^2}{(\omega - k_z u)^2} + \frac{\omega_d^2}{\omega^2} \quad (3.3)$$

where

$$\hat{\omega}_e^2 = \frac{k_z}{k}\left(\frac{k_z}{k} + \frac{\partial u/\partial x}{\omega_{ce}}\right)\omega_e^2 \quad (3.4)$$

If $\hat{\omega}_e^2$ is positive, equation (3.3) becomes the familiar two-stream instability. The threshold for instability to occur is

$$k_z u < \hat{\omega}_e \frac{(1+B^{1/3})^{3/2}}{\sqrt{1+A^2}} \quad (3.5)$$

where $B = \frac{\omega_d^2}{\hat{\omega}_e^2}$.

However, solving equation (3.3) for the wave mode, ω_r and growth rate, γ, we obtain

$$\omega_r - k_z u_e \approx \gamma \approx \frac{\left(\omega_d \widehat{\omega}_e^2\right)^{1/3}}{\sqrt{(1+A^2)}} \quad , \quad if \;\; \omega_d^2 >> \widehat{\omega}_e^2 \tag{3.6}$$

or

$$\omega_r \approx \gamma \approx \frac{\left(\omega_d^2 \widehat{\omega}_e\right)^{1/3}}{\sqrt{(1+A^2)}} \quad , \quad if \;\; \widehat{\omega}_e^2 >> \omega_d^2 \tag{3.7}$$

The most unstable modes [see also Lapuerta and Ahedo, 2002] are:

$$\omega_r \approx \frac{(\omega_i^2 \hat{\omega}_e)^{1/3}}{2^{4/3}\sqrt{(1+A^2)}} \tag{3.8}$$

$\gamma \approx \frac{\sqrt{3}(\omega_i^2 \hat{\omega}_e)^{1/3}}{2^{4/3}\sqrt{(1+A^2)}}$ and the wave number k at this maximum growth rate satisfies the condition

$$k_z u \approx \hat{\omega}_e \tag{3.9}$$

If, instead, we have that

$$k_z u_e \approx \omega_d \tag{3.10}$$

then all the three roots of equation (3.3) are real if

$$k_z u_e - \omega_d > \frac{3}{2\sqrt{(1+A^2)}}\left(\widehat{\omega}_e^2 \omega_d\right)^{1/3} \tag{3.11}$$

and this gives the instability region.

If, on the other hand, the charged dust is treated as massive point particles, whereby we have only two components plasma in a background of dust particles, and we neglect the ion diamagnetic velocity, then dust term would be replaced by ion in (3.5) - (3.8). Therefore, the maximum growth is on the order of the dust lower frequency hybrid defined by

$$\gamma \approx \frac{\omega_d}{\sqrt{(1+A^2)}} \tag{3.12}$$

And since $\omega_{ce}^2 >> \omega_{ci}^2$ we obtain the maximum wave mode and growth rate as

$$\omega_r \approx \gamma \approx \omega_i \tag{3.13}$$

which was estimated by Scales et al [1997] from the results of their simulations. The value of k at this maximum growth is

$$k_y L_E \approx \frac{k_z}{k_y}\frac{n_e}{n_i}\frac{m_i}{m_e} \approx 1 \tag{3.14}$$

where L_E is defined as the scale length of the electron velocity shear :

$$L_E = \left(k_y \,\partial u / \omega_{ce} \partial x\right)^{-1} \tag{3.15}$$

4.0 Conclusions

We have found a resonant instability for velocity-sheared charged dust streaming in plasma condition. The wave which is near the lower hybrid frequency is excited by the velocity gradient. The threshold for the onset of the instability in cold plasma limit was determined. The theory presented here should be applicable in wide ranges of phenomena in space plasmas occurring due to charged dust with low but sheared relative velocity, and weak density gradient.

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